

An Error Analysis of Matriculation Students' Permutations and Combinations

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Abstract

Background/Objectives: This study aims to identify errors in Permutations and Combinations topic among matriculation college students. **Methods/Statistical Analysis:** A total of 83 students consisting of 46 module I's students, 23 module II's students, and 14 module III's students of a Two-Year Program participated in this study. A diagnostic test on Permutations and Combinations was used as an instrument to gather types of errors students made. The errors were classified based on Carmen Batanero's classification. Data were analysed descriptively using frequency and percentage to diagnose the types of errors. **Findings:** The findings show that students made five major errors: 1. Wrong question interpretation. 2. Wrong identification of the type of object used either identical or different, especially letters, numbers and non-living things. 3. Wrong arithmetic operations used in finding solution. 4. Incorrect use of formulas and 5. Wrong or meaningless answers given. **Applications/Improvements:** The results of this study can help educators to plan more effective teaching and learning strategies in making the classroom activities for Permutations and Combinations meaningful.

Keywords: Combinatorics, Error Analysis, Permutations and Combinations

1. Introduction

Imagine in a library with no numbering system, how can one organize the books in layout and be accessed by many people? This is a problem that can be solved using knowledge of Combinatorics. The Combinatorics topic has a broad scope, as it does not only involve in solving the problems of arrangement, permutation and combination¹. In Malaysia, Combinatorics is included in the syllabus of Additional Mathematics in secondary, high school and Matriculation College. There are two sub-headings which are the Permutations and Combinations. Permutations and Combinations are included in the seventh topic to be covered in the second semester of One-Year Program and the second topic in the fourth semester of Two-year program for college matriculation in Mathematics Syllabus Two-Year Program Matriculation Division Ministry of Education Malaysia. Basic knowledge from secondary

school is expected to help students to understand and master the topic better. However, previous studies found that students' achievement in this subject at school level is weak¹⁻⁴. Similar results were found at the higher level of education institutions⁵.

This topic also has a big role for the application of science and technology such as thermodynamics and statistical mechanics in Physics, counting the number of organic molecules in Chemistry, and the spread of epidemics in Biology⁶. Because it is closely related to the calculation of the probability and the theory of cognitive development theory³, it is important for students to understand and master the topic.

2. Error Analysis

Analysis of student errors in mathematics topics should be done to help lecturers improve the quality of

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teaching and learning. Through problem solving strategy applied on students, lecturers can make error analysis of a topic which should be done more specific so that misconceptions among students can be tracked. The error analysis can help teachers to better understand the causes of the mathematical difficulties experienced by students, eventually building a recovery model that is suitable with the weakness levels of students⁷.

There are various error analysis models which have been proposed by researchers and educators in the statistics and mathematics. Among the models are the Error Hierarchy Model by Newman¹⁷ which classified the types of errors made by students into careless errors, systematic errors and random errors on differentiation topic. Empirical Classification Model by Nitsa Moshovitz-Hadar¹⁷ coded mathematical errors according to categories of technical error, theorem misuse or definition errors in interpreting language in the title of Coordinate Geometry⁷. Casey Model identified mathematical errors according to known and unknown reasons and Cox model categorized errors into random errors, systematic errors and errors of negligence in the operation of addition and subtraction of fractions⁸.

Previous research data showed that many students made mathematical errors such as mathematical calculations and algebraic manipulation operations involving indexes and this error was related to their weaknesses in the basic concept of index⁹. Norasiah¹⁸ found students' errors in understanding terms and skills in the subject matter topics formula, quadratic equations and simultaneous equations. In fact, understanding and transformation are two major errors in solving mathematical word problems.

In the case of Combinatorics topic, common students' errors were identified by Batanero et al.² and Hadar and Badass¹⁰. There were fourteen types of errors made by students when solving problems of Combinatorics¹, for instances, errors in basic calculations¹¹ and errors in the listing of possible responses¹². Therefore, this study hopes to add literature on student errors while applying problem solving in Combinatorics, especially in the topics of permutations and combinations.

3. Permutations and Combinations

Combinatorics has some basic models such as combination, arrangement, permutations, notation,

concept and formula¹. There are certain procedures in Combinatorics such as logical procedures and graphics procedures related to the graph charts and tree diagram, as well as numerical procedures regarding addition, Pascal's triangle, combinatorics and factorial¹.

Among the operations that are often problematic to students are permutations with repetition², the product of permutations, Combinations with repetition and Combinations Reduction¹³. At the matriculation college level, there are seven learning outcomes that students need to understand and master. Among them are the permutations of n distinct objects, permutations of r objects from n different objects, Permutations of the identical objects and Combinations of r objects from n objects¹⁴. Each of these subheadings has different solution processes and strategies. Weakness in distinguishing each process and learning strategy causes students to answer questions incorrectly.

Questions in the topic of Permutations and Combinations are usually presented in word problems. Mathematical word problem has been introduced since primary school and become an important component in the new curriculum for primary schools in Malaysia. Problems in solving word problems arose when students could not understand the question because of the long wordings. Hence, the students find it difficult to answer the questions properly. A high degree of difficulty is also related to conceptual error¹ in answering word problems on this topic. Researches in the field of Combinatorics education had been widely conducted and focused on Combinatorics reasoning among children³, students' thinking about Combinatorics from oriented sets and perspective-oriented process¹⁵ and the Combinatorics classification model^{1,2}. These studies found that students of all ages were problematic in solving the problem that involved calculation¹⁻³. In fact, there are also studies that found students' success rate was still low in many kinds of Combinatorics problems either before or after instruction^{1,4}. Several studies have been carried out in other countries such United States and United Kingdom to investigate the students' errors in Combinatorics^{1,2,10,16}. In Malaysia, the analysis of students' errors in combinatorics is still given less attention.

4. Research Objectives

The objective of this study is to analyse the types of errors made by students in the topic Permutations

and Combinations. The study also attempts to answer questions related to the subtitles in Permutations and Combinations, which are arranging n number of distinct objects, arranging r objects from n different objects, arranging r from some n of the identical object, select r objects from n different objects and the problems associated with both Combinations and Permutations subheading.

5. Methodology

The research uses a survey method to explore a situation by collecting data for identifying the forms of common errors made by students when solving Permutations and Combinations problems. The instrument used was Permutations and Combinations Diagnostic Test taken from semester 4 final trial examination of Two-Year Program (PDT) that was constructed by the lecturers of the Matriculation College. The set of questions consists of 10 subjective items, which most were from the past exam questions of a Semester Matriculation Level Examination (PSPM). However, the researchers only analysed the items related to the topic Permutations and Combinations.

The items for Permutations and Combinations have features such as object questioned a letter, the selection and arrangement of a certain object from the overall number of objects and solving problems related to the two sub-headings. The following Table 1 presents subtopics tested in the final trial examination of semester 4 PDT. The letter is identical objects, which students should take into account the same letters because each is indistinguishable. However, the word given has no alike letter; therefore, the students did not have to take into account of special operations for identical object.

In order to ensure the questions used in the test had covered all skill levels to be assessed, a Test Specification Table was constructed. The instrument content validity followed the Mathematics syllabus of Two-Year Program and was also reviewed by two experienced lecturers that has exceed ten years in teaching mathematics in matriculation colleges for content validity and face validity. One of the researchers examined the answer scripts with another lecturer for inter-rater reliability. Mackinnon¹⁹ stated that the examination between two examiners can be used to avoid bias and prevalence. Several discussions were made to achieve 95% of agreement between the two examiners.

Table 1. Items of permutations and combinations diagnostic test

Subtopics of Permutations and Combinations Test	Item: Given a word LOGARITHMS. Find the number of different ways to
Permutation: Find the number of permutations of r objects from n different objects	(a) arrange four letters
Permutation: Find the number of permutations of n distinct objects	(b) Arrange all letters such that all the vowels are next to each other.
Combination: determine the number of ways to form combinations of r objects from n different objects	(c) Choose four letters such that all of them are consonants.
Solve problems associated with both Combinations and Permutations	(d) Arrange four letters such that the number of vowels and consonants are equal.
	(e) Arrange four letters such that the number of consonants are more than the number of vowels.

The researchers utilized a convenience sampling in selecting 83 semester 4 Two-Year Program (PDT) students aged 18 and 19 years old from one of matriculation colleges in Malaysia. The respondents were 26 (31.33%) males and 57 (68.67%) females from various fields and 46 of the students studied module I, 23 students used module II and 14 students utilized module III. All students were required to take Mathematics and Chemistry as compulsory subjects and two optional subjects. For instance, a module I student could take Biology and Physics, a module II student could study Physics and Computer Science and a module III student could choose Biology and Computer Science for the optional subjects.

During the study, the students have finished their teaching and learning sessions of mathematics in 4th semester covering various topics including Permutations and Combinations. The college teaching period for this topic was 12 hours within 3 weeks while the tutorial sessions were also held in the same period. The administration of the test to gather data was done after the class sessions of the selected topic and it was hoped the data collected could be utilized to answer the research questions.

6. Findings and Discussion

The student errors were classified according to Carmen Batanero's¹ categories as follow:

- Misinterpretation of questions (STATEMENT),
- Confusion in distinguishing characteristic of arrangements, permutations and combinations (ORDER),
- Repetition of error (REPETITION),
- Misrecognition of certain types of objects used either identical or different, especially in the case alphabetic, numeric, non-living objects and persons (OBJECTS),
- Creation of solutions using wrong arithmetic operations (OPERATIONS),
- Wrong formula used (FORMULA),
- Disability to remember the meaning of values for parameters in the Combinatorics formula (PARAMETERS) and
- Answers given with no correct value (INTANSWER).

Overall, the analysis of errors made by students indicates that their achievements in the topic Permutations and Combinations were still low. This can be seen based on the percentages of students who got the correct answers which were different for each question, where there was a large range between the maximum percentage, 86.65% and the minimum percentage, 3.61%. This finding was parallel with the study of Godino et al⁵.

Table 2. Errors for item 1

Types of errors	Examples	Frequency	%
ORDER	Using Combinations formula ${}^{10}C_5 = 210$	2	2.41
REPEATED	Finding Permutations using repeated operations $10 \ 10 \ 10 \ 10 = 10000$	2	2.41
PARAMETER	Taking wrong value of r ${}^{10}P_3 = 720$	1	1.20
INTANSWER	Arranging the whole object $10! = 3628800$	6	7.23

The descriptive statistics shown in Item 1 of the instrument was a question related to permutations in which students had to find the number of ways to arrange the order of 4 letters from 10 different letters. A total of

86.75% of the students could answer this question correctly and 13.25% answered incorrectly. The errors are presented in Table 2. The highest number of error made was giving wrong answer or did not give any indication to get the answer for the question, such as not choosing the number of objects needed. Mistakes like this can be considered as trial and error³. However, questions on permutations without repetition have been mastered by most of the students because 80% were able to answer with excellence.

Table 3 specifies the types of errors made in response to Item 2 in which students were asked to find the number of ways to construct whole different objects with regard to certain requirements. Only 30.12% of students could answer this question correctly, while 69.88% was wrong. For this kind of problem students should take into account the necessary requirements before answering. The highest error in this question was not remembering the meaning of the values for parameters (PARAMETERS) such as consonants or misconception of the term, which has accounted for 27.71%.

Item 3 is on Combinations subtopic in which students had to choose only 4 letters of 10 different letters. A total of 50.6% of the students could answer this question

Table 3. Errors for item 2

Types of Errors	Examples	Frequency	%
OBJECTS	Using division where this operation is only used when there are similar letters $(10! / 3!x7!) x2! = 240$	10	12.05
ORDER	Using Permutation and Combinations formula $8!x{}^{10}C_7 = 4838400$	3	3.61
PARAMETER	Not considering the arrangement for consonants $3!x7 \ 6 \ 5 \ 4 \ 3 \ 2 \ 1 = 30240$	23	27.71
OPERATIONS	Not necessary to use addition ${}^{10}P_3 + {}^{10}P_7 = 605520$	4	4.82
INTANSWER	Did not choose the number of consonants needed $7! \times {}^8P_3 = 1693440$	13	15.66
No answer		5	6.02

Table 4. Errors of item 3

Types of Error	Examples	Frequency	%
ORDER	Using Permutations formula ${}^7P_4 = 840$	31	37.35
PARAMETER	Taking the wrong value of r ${}^{10}C_7 = 120$	9	10.84
REPEATED	Using repetition operation $7777 = 2401$	1	1.2

correctly and 49.4% were incorrect. The analysis of errors is presented as in Table 4. From the analysis, some students were still not able to distinguish between Permutations and Combinations, totalling 37.35% which consistent with findings from a study by Batanero et al.¹. According to Batanero et al.¹, students must master the concepts of Permutations and Combinations where both topics have different concepts and procedures.

For item 4 and 5, students had to solve problems related to both subtopics of permutations and combinations. The given problems must be analysed as thoroughly as possible because students should distinguish between methods of advanced sorting or choose first and arrange later. Questions like these are quite confusing and students always have problems in answering them. This was evident when only 6.02% and 3.61% students could solve correctly questions item 4 and 5. For item 4, 93.98% of the students did not answer accurately, while 96.39% performed incorrectly for item 5.

Overall, the main types of errors made by the matriculation students are confusion in distinguishing the characteristics of Permutations and Combinations (ORDER), which are 132 (53.01%) of the total number of errors, while not considering the meaning of the values of the parameters in the formula (PARAMETERS) was 53 (21:29%). This means that half of the students are still weak in distinguishing each subtopic or do not understand the concept of Permutations and Combinations. This finding is consistent with the study by Batanero et al.¹ that showed students always memorize the formula and use it without understanding the true meaning of the formula¹⁶. Both findings showed similar results to the study by Batanero et al.¹ where the errors are the most common mistakes made by students.

In addition, another type of error made by the students is giving meaningless answers (INTANSWER), which was

Table 5. Errors of item 4

Types of Error	Examples	Frequency	%
OPERATIONS	Using addition only without multiplication ${}^3P_2 + {}^7P_2 = 48$	11	13.25
ORDER	Not choosing and arranging vowels and consonants as required	51	61.45
REPEATED	Using repetition 6 1 0 $1 0 1 0 = 6000$	1	1.2
INTANSWER	Not choosing correctly $7P_2 \times 7P_2 = 84$	11	13.25
No answer		4	4.82

Table 6. Errors item 5

Types of Errors	Examples	Frequency	%
ORDER	Only using Combinations formula and not arranging the chosen letters $({}^{10}C_3 \times {}^3C_1) + ({}^7C_4 \times {}^3C_0) = 140$	45	54.22
OPERATIONS	Using only addition without multiplication ${}^4P_4 + {}^4P_0 + {}^4P_3 + {}^4P_1 = 53$	2	2.41
PARAMETER	Only answer parts of the question ${}^7C_3 \times {}^3C_1 \times 4! = 2520$	20	24.10
INTANSWER	Only choosing some letters and not arranged ${}^7C_3 \times {}^3C_1 = 105$	3	3.61
No answer		10	12.05

made by 33 (13.25%) students. Most of the students who do this kind of mistake are using the trial and error method³. Besides, about 17 (6.83%) students made a solution using wrong arithmetic operations (OPERATIONS) such as multiplication and addition. This finding is consistent with the study by Lockwood, Swinyard and Caughman¹¹ in which students have to master basic calculations to facilitate students in mastering this topic.

The number of errors in identifying the type of object used either identical or different especially the case of

alphabetic, numeric, non-living objects and person (OBJECTS) was 10 (4.02%). Alphabet, numbers and non-living objects are categorized as an identical object where there are objects that are common in these groups; hence, the students must perform operations division. This is because these objects cannot be identified whether they have or have not been selected and arranged. The final is an error in making repetition, by 4 students (1.6%). The findings are in contrast to Batanero et al.¹, this kind of error made by the students is somewhat less likely to occur due to the items given did not have a question in relation to the case of repetition.

7. Conclusion and Suggestion

The teaching and learning plan based on student errors can help matriculation lecturers especially in helping students to overcome their weaknesses. In addition, the lecturers can also give more attention to any kind of errors and weaknesses that are less done by the students such as Permutations and Combinations basic concepts. However, this basic concept should be taught as thoroughly as possible because students need to be able to distinguish and to answer these questions profoundly¹. At the tertiary level, the problem of Combinatorics is not only to distinguish between Permutations and Combinations, but to apply them in other related topics and subjects^{3,5,6}. This study can be used as a guideline for teachers and especially matriculation lecturers in repairing and improving the level of understanding in the topic Combinations and Permutations. Moreover, it is hoped that in the future more studies in this topic to be undertaken because it is closely related to other topics such as probability^{1,3} and inferential statistics.

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