ISSN (Print) : 0974-6846
Indian Journal of Science and Technology, Vol 9(21), DOI: 10.17485/ijst/2016/v9i21/95209, June 2016 ISSN (Online) : 0974-5645

Determination of the Appropriate Geometric Moment Invariant Functions for Object Recognition

Vazeerudeen Abdul Hameed

Asia Pacific University of Technology and Innovation,Technology Park Malaysia, Kuala Lumpur, 57000, Malaysia; vazeer@apu.edu.my

Abstract

Objectives: Moment invariants have been largely deployed to solve object recognition problems often without analyzing their appropriateness. This paper presents a heuristic method to determine the most appropriate geometric moment invariant functions for object recognition. **Methods:** The mathematical structure of the moment invariant functions was studied. The elements in the functions that contribute to amplification of noise in the images were identified through derivations. The moment invariant functions were then classified into noise susceptible and noise resilient categories. Sample functions from each of these categories were applied over standard images subjected to different types of noises to appraise the performance of the moment invariant functions. **Findings:** Research work accomplished using geometric moment invariant functions always showed evidence of false positives and false negatives due to the inherent weakness of some of the moment invariant functions. Most or all of the works using geometric moment invariant functions used only the first seven Hu moment invariants. Upon expansion of the moment invariant functions, it became essential to effectively choose the functions. Existing research explains that the higher the order of the moment functions the more susceptible are they to noise in the images. The research accomplished in this paper proves that not all higher order functions are susceptible to noise. This paper proves that there are higher order geometric functions with smaller exponents which are highly resilient to noise. **Applications/Improvements:** The research contributed towards identifying the most suitable moment invariant functions for object recognition. This approach helps to minimize false positives and false negatives and choose noise resilient moment invariant functions.

Keywords: Invariant, Moment, Rotation, Scaling, Skew, Transform, Translation

1. Introduction

Image processing is a field of study that comprises several stages such as Image acquisition, Image enhancement, Image restoration, Image representation, Image compression and Object recognition. Among these stages object recognition has been a challenging goal for the researchers to accomplish. Moment invariant functions have been used as a viable option for various object recognition problems as explained in Section 3 below.

Object recognition has been accomplished on specific classes of objects such as iris, thumb print face recognition and many others. Iris recognition is a biometric identification technique which involves unique recognition of irises. For such a specific problem, careful choice of measures that have impact on iris analysis is made. Moment invariants of different forms such as orthogonal or non-orthogonal functions have been deployed. Thumb print recognition, palm recognition are some of the other specific problems of object recognition that have been widely addressed. Moment invariants as the name implies are expected to be invariant of several transformations that an image could undergo.

An image could undergo various kinds of

* Author for correspondence

transformations. Rotation is a form of transformation where the pixels of an image are rearranged from a position (x, y) to a new position (x cos(t) – y sin(t) , x sin(t) + y cos(t)). Damages incurred by the pixels in an image during a rotation are that they are relocated. The amount of damage incurred can be estimated and remains minimal since the position of the pixels change with minor loss of data. A uniform scaling of an image is a transformation where the number of pixels is increased or decreased along both the x and y axes equally. If (X, Y) are the length and breadth of an image then the image is scaled by a factor of a constant A. This results in an image of dimensions (AX, AY). This causes an enlarged image or a diminished image. Negative scaling of an image causes loss of information. This may render the image unintelligible when the degree of scaling is tremendously high.

Scaling, rotation and translation are transforms that affect the x axis and y axis of an image uniformly. However transformations such as stretching and skewing affect the x and y dimensions of an image by different proportions. Therefore the transformations cause large amount of change in the images. These changes might be invisible at times to human eye and easy to recognize due to inherent human intelligence. However for computerized object recognition, these transformations to images are a huge burden. Moment invariant functions need to be designed and used in accordance with the nature of the object recognition problem at hand.

2. Moment Invariant Functions

Moment invariants were first explained[1], a German mathematician. The moment M_{pq} of an image f (x, y) is defined as:

$$M_{pq} = \iint_D P_{pq}(x,y) f(x,y) dxdy \tag{1}$$

In Equation (1), r = p+q is the order of the moment and p> = 0, q> = 0. Ppq is a polynomial basis function that could be either orthogonal non–orthogonal in nature. Non–orthogonal moment functions are also called as geometric moment functions. A number of non–orthogonal polynomial basis functions could be drawn.

