ISSN (Print) : 0974-6846
ISSN (Online) : 0974-5645
Indian Journal of Science and Technology, *Vol 9(6), DOI: 10.17485/ijst/2016/v9i6/84699, February 2016*

Calculation of the New Gravity Wall Composed of Three Flat Slabs by the Equations of Bending Theory of Shells

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Abstract

Background/Objectives: In this paper, for the weight reduction of concrete gravity walls and reduction of wall thickness, flat slabs with the retaining walls have been matched and offered a new wall. **Methods/Statistical Analysis:** In this article, flat slabs are adapted with concrete gravity walls. Then to calculate new retaining wall, a new method, based on the principles of mathematics and engineering mechanics, is presented. In solving the new retaining wall, Fourier series and shells theory and the classical analysis method are used. **Findings:** In this research, a new structure form to reduce the material used in concrete gravity retaining walls and a new method for analyzing the new retaining walls have been offered. In addition, equations of bending theory of cylindrical shells considering with physical and mathematical aspects in solving the new retaining wall consisting of three flat slabs as well as engineering mechanics bases and classical analysis method have been indicated. Moreover, Fourier series to solve the new retaining wall problem have been used. Also, the studies showed that the use of the new retaining wall is more economical. **Applications/Improvements:** In various projects such as road, bridge building, landscaping, new retaining wall can be used. In general, new retaining wall can be used to stop the soil from falling everywhere.

Keywords: Bending Theory, Flat Slabs, Gravity Wall, New Wall, Theory of Shells

1. Introduction

Retaining walls in different civil engineering projects like roads, bridges and buildings are used. The widespread uses of these systems have caused new problems, in spite of many studies that have been achieved, and made the background for the new studies. In this study, the retaining walls are made from flat slabs. Then, it was tried based on the shells theory and physical and mathematical aspect to solve the problem of the new retaining wall. Also, the equations applied on the shells necessary for the suggested problem mode were extended. In general, in this research, it was tried to provide the method of computation and analysis of lightened retaining wall with three flat slabs.

Retaining structures design should address at least three basic requirements: geotechnical requirements, structural requirements and economic[1]. Traditional methods for design of retaining structures are based on trial and error approach, in which a trial design is proposed and is checked against the geotechnical and structural requirements, which is followed by revision of the trial design, if necessary. Moreover, there is no guarantee that final design is economical[1].

Cylindrical shells are increasingly used in the engineering and aero space industry. Love and Flügge are valid for small displacements used for the analysis of cylindrical shells vibration[2]. The studies on cylindrical shells vibration on the basis of 3D elasticity theory have been done by Herr and Mirsky for infinite long cylindrical shells and by Ye and Soldotas for finite long cylindrical shells[2]. Timoshenko and Gere have presented a classical solution of buckling pressure of long cylindrical shells with uniform thickness under external hydrostatic pressure by considering plane strain for cylinder and solving buckling load of a ring[3]. Jonaidi et al. proposes an expression

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used to calculate the critical stress of curved panels under sinusoidal loading of the bottom edge[3].

For the first time, basic assumptions on shells and slabs engineering theories were stated by Love. These assumptions are known as Kirchhoff and Love's assumptions[4].

Since 1800s, several modes in shells and slabs were studied by many people like Lure, Novozhilov, Goldenveizer, Dikmen, Calladine, Vlasov, Drucker-Prager, Timoshenko, Galerkin, Courant, Boeing, Clough, Zienkiewitcz, Cheong, Ugural and others. The literature related to shells and slabs analyses are various. Some of the literatures related to shells and slabs theories are given in the reference section[5-19].

Many studies have been achieved on counterfort retaining walls, buttress retaining walls and multi cylindrical shells by this researcher but there is not adequate research on concrete gravity retaining walls[20-31].

