



## Percolation approach to soil compressibility analysis

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### Abstract

Soil compressibility is a phenomenon linked to the partially sinking of structures, which in turn, produces fissures in floors, walls, columns, and other structural elements. Also it involves the development of pressures generated in the water (primary phase) and the soil (secondary phase) on time. These can be explained using percolation principles. In this paper a novel alternative for the analysis of the abovementioned phenomena is presented. Indeed, via percolation, according to the water being expelled from the soil under different loading conditions is considered. Nets of sites of given dimension with a determined number of occupied sites (amount of soil particles) and other of empty sites (initial amount of water in the soil) are defined. Each net simulates the soil stresses by means of the cumulative probability after each load. Finally, the necessary nets were constructed with less amount of empty sites according to the water being expelled from soil on time under each load and therefore, of lesser dimensions to determine the critical percolation (starting of structural friction of soils).

**Keywords:** Compressibility, soils, percolation.

### Introduction

After compressibility, soils increase density and deformations arise like sinking of the overlaying ground surface. This occurs because of the drainage of water driven by excess of pore-water stress at the first phase, and structural viscosity of soil at the second phase. Compressibility is the decrease process of soil volume triggered by a transient flow. The excess of pore water pressure, *i.e.*, load-induced pore water pressure, generated in both cases, causes the water to be squeezed out of the soil mass, seepage flow, and therefore the starting of volume changes. This movement of water continues at a decreasing rate until all excess pressure has dissipated and steady-state conditions are regained again.

Furthermore, clay content and mineralogy affect the process of soil compression (Larson *et al.*, 1980; Horn, 1988; McBride & Watson, 1990; Smith *et al.*, 1997). Coarse-textured soils are less susceptible to compaction than those with a fine texture (Horn, 1988; McNabb & Boersma, 1993; Horn & Lebert, 1994; McBride & Joosse, 1996). Moreover, soil resistance to compaction depends on the intrinsic soil attributes, texture being one of the most relevant (Larson, *et al.*, 1980; Horn, 1988; McBride, 1989). Soil response to compression is also influenced by organic matter content (Larson, *et al.*, 1980; McBride, 1989; McBride & Watson, 1990; Soane, 1990). The susceptibility to compression decreases as soil organic content increases (O'Sullivan, 1992; Zhang *et al.*, 1997). However, the effect of organic matter on the reduction of

soil compressibility seems to be dependent on soil moisture at the time of load application (Soane, 1990). Soil moisture has been widely recognized determinant for soil compressibility (Larson & Gupta, 1980; McBride, 1989; Soane, 1990; O'Sullivan, 1992; McNabb & Boersma, 1996; Sánchez *et al.*, 1998). However, uncertainty exists regarding the effect of soil moisture on the susceptibility to compaction. Larson *et al.* (1980) and O'Sullivan (1992) indicated that susceptibility to compaction is independent of soil water content, which is in direct contrast to the findings of Sánchez *et al.* (1998, 2001). The differences in the soil susceptibility to compaction seem to be related to the mechanism whereby a decrease in soil water content increases the number of contacts between particles, which, in turn, is directly dependent on soil texture (McNabb & Boersma, 1996). Crystalline clay minerals may lose more water from the external surface than non-crystalline minerals. As a result, the number of contacts between particles increases at a faster rate in the former than in the latter as soils dry (McNabb & Boersma, 1996). Soil compressibility defined as the volume decrease when soil is subjected to a mechanical load, has been described by the shape of the soil compressibility curve (Horn & Lebert, 1994). Several nonlinear models that describe the entire compressibility curve and account for the influence of soil properties, such as differences in initial bulk density or in soil water content on soil compressibility, have been proposed (McNabb & Boersma, 1993). Soil deformation



occurs when some individualized (crystals) or grouped (domains) particles are able to separate and move in relation to each other. This movement is restricted by friction forces and by bonds existing between particles. The denser the soil and the more intricate the particle arrangement, the smaller the pore space available for particle movement, and the higher the friction forces between them. Thus, displacement and rearrangement of solid particles to closer positions (deformation) become more difficult as void ratio increase (Pérez-Rea *et al.*, 2005).

As compressibility takes place, settlement occurs, and continues at a decreasing rate until steady-state conditions are regained again. Usually, compressibility test measures primary compression as the first reported deformation and secondary compression as the last one according primary has to finish for secondary appears for each load increment.