Complex Zernike moment functions were one of the first orthogonal moments and invariant functions proposed in 1934. Equation (2) shows the Zernike moment functions on polar co-ordinates.

$$A_{mn} = \frac{(m+1)}{\Pi} \sum_{x=-\infty}^{\infty} \sum_{y=-\infty}^{\infty} I(x,y)[V_{mn}(x,y)]^*$$

$$V_{mn}(x,y) = V_{mn}(r,\theta) = F(r)e^{jn\theta},$$

$$F(r) = \sum_{s=0}^{s=[m-|n|]/2} \frac{(m-s)!r^{m-2s}}{[s![\frac{(m+|n|)}{2}-s]![\frac{m+|n|}{2}-s]!]} \tag{2}$$

Zernike moments have been known to have minimum amount of information redundancy and invariance to rotation. Calculating the higher order Zernike moment invariants has always been computationally complex.

Legendre moments are yet another set of orthogonal moments which are invariant functions calculated as in Equation (3). Legendre moment functions were inherently invariant to scaling and translation. They are better over Zernike moments but are computationally expensive.

$$\lambda_{mn} = \frac{(2m+1)(2n+1)}{4} \sum_x \sum_y P_m(x) P_n(y) P_{xy} \tag{3}$$

Non-orthogonal moment invariant functions have been found to be computationally easier when compared to the orthogonal functions.

The Hu moment invariants were generated with non-orthogonal basis functions which are also called as geometric moments based on the Equation (4). The moment functions were further normalized into central moments that were invariant to translation.

$$M_{pq} = \iint_D x^p y^q f(x,y) dxdy \tag{4}$$

Seven moment invariants were firstly founded[2]. These invariants were made of second and third order moments as in Equation (5). It is to be noted that these seven invariants have been abundantly used over several problems of object recognition either in isolation or in combination with other techniques to achieve a high degree of accurate object recognition. Some evidences of its abundant application have been discussed in the following sections.

The seven of the Hu moment invariants were the only available geometric moment invariant functions of highest order three. No further extension to these functions in Equation (5) was provided.

Research works have established a generalized algorithm to expand the order of the Hu moment invariants from three to infinitely large higher order moment invariant functions[3]. With this research more number of higher order moment invariant functions could be generated according to the need of the object recognition problem.

$$
\begin{aligned}
I_1 &= \eta_{20} + \eta_{02} \\
I_2 &= (\eta_{20} - \eta_{02})^2 + (2\eta_{11})^2 \\
I_3 &= (\eta_{30} - 3\eta_{12})^2 + (3\eta_{21} - \eta_{03})^2 \\
I_4 &= (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2 \\
I_5 &= (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] + \\
&\quad (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \\
I_6 &= (\eta_{20} - \eta_{02})[(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2] \\
&\quad + 4\eta_{11}(\eta_{30} + \eta_{12})(\eta_{21} + \eta_{03}) \\
I_7 &= (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12})[(\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2] \\
&\quad - (\eta_{30} - 3\eta_{12})(\eta_{21} + \eta_{03})[3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2]
\end{aligned} \tag{5}
$$

3. Work done using Hu Moment Invariants

Naser Zaeri et al. demonstrated thermal face recognition system using geometric moments and thermal images[4]. The authors have explained the difficulties of face recognition due to illumination, noise and variability in facial expressions. These difficulties have led them to use thermal images. However a careful consideration of the choice of the moment functions could have provided better results.

Iris recognition is a commonly known biometric authentication technique where Hu moments were deployed for a successful recognition[5]. The research accomplished a False Acceptance Rate of 0.0% and a False Rejection Rate of 2.5%. Accuracy, Speed and the template size had been of major concern. It must also be noted that images of size 512 x 512 had been used. They were decomposed into four regions each of size 256 x 256. Further the images were decomposed into sizes of 128 x 128. The Hu moment invariants had been applied to these minimum variance sub regions in the iris image. The Hu moment invariants contributed to the cause of accuracy. However no analysis has been presented on the selection of the Hu moment invariants. All the seven invariants were deployed.

Hui Zhang et al. demonstrated that blurred image recognition could be achieved with a high degree of accuracy using Legendre moment functions[6]. The Legendre moments were deployed in combination with the Point Spread Function of the image. The authors have proved that the performance of the orthogonal Legendre moment is better in comparison with a complex or geometric moment invariant. However the invariant functions have been invariant to translation but not rotation or scaling. Moreover Legendre moments of orders up to seven have been used.

K. Vaidehi et al. accomplished breast tissue classification using k-Nearest Neighbours algorithm[7]. This research extracts six important statistical features used for image classification. The work achieved an accuracy of 80%. However deploying geometric moment invariant functions could retrieve more number of features from mammogram images and help achieve better accuracy using the proposed invariants[3].