2. Materials and Methods

2.1 Solving the Problem of the New Wall Composed of Three Flat Slabs

To solve the problem of the new lightened retaining wall consisting of three flat slabs, bending theory equations of cylindrical shells were matched with the new retaining wall then the problem was solved. Based on the lateral soil pressure, the new retaining wall was pressed under the triangular active soil pressure. Also, the size of offered retaining wall was computed like the size of concrete gravity retaining wall. Geometry proportions, in this paper, were shown like Figure 1. Differential equations of bending theory of cylindrical shells were presented as follows. For the front cylindrical shells we have:

$$D\,\frac{d^4w}{dy^2}-\frac{1}{R}N_y=q(y)$$
$$\frac{1}{Eh}\frac{d^2Ny}{dy^2}+\frac{1}{R}\frac{d^2w}{\partial y^2}=0 \qquad (1)$$

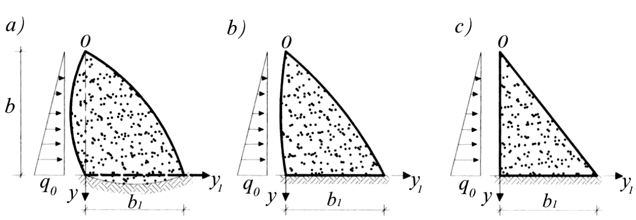

Figure 1. The geometrical parameters for the new retaining walls.

With the use of second Equation, we can simply obtain:

$$N_y=-\frac{Eh}{R}\cdot w \qquad (2)$$

By putting N_y value in the first differential Equation, we can obtain this definite differential Equation:

$$D\frac{d^4w}{dy^4}+\frac{Eh}{R^2}w=q(y) \qquad (3)$$

In this way, the differential cylindrical shells Equation below is obtained:

$$D\frac{d^4w}{dy^4}-K_s\frac{d^2w}{dy^2}+\left(K+\frac{Eh}{R_1^2}\right)w=q_1(y) \qquad (4)$$

If we put thick flat slabs instead of lower cylindrical shells in the above equation in the low weight retaining wall, we can have $R_1=\infty$ in the differential equation. In this situation, we make the boundary condition much simpler. If we substitute all of the shells with flat slab, the equation will be much more solvable. If we consider all cylindrical shells of retaining walls including interior cylindrical shells as flat slabs, we can make the above cylindrical Equation simpler. Then they are written as:

For the front flat slab we have:

$$D\frac{d^4w}{dy^4}=q(y) \qquad (5)$$

For the lowest flat slab we have:

$$D\frac{d^4w}{dy^4}+Kw-K_s\frac{d^2w}{dy^2}=q_1 \qquad (6)$$

The above differential equations are solvable with the help of definite mathematical principles and the constant values of integral in the continuous condition and boundary conditions are determined. Boundary conditions of the front cylindrical shells will be considered as follows:

For $y=0$, we have:

$$w\big|_{y=0}=0;\ \frac{d^2w}{dy^2}\bigg|_{y=0} \qquad (7)$$

For $y=b$, we have:

$$w\big|_{y=b}=0;\ \frac{dw}{dy}\bigg|_{y=0}=\frac{dw_1}{dy_1} \qquad (8)$$

For solving the problem, we use Fourier series:

$$w=\sum_{n}A_n\sin\mu_n y-\frac{1}{D}F(y)M_o \qquad (9)$$

To determine the constant value of the problem, we use boundary conditions and continuous condition. By using the above equation in the differential equation 3 we find:

$$\sum_n \left(D\mu_n^4 + \frac{Eh}{R^2} \right) A_n sin\, \mu_n y - \frac{Eh\, F(y)}{R^2 D} M_o = \frac{q_o y}{b} \tag{10}$$

We can write the equation as:

$$\sum_n \Delta_n^2 A_n sin\, \mu_n y = \frac{q_o y}{b} + \frac{Eh}{R^2 D} M_o F(y) \tag{11}$$