### Percolation principles analysis

Many phenomena in physics and chemistry can be modeled by stochastic analysis. Cumulative probability distribution function and percolation principles can be used. Typically, these systems involve a collection of particles laying on a lattice and evolving in a random way, interacting with neighboring sites on the lattice. Percolation general formulation involves elementary geometrical objects such as spheres, bounds, sites, and others, which are placed either in a net of finite dimension or a continuum. One can distinguish one-, two-, or three-dimensional nets. In any case, in a net, the capillary (bonds in current technology) connecting two adjacent nodes (sites) can have a uniform diameter, and they can be surrounded by a number of capillary segments with different diameters (Pérez-Rea *et al.*, 2005). Dullien in 1992 studied several two-dimensional nets with 20 to 40 porous-segments wide and depth ranges from 15 to 80 segments. The non-wet phase was studied inside the net through one of the faces; the two perpendicular sides to the penetration face were supposed to be non-permeable and the fourth face was open (Dullien, 1992). Initially, it was assumed an arbitrary distribution of the porous diameters. Then, they were assigned to different net bonds in a random fashion. Net bonds or sites are numbered sequentially from 1 to  $N$ , where  $N$  is the total number of net bonds or sites. This procedure can be implemented in a computer where bonds or sites locations can be generated by pseudo-random numbers. The cumulative probability function,  $p_k$ , accounts for the cumulative number of porous of size  $j \leq k$  in the net, that is,

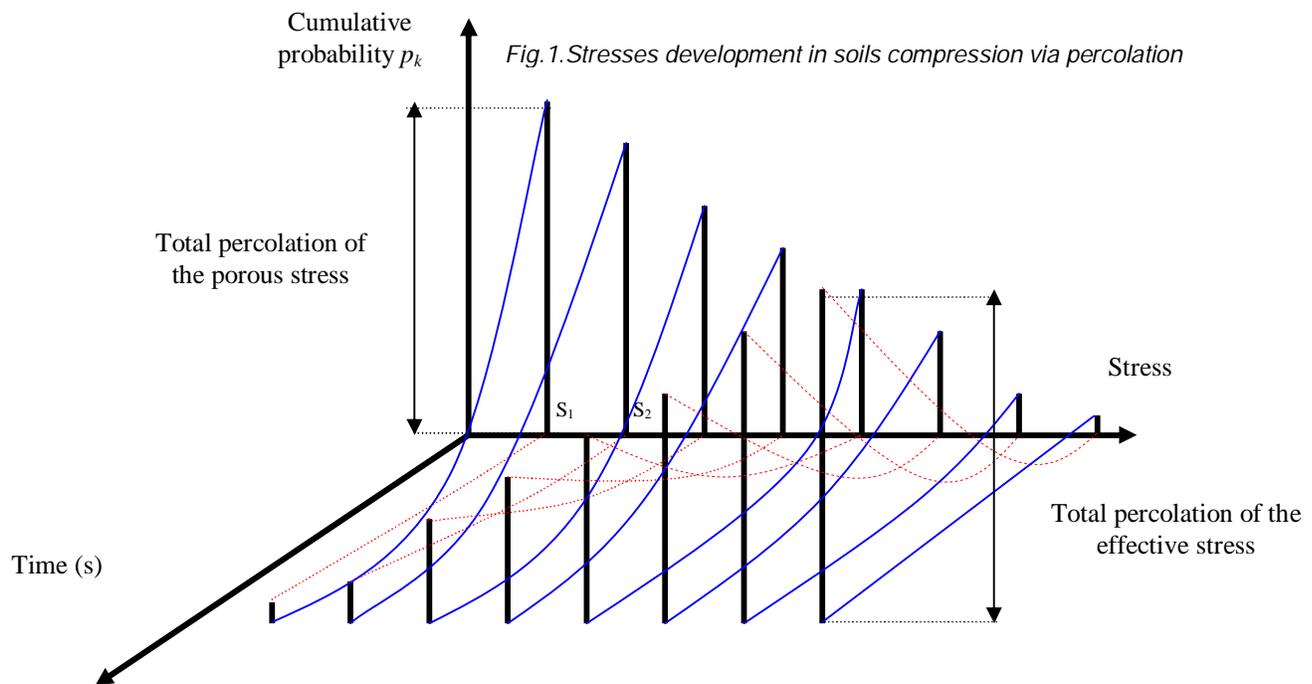
$$p_k = \frac{\sum_{j=1}^k N_j}{\sum_{j=1}^n N_j} \quad (1)$$

