

Image Segmentation using Level Set Evolution Driven by Complex Wavelets

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Abstract

Level set segmentation is an efficient tool for segmenting the images by evolving the level curves. By means of region based image segmentations, the region of object is given more importance over edge information. In this paper a new region based level set is formulated with the direction and evolution flow of contour will be controlled by the complex wavelets. Real and imaginary parts of wavelet decomposition are taken into consideration in defining the new signed pressure force henceforth the level set formulation which leads to faster and accurate image segmentation. This method has been successfully implemented on biomedical and satellite images and proves better over the conventional methods.

Keywords: Active Contours and Image Segmentation, Complex Wavelets, Hilbert Transform, Level Sets

1. Introduction

Discrete Wavelet Transform (DWT) is an efficient tool which will uses in various fields of engineering and other applications. Dual complex wavelets are improvements of DWT and can be achieved by expansive discrete wavelet transform in place of critically sampled one¹.

Dual Tree Complex Wavelet Transform (DTCWT) can be applied in many areas of signal processing and image processing like denoising, feature extraction and image segmentation with its shift invariance and directionality. In this paper we present a segmentation of images using level set formulation which is modified by adding the directionality of complex wavelet transform.

Image segmentation plays a vital role in the field of image understanding, image analysis, and pattern identification. The foremost essential goal of the segmentation process is to partition an image into regions that are homogeneous with respect to one or more self-characteristics and features. It is a pre-processing technique for object recognition. Level Set image segmentation using contour evolution can be basically classified into two categories edge based and region based models.

In edge based level set model one of the most prominent method is Geodesic Active Contour (GAC) proposed by Casallas². Edge based active contours or level set method uses balloon force to evolve the contour and it is complex to design it. The balloon force either shrink or expand the contour by the force acting inside or outside respectively. If defined force is large contour will pass through weak edges and the contour may not be allowed if it is small. Due to the property of local minima edge based model fails to detect interior and posterior boundaries of the objects if the initial contour defined is far from the desired object.

Local minima as discussed in edge based models can be overcome in region based models. Moreover these models are less sensitive to noise and give good performance for images with weak or without edges as they utilizes the statistical information within or outside the contour. Even though, initial contour is defined far away from the object, it has ability to detect interior and exterior boundaries of the desired object. One of the popular active contours without edges is implemented by Chan and Vese (C-V) et al³. which showcases the level set model that minimizes the Mumford-Shah function⁴.

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The C-V model has the property of global segmentation to segment all the objects within an image irrespective of the initial contour defined. Whereas the GAC model detects interior boundaries if the contour is defined inside the object and detects exterior boundaries if it is outside, so that it can be consider as a local segmentation model⁵. In this paper we introduce a combined approach of level set and wavelets for segmentation of images. Here the proposed ACM is based on the advantages of GAC and C-V models with additional wavelet energies embedded to it.

This paper will be organized as follows Section 2 will give information regarding previous work done; Section 3 is about Dual tree complex wavelet transform. Section 4 gives the detailed information about our proposed algorithm by combing wavelets and level set. In section 5 experimental results are given.

2. Level Set Background

The level set method was initially proposed to track moving fronts in 1988 and has spread across various imaging domains. Level sets with implicit geometries and if the surface is evolved by solving the partial differential equations called as implicit level sets. The central idea is represent the evolving contour using a signed function, where its zero level corresponds to initial contour. Then, according to the motion equation of the contour, one can easily derive a similar flow for the implicit surface that when applied to the zero level will reflect the propagation of the contour⁶⁻⁹

The existing active contour models can be broadly classified as either parametric active contour model or geometric active contour models according to their representation and implementation. It can be used efficiently to address the problem of curve or surface propagation in an implicit manner The GAC model utilizes the gradient of an image to get the Edge Stopping Function (ESF) to locate the contour around the boundaries of desired object. Generally, a positive, decreasing regular ESF $g(\nabla I)$ is used such that $\lim_{t \rightarrow \infty} g(t) = 0$.

$$g(\nabla I) = \frac{1}{1 + |\nabla G_\sigma * I|^2} \tag{1}$$

The denominator term $G_\sigma * I$ in equation 1 gives

convolution of an image I with a Gaussian kernel having the standard deviation σ . The evolving curve C that represents the planar curve in ω . Let Ω is bounded open subset of R^2 in the given image $I : [0, a] \times [0, b] \rightarrow R$. The GAC model is formulated by minimizing the following energy functional

$$E(c) = \int_0^1 g(|\nabla I(C(q))|) |C'(q)| dq \tag{2}$$

where g is the energy function represented in Equation 1. Using the calculation of variation we could get the Euler Lagrange Equation of Equation 2 is as follows.