We multiply the equation 11 by $sin\ \mu_n y$ and then we use integral and then we reach to the following equation:

$$\Delta_n^2 A_n = \frac{2}{\mathrm{b}} \int_0^b \left(\frac{q_o y}{b} + \frac{Eh\, F(y)}{R^2 D} M_o \right) sin\, \mu_n y\, dy \tag{12}$$

Whence:

$$A_n = \frac{2}{\mathrm{b}\, \Delta_n^2} \int_0^b \left(\frac{q_o}{b} y + \frac{Eh\, F(y)}{R^2 D} M_o \right) sin\mu_n y\, dy \tag{13}$$

Whence:

$$\Delta_n^2 = D\mu_n^4 + \frac{Eh}{R^2};\ \mu_n = \frac{n\pi}{b} \tag{14}$$

After using integral, series coefficients are determined as:

$$A_n = -\frac{2}{\mathrm{n}\pi\, \Delta_{\mathrm{n}}^2} \left[q_o - \frac{Eh}{R^2 D\, \mu_n^2} M_o \right] (-1)^n \tag{15}$$

By considering restring condition in continuity place of line $y = b$, the cylindrical shells and boundary condition of the above equation are obtained:

$$\sum_n (-1)^n n\pi\, A_n - \frac{b^2}{3\ \mathrm{D}} M_o = 0 \tag{16}$$

By putting series coefficients (A_n) in equation 16, we can have the following solution:

$$M_0 = \frac{\delta_n}{a_n} = \frac{2\, q_o}{a_n} \sum_n \frac{1}{\Delta_n^2} \tag{17}$$

Whence:

$$\delta_n = 2\, q_o \sum_n \frac{1}{\Delta_n^2} \tag{18}$$

$$a_n = -\frac{1}{3} \frac{b^2}{D} + \frac{2Eh}{R} \sum_n \frac{1}{\mu_n^2 \Delta_n^2} \tag{19}$$

The coefficient series are calculated as:

$$A_n = -\frac{2\, q_o}{n\pi\, \Delta_n^2} \left[1 - \frac{2\, Eh}{R^2 D\, \mu_n^2 a_n} \cdot \sum_n \frac{1}{\Delta_n^2} \right] (-1)^n \tag{20}$$

In practical example we can use only $n = 1, 2$.

By considering hinge connection condition in line y = b, between cylindrical shells and base slab, we can find:

$$w = \sum_n A_n sin\, \mu_n y \tag{21}$$

$$A_n = -\frac{2\, q_o}{n\pi\, \Delta_n^2} (-1)^n \tag{22}$$

By accepting $R = \infty$ in 22th equation, we determine flat slabs by solving the equation in rectangular flat slabs with integral:

$$w = \frac{1}{D} \left(\frac{q_o y^5}{120\, b} + \frac{cy^3}{6} + \frac{dy^2}{2} + ey + f \right) \tag{23}$$

By using boundary condition and integral, the constant value is determined. In decant coordinate system, boundary condition in $y = 0$ is equal to:

$$w = 0,\ M_y = 0 \tag{24}$$

By using the above boundary condition $f = 0$ and $d = 0$ will find. And the boundary condition in y = b is:

$$w\ =\ 0;\ \frac{dw}{dy} = 0 \tag{25}$$

Whence:

$$c\ =\ -\frac{q_o b}{10};\ e\ =\ \frac{q_o b^3}{120} \tag{26}$$

In this way, the rigid bar in line $y=b$ we can find:

$$w = \frac{1}{D} \left(\frac{q_o y^5}{120\ \mathrm{b}} - \frac{q_o b y^3}{60} + \frac{q_o b^3 y}{120} \right) \tag{27}$$

Or:

$$w = \frac{q_o b^3 y}{120\ D} \left(\frac{y^4}{b^4} - \frac{2y^2}{b^2} + 1 \right) \tag{28}$$