where  $N_j$  is the total number of porous in the net characterized by a porous size  $j$ , for  $j = 1, 2, \dots, n$ , being  $j=1$  the greatest and  $j=n$  the smallest;  $k$  ( $k=1, 2, \dots, n$ ) is the smallest porous being traversed at some time while the percolation process. As a first step in the process, only the largest porous ( $k = 1$ ) and the ones accessible through the net face are penetrated. Then, the second largest ones ( $k = 2$ ) are penetrated, and so forth, until some value of the cumulative porous size distribution, corresponding to a porous of intermediate size. Percolation is denoted then to the particular point in which the fluid penetration reaches the net opposite face. Before this, the penetrated porous are unable of transporting the fluid between the injection and the output face of the sample. In percolation, there is a cumulative probability,  $p_{cr}$ , called the *critical probability*. It has been found that the net size of approximately  $40 \times 40$  porous segments is the minimum for representing a true value of  $p_{cr}$  with a probability of  $\pm 0.01$ . The probability of critical bonds or sites percolation,  $p_{cr}$  specifies the minimum fraction of net bonds or sites required to be opened and, thus, the fluid can penetrate from one net side to the other. In other words,  $p_{cr}$  is the *percolation threshold* and corresponds to the minimum concentration at which an infinite cluster traverses the space. The main key-point of the theory is that, for each net, there exists a critical probability  $p_{cr}$ , where  $0 < p_{cr} < 1$ , at which an infinite cluster appears definitely (Dullien, 1992).

Two sites are *connected* if there exists at least a path between them that contains all of its bonds occupied. A set of connected (occupied) sites, communicated via the rule of the closest neighbor, surrounded by void bonds is called a cluster. If the net is very large and if  $p_k$  is sufficiently small, the size of any connected cluster is small. However, if  $p_k$ , as given in (1), is close to 1, the net must be almost completely connected (Newman & Schulman, 1981).

### Soils compressibility analysis

The soil compressibility consists of a primary compression that takes place during the increase in effective vertical stress and a secondary compression that follows at constant effective vertical stress. The first one is due to the hydrodynamic delay caused by gravitational water; this takes into account only the delay of the elastoplastic deformation. The second one can be represented as a compression phenomenological law, due to viscous effects (Zeevaert, 1973). A soil-structure-change starts as soon as the increase in effective vertical stress arises. It continues with time to establish internal equilibrium under the new external condition (Veenhof & McBride, 1996).