$$C_t = g(|\nabla I|) k \vec{N} - \left(\nabla g \cdot \vec{N} \right) \vec{N} \tag{3}$$

where the curvature of the contour is k and the normal inward to the curve is \vec{N} . To increase the propagation speed, a constant velocity term α is added. By including the constant velocity term α in the above equation can be rewritten as

$$C_t = g(|\nabla I|) (k + \alpha) \vec{N} - \left(\nabla g \cdot \vec{N} \right) \vec{N} \tag{4}$$

The corresponding level set formulation based on Equation 4 is as follows

$$\frac{\partial \phi}{\partial t} = g|\nabla \phi| \left(\text{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right) + \alpha \right) + \nabla g \cdot \nabla \phi \tag{5}$$

where α is the balloon force which controls the shrinking or expanding of the level set evolution Chan and Vese model is a special case of Mumford-Shah functional. Let I be the given image with c_1 and c_2 the mean intensities inside and outside the contour respectively. For a given image $I(x)$ in the domain Ω . The C-V model is formulated by minimizing following energy functional.

$$E(c_1, c_2) = \lambda_1 \int_{\text{inside}(c)} |I(x) - c_1|^2 dx + \lambda_2 \int_{\text{outside}(c)} |I(x) - c_2|^2 dx, \quad x \in \Omega \tag{6}$$

Based on level set method it has been assumed as

$$\phi(x) = \begin{cases} 1 & \text{inside}(c) \\ -1 & \text{outside}(c) \\ 0 & \text{otherwise} \end{cases} \quad x \in \Omega$$

Mean intensities of an image inside and outside of the contour are given by C_1 and C_2 respectively.

$$C_1(\phi) = \frac{\int_{\Omega} I(x) \cdot H(\phi) dx}{\int_{\Omega} H(\phi) dx} \tag{7}$$

$$C_2(\phi) = \frac{\int_{\Omega} I(x) \cdot (1-H(\phi)) dx}{\int_{\Omega} (1-H(\phi)) dx} \tag{8}$$

Solving for minimizing the corresponding variation level set formulation including length and area terms is

$$\frac{\partial \phi}{\partial t} = \delta(\phi) \left[\mu \nabla \left(\frac{\nabla \phi}{|\nabla \phi|} \right) - v - \lambda_1 (1-c_1)^2 + \lambda_2 (1-c_2)^2 \right] \tag{9}$$

where $\lambda_1, \lambda_2, \mu$ and v are fixed parameters such that $\lambda_1, \lambda_2 > 0$ and $\mu, v \geq 0$. and μ, v are used for controlling the smoothness of zero level set and increasing the propagation speed respectively and ∇ is gradient operator, $H(\phi)$ is the Heaviside function and $\delta(\phi)$ is the Dirac function.

3. Dual Tree Complex Wavelet Transform

General wavelet decomposition decomposed the image into four types of lower-resolution coefficient images: the approximation produced by applying two Low-Pass filters (LL), the diagonal details, computed with two High-Pass filters (HH), and the vertical and horizontal details, output of a High-Pass/Low-Pass combination (LH and HL). Figure 1 is an example of two levels wavelet decomposition is reported¹⁰⁻¹².

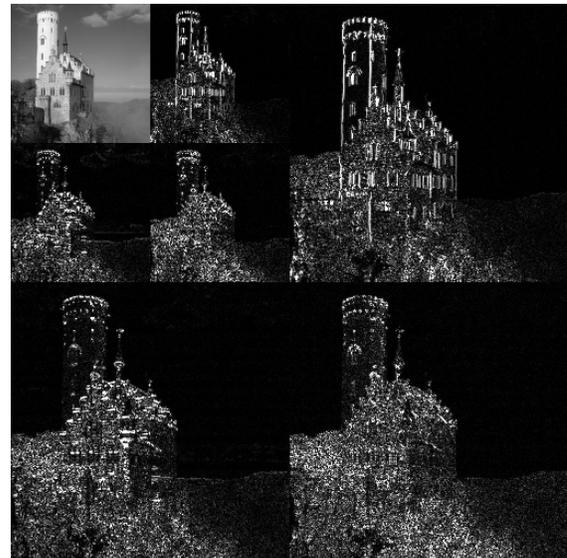


Figure 1. Two level wavelet decomposition of 2 D image.

Although DWT introduced by Mallat has become a major tool for signal and image analysis, it has two major disadvantages. 1. Lack of shift invariance, 2. Poor directional selectivity. These problems can be solved by DTCWT which has approximately shift invariant and good selectivity and directionality.