When $y = 0, 5b$, then:

$$w = 0{,}00239\ \frac{q_o b^4}{D} \tag{29}$$

And for bending moment we have:

$$M_y = -\frac{q_o by}{2}\left(\frac{y^2}{3\,b^2} - \frac{1}{5}\right) \tag{30}$$

When $y = b$:

$$M_y = -\frac{q_o b^2}{15} = -0{,}0667\, q_o b^2 \tag{31}$$

When $y = \frac{b}{2}$, we have:

$$M_y = 0.0292 q_o b^2 \tag{32}$$

By this method and considering hinged condition in $y = b$ it means $w = 0$, $w = 0$, $w'' = 0$:

$$f = 0;\ d = 0;\ c = -\frac{q_o b}{6};\ e = \frac{7}{360} q_o b^3 \tag{33}$$

So the equation will solve like:

$$w = \frac{q_o b^3 y}{6\,D}\left(\frac{1}{20}\frac{y^4}{b^4} - \frac{y^2}{3\,b^2} + \frac{7}{60}\right) \tag{34}$$

$$M_y = -\frac{q_o by}{6}\left(\frac{y^2}{b^2} - 1\right) \tag{35}$$

If $y = 0.\,5b$, we have:

$$w = 0{,}00651\,\frac{q_o b^4}{D};\ M_y = 0{,}0625\, q_o b^2 \tag{36}$$

3. Numerical Results and Discussion

By considering boundary condition between lower flat slab and fore flat slab for the new retaining wall the values of internal forces and bending moment have been shown in Figure 2.

In this way, if we consider interior part of the retaining wall as cylindrical shell, the flexure forces and bending moments are presented in Figure 3.

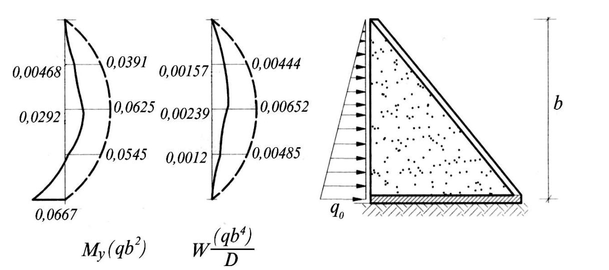

Figure 2. The obtained internal forces for the front slab of the new retaining wall.

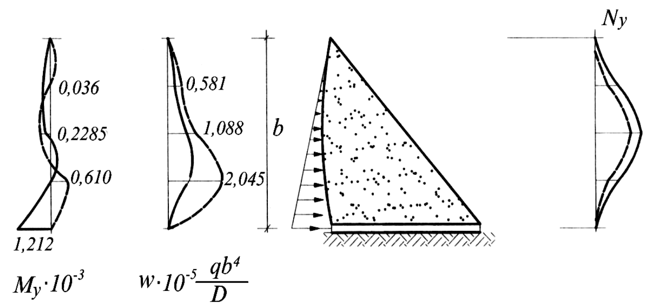

Figure 3. The obtained internal forces for the front shell of the new retaining wall.

The achieved equations for the new retaining wall consisting of flat slab will completely analyze the mechanical behavior. In the stresses and deflections and bending moments we have the same results too. We also find out the internal forces and bending moment in flat slab is more than cylindrical shell shown in Figure 2 and Figure 3.

4. Conclusion

In this research, thin flat slabs with concrete gravity retaining walls are matched and a new form of retaining wall is offered and then a new way to analyze the new wall is considered.

The analysis in this study shows that using shell in the front portion of the new retaining wall instead of flat slab is more profitable, because against the equal forces, the shells are thinner than flat slabs.

The value calculated for concrete used in the new retaining wall is much less than old retaining wall. The analysis shows that in the new retaining wall 90 percent of the old retaining wall is employed. For this purpose, if the surface of the consumed concrete of the gravity retaining wall is about $0.5b_1b$, then it will lead to $0.04b_1b$ in new retaining wall. Therefore, the employment of the new retaining wall is more economical.

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