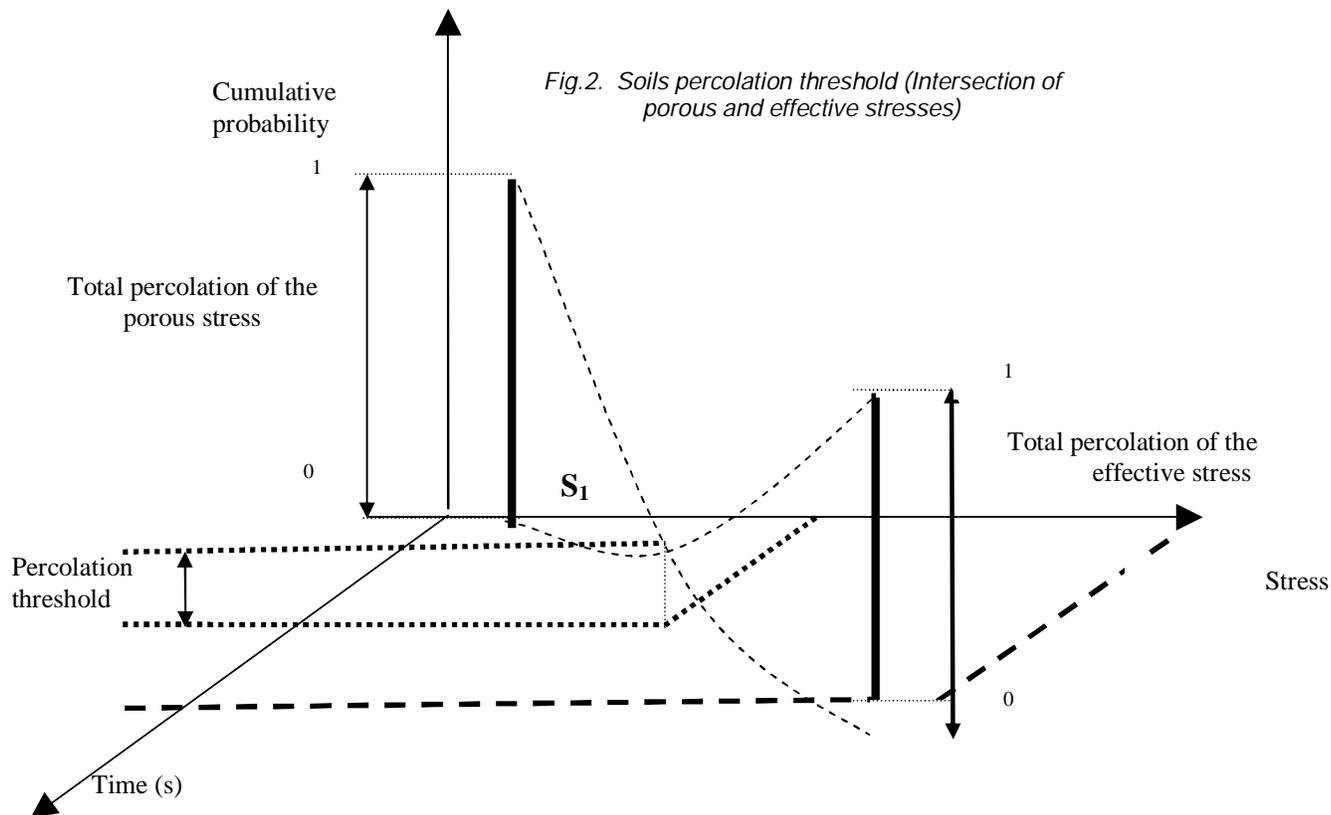
The compression-soil process develops two stresses, namely, the porous stress and the effective stress. For any time, the sum of these two stresses remains constant. At the beginning, the porous stress is the only one that is present. After applying a load,  $\Delta p$ , in the water rises a stress,  $u$ , in excess of the hydrostatic such that  $\Delta p = u$ . At a later instant, the effective stress increases whereas the porous stress decreases. Finally, at an infinite time,  $\Delta p$  is equal to the effective stress since the porous stress vanishes. Based on the above, and regarding the developing stresses for a given load, at  $t = 0$ , there exists percolation due to porous stress, and at  $t = \infty$ , there exists percolation due to the effective stress. Therefore, there exists a critical time,  $t = t_{cr}$ , at which there is an intersection of the former with the latter. This percolation analysis to soil compression is better explained in Fig. 1. In that figure, for a first stress,  $S_1$ , applied to the soil at  $t = 0$ , all the stress is taken by the water, i.e., porous stress; at this instant, the percolation value tends to 1. At a later instant,  $t = t$ , the porous stress percolation diminishes, there exits water flow and the effective stresses rise with an cumulative probability,  $p_k$ , very likely less than the threshold value. At  $t = \infty$ , the cumulative probability of the porous stress vanishes and the cumulative probability of the effective stress takes a value greater than or equal to the critical percolation or percolation threshold.

For a second stress  $S_2$  applied to the soil, all the exerted stress will be taken by the water again, porous stress. However, since there is less water, due to the first load, the percolation value is less than 1. At a later time, the percolation value of the porous stress decreases because the water is gone. On the other hand, effective stresses appear in lesser times than in the first load, with

a cumulative probability. Finally, at  $t = \infty$ , the cumulative probability value of the porous stress vanishes and the corresponding to effective stresses takes a value greater than in the above load condition. For  $t = 0$ , and greater soil stresses, these will be taken for the remaining water. Thus, the porous stress cumulative probability will decrease for each stress increase. At later times, the cumulative probability of the porous stress decreases whereas the percolation of the effective stress rises as the stress increases, and sooner because of the constant decrease of the water soil content. Finally, for  $t = \infty$ , the porous stress cumulative probability vanishes and the percolation of the effective stress tends to 1.

Based on the above, it could be thought of that for each stress increase, an effective stress percolation threshold can be found. However, this idea was discarded because for small loads, the most likely is that only part of the soil skeleton of given thickness is working without really existing a threshold. On the other hand, for heavier loads, it is likely that the effective stresses had already percolated, that is, the threshold had already reached. Thus, the percolation threshold value, appears at the intersection of both as shown in Fig. 2.