However, shift invariance can be achieved with an approximate a real DWT by doubling the sampling rate at each level of the tree. For this, the samples should be evenly spaced. Up sampling by 2 or doubling the sampling rate is achieved by eliminating the down-sampling by 2 after the level 1 filter, H_{0a} and H_{1a} as shown in Figure 2. This is

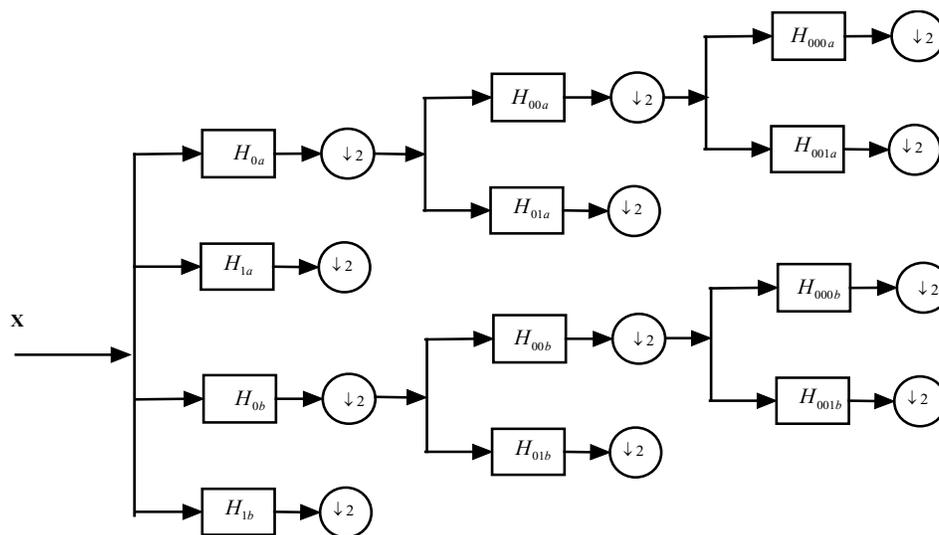


Figure 2. Dual tree filters in complex wavelet transform.

equivalent to two parallel fully-decimated trees provided that the delays of H_{ob} and H_{1b} are one sample offset from H_{0a} and H_{1a} . The filters in one tree must provide delays that are half a sample different from those in the other tree. To obtain linear phase response, it is required to consider odd-length filters in one tree and even-length filters in the other. The dual-tree complex DWT of an input signal is implemented using two critically-sampled DWTs in parallel as shown.

The transform is 2-times expansive over DWT because for an N-point signal it gives 2N DWT coefficients. If the filters in the upper and lower bands are the same, then no advantage will be gained. However, if the filters are designed in a specific manner, then the sub band signals of the upper DWT band can be interpreted as the real part of a complex wavelet transform, and sub band signals of the lower DWT band can be interpreted as the imaginary part. Equivalently, for specially designed sets of filters, the wavelets associated with the upper DWT band can be an approximate Hilbert transform of the wavelets associated with the lower DWT band. With this type of design, the dual-tree complex DWT is almost shift-invariant, in contrast with normal DWT. Moreover, the dual-tree complex DWT can be used to implement 2D wavelet transforms where each wavelet is oriented in a direction, which is especially useful for image processing applications. Each of the six wavelets is oriented in a distinct direction. Unlike the critically-sampled separable DWT, all of the wavelets are free of checker board artifact. Each sub band of the 2-D dual-tree transform corresponds to a specific orientation as shown in Figure 3.

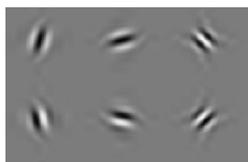


Figure 3. Directional wavelets for reduced real DWT.

The wavelets are oriented in the same six directions as those of the real 2-D dual-tree DWT. However, here we have two in each direction. If the six wavelets displayed on the first row are treated as the real part of a set of six complex wavelets and the six wavelets displayed on the second row are treated as the imaginary part of a set of six complex wavelets. Then the magnitudes of the six complexes are shown on the third row. As shown in the

Figure 4, the magnitude of the complex wavelets does not have an oscillatory behavior and they are bell-shaped envelopes and Figure 5 shows real and imaginary filter output images with 1st level of decomposition.

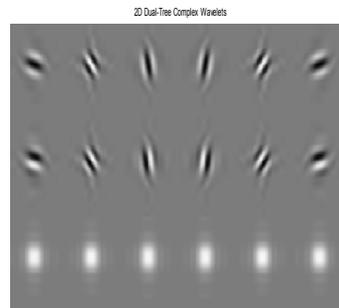


Figure 4. Directional dual tree complex wavelets.



Figure 5. Real and imaginary filter outputs of level 1.

2-D CWT produces three sub images in each of spectral quadrants 1 and 2, giving six band pass sub images of complex coefficients at each level, which are strongly oriented at angles of $\pm 15^\circ, \pm 45^\circ, \pm 75^\circ$.

4. Proposed Method

In this model wavelets based Level set evolution is introduced with the modified Signed Pressure Force (SPF) functions wavelet energies of