Farhood et al. demonstrated liveliness detection using Zernike and Frensel transforms[8]. It is important to note that Zernike moments are orthogonal but computationally complex. Deploying geometric moment invariant functions to replace Zernike moments could reduce the computational complexity and provide any number of features required for liveliness detection ensuring careful choice of the functions.

4. Heuristic Method to Select Moment Invariant Functions

The existing seven moment invariant functions proposed so far have been the most widely used geometric moment invariants in image processing algorithms involving object recognition[2]. The skew invariant moment invariant functions proposed have been used in applications where skew invariance is important[9]. The seven fundamental rotationally invariant moment invariant functions were further expanded[3]. This research work expanded the number of rotationally invariant moment invariants from six to infinite numbers. There has been less importance given towards the selection of the right moment invariant functions for a problem at hand. The moment invariant functions possess some characteristics that have to be carefully considered before applying them over a problem. Fundamentally the moment invariant functions possess two important qualities in them namely the invariance and the discriminatory ability. These two qualities are opposite of each other in that, the higher the quality of invariance the lower the quality of discrimination and vice-versa. Every individual moment invariant function has one of these qualities overlaying the other. Hence it is important to analyse the functions before deploying them over solving an object recognition problem.

Every moment term in a moment invariant function measures a characteristic of the image over which the term is evaluated. A commonly found opinion from the researchers is that the higher the order of the moment invariant functions the better their discriminatory ability but they are more likely to be sensitive to noise and may not perform well. The cause of the susceptibility of the moment invariant functions to the noise shall be explored.

The moment invariant functions always have moment functions of the form shown in Equation (6).

$$n_{ab}^{z} = \left(\sum_{x} x^{a} \sum_{y} y^{b} f(x,y) \right)^{z} \qquad (6)$$

In Equation (6), (a + b) is the order of the moment term and z is the exponent. Increasing the value of (a + b) increases the order of the moment terms. In the equation above, it must be noted that f (x, y) may be corrupted by noise which shall be measured by n_{ab}. With increase in the value of z, the impact of the noise is amplified and thereby causing deviation in the value of the moment term n_{ab}. The following derivation evaluates the impact of the exponent over the moment terms. The moment terms of the form n_{10} and n_{20} are considered for z = 1 in Equation (7).

$$n_{10} = \sum_{x=1}^{m} x f(x,y), \qquad (7)$$

$$n_{20} = \sum_{x=1}^{m} x^{2} f(x,y).$$

Let f(x)=1.

$$n_{10} = \sum_{x=1}^{m} x = \frac{m(m+1)}{2} = O[m^{2}] \qquad (8)$$

$$n_{20} = \sum_{x=1}^{m} x^{2} = \frac{m(m+1)(2m+1)}{6} = O[m^{3}]$$

The above expressions in Equation (8) show that n_{10} is asymptotically less complex than n_{20}. Hence n_{10} would be more resilient to noise than n_{20}. However if the exponent of n_{10} is increased from 1 to 2 we find that:

$$n_{10}^{2} = \left(\sum_{x=1}^{m} x \right)^{2} = \left(\frac{m(m+1)}{2} \right)^{2} = O[m^{4}] \qquad (9)$$

Now n_{20} in Equation (8) outperforms $(n_{10})^2$ in Equation (9) as it is asymptotically less complex.

Hence the analysis clarifies that increasing the order of the moment and decreasing the exponent of the moment term is important to ensure that the moment invariant functions are asymptotically less complex in calculation.

Making a decision over the choice of the moment invariant functions was difficult for the researchers as there were initially only seven moment invariant functions as formulated in (Hu). However with the expansion in the number of moment invariant functions in (Vazeer) there is a wide choice of the type of moment invariant functions that can be chosen from. As explained earlier, choice has to be made to measure the invariance or the difference in the images before applying the moment invariant functions over the images. The following experiment demonstrates the impact of choosing the correct and inappropriate moment invariant functions for a given problem.

5. Experiment

An image might have been affected by some random noise. Identification of the image requires a moment invariant function that has a high degree of invariance and not the discriminatory ability. In this experiment, Figure 1 shows a grey scale camera man image that is commonly used in image processing research. Figure 2 was created from Figure 1 by introducing some salt and pepper noise. Similarly Figure 3 and Figure 4 were created by introducing Poisson noise and Speckle noise respectively upon Figure 1. Two moment invariant functions namely I_2 and I_a were calculated over the images. I_2 is a function of order two with largest exponent two and I_a is a function of order 4 with largest exponent of one. As explained above, the moment invariant function I_2 has higher discriminatory ability while I_a has high invariance property. Table presents the measure of I_2 and I_a for Figure 1. The table presents the measure of five iterations of randomly introduced salt and pepper noise, Poisson noise and Speckle noise over the image in Figure 1, as shown in Figure 2, 3 and 4 respectively.