By joining the corresponding values of the cumulative probability for all load increments, the plot shown in Fig. 2 can be drawn. At the beginning, percolation due to the porous stress tends to 1. Then, porous stress decreases until it vanishes, and the effective stress rises as time and stress increase. According to soil compression fundamentals, this point would represent the starting of the secondary compression. On the other hand, the last moment of percolation is due to porous stress. This means that when the secondary compression starts,

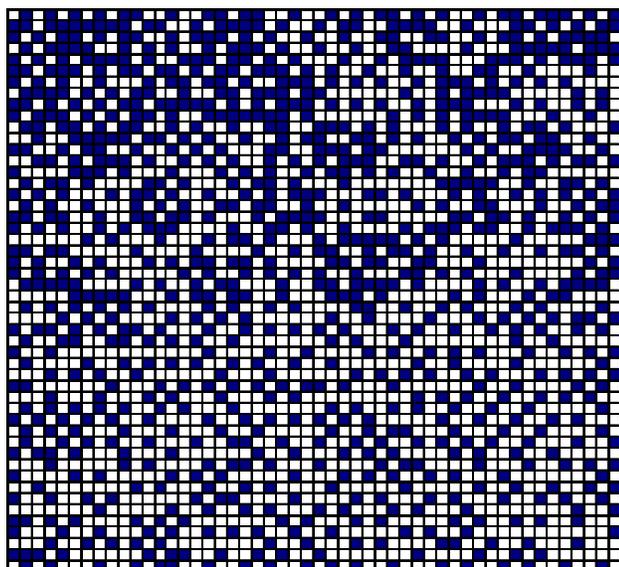


there is already percolation of some part of the soil particles. Thus, as secondary compression goes on, the next behavior of the effective stress would correspond to form an infinite cluster in the soil with a total percolation that tends to 1. Ideally, at this moment, the soil compression would stop, in agreement with the hypothesis that the soil is incompressible (Juarez, 1974).

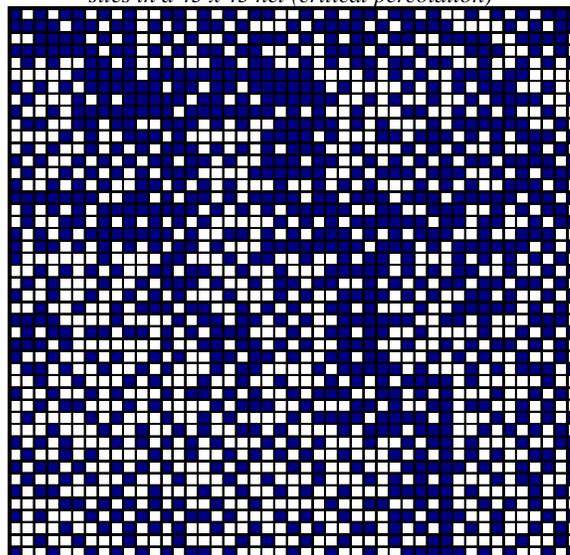
**Random media analysis via percolation**

Based on soil behavior abovementioned, the proposed random media analysis, using the percolation approach, is a proposed square net, Fig. 3, where the occupied squares are the soil particles. It was assumed that all the particles had the same diameter, and were randomly distributed, with a large amount of empty

*Fig.3. Supposed random arrangement of 1000 sites in a 50 x 50 net*



*Fig.4. Supposed random arrangement of 1000 sites in a 45 x 45 net (critical percolation)*





squares. These represent the space filled with water.

To start with, it was proposed to evaluate the cumulative probability,  $p_k$ , in a net of given dimension, greater than  $40 \times 40$ , to be considered valid (Dullien, 1992). Then, holding the same particles number (1000 per example) and according to laboratory data (reduction of water volume being leaked for each applied stress and time increment), the net was shrunk because of the reduction of voids, arouse the critical percolation, as shown in Fig. 4, where the secondary stage can be identified as well. If the size net continues diminishing one gets the complete sites percolation, i.e., the total developing of the secondary compression.

This random media analysis, together with laboratory data, will help know the percolation threshold, i.e., the approximate time at which the secondary compression appears.

### Conclusions

A novel alternative for the analysis of soils compressibility that could also help to solve it, was presented. In agreement with stress and time increments distributions during the compression processes, it can be said that when a first stress increment is applied, at  $t = 0$ , there exists a total percolation (that tends to 1) due to porous stress. At  $t = t$ , this decreases until it vanishes, whereas the effective stress rises tends to 1 (total percolation in  $t = \infty$ ) as stress increases. The intersection of both cumulative probability paths (porous and effective stress) at different stress and time increments would be equivalent to find the value of the critical percolation. Following the proposed procedure, the necessary nets were constructed with the same number of occupied sites (amount of soil particles) and less amount of empty sites (amount of water) according to the water being expelled under each load and therefore, of lesser dimensions to determine the critical percolation (ending of the primary compression and starting of the secondary compression or structural friction of soils). Finally, the total percolation (ideal instant of total compression) can be determined.

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