Figure 1. Original image.

Figure 2. Image with Gaussian noise.

Figure 3. Image with Poisson noise.

Figure 4. Image with Speckle noise.

Table 1. Measure of invariants for Figure 1, 2, 3 and 4

	Iteration	I2	Ia
Figure 1	1	1.28E+19	6.74E+14
	1	1.02E+19	6.74E+14
	2	1.03E+19	6.74E+14
	3	1.00E+19	6.75E+14
	4	1.02E+19	6.74E+14
Figure 2	5	1.03E+19	6.75E+14
	% Standard Deviation	1.21E+00	6.18E-02
	Min % Difference	18.75	0.0142
	Max % Difference	22.1094	0.0611
	1	1.28E+19	6.74E+14
	2	1.27E+19	6.74E+14
	3	1.28E+19	6.74E+14
	4	1.28E+19	6.74E+14
Figure 3	5	1.27E+19	6.74E+14
	% Standard Deviation	5.05E-01	2.37E-02
	Min % Difference		
	Max % Difference	0.3235	0.045
	1	1.32E+19	6.71E+14
	2	1.32E+19	6.69E+14
	3	1.35E+19	6.70E+14
	4	1.32E+19	6.71E+14
Figure 4	5	1.32E+19	6.69E+14
	% Standard Deviation	8.38E-01	9.98E-02
	Min % Difference		
	Max % Difference	2.956	0.436

Table 1 shows the measure of I_2 and I_a for the original image in Figure 1. The table shows that the values of I_a for Figure 2 are very closely similar to I_a of Figure 1, while the I_2 values are largely varying across the iterations. The percentage standard deviation among I_a measures of the five iterations is 0.0618% as against 1.21% for I_2. Similarly the minimum and maximum difference across the iterations for I_a is lower than that of I_2.

The same characteristic can be observed in the measure of the invariants I_2 and I_a for the Figure 3 and Figure 4 as presented in the table. Hence the experiment clarifies that

I_a is a better choice of invariant when compared to I_2 for identification of similar images.

6. Conclusion

The paper emphasizes the use of selective higher order moment invariant functions for effective object recognition. A heuristic method of selecting an appropriate moment invariant function has been elaborated. The experimental results prove that careful selection of moment invariant functions can largely impact the object recognition and that the higher order moment invariant functions need not be always susceptible to noise. A careful selection of a higher order moment invariant function can enable effective object recognition.

7. References

1. Hilbert D. Theory of algebraic invariants. Cambridge, U.K: Cambridge University Press; 1993 Nov.
2. Hu MK. Visual pattern recognition by moment invariants. IRE Transactions on Information Theory. IT-8. 1962 Feb; 8(2):179–87.
3. Hameed HA, Shamsuddin SM. Generalized rotational moment invariants for robust object recognition. Int J Advance Soft Compu Appl. 2013 Nov; 5(3):1–23.
4. Zaeri N. Thermal face recognition using moments invariants. SCIEI International Conference on Digital Signal Processing (ICDSP 2014); Milano, Italy. 2014 Nov.
5. Sheela SV, Vijaya PA. Non-linear classification for iris patterns. Proceedings of the International Conference on Multimedia Computing and Systems (ICMCS); Ouarzazate, Morocco. 2011 Apr 7-9. p. 1–5.
6. Zhang H, Shu H, Han GN, Coatrieux G, Luo L, Coatrieux J. IEEE blurred image recognition by Legendre moment invariants. IEEE Trans Image Proc. 2010 Mar; 19(3):596–611.
7. Vaidehi K, Subashini TS. Breast tissue characterization using combined K-NN classifier. Indian Journal of Science and Technology. 2015 Jan; 8(1):1–4.
8. Mousavizadeh F, Maghooli K, Fatemizadeh E, Moin MS. Liveness detection in face identification systems: Using Zernike moments and fresnel transformation of facial images. Indian Journal of Science and Technology. 2015 Apr; 8(8):523–35.
9. Suk T, Flusser J. Affine moment invariants generated by graph method. Pattern Recognition. 2011 Sep; 44(9):2047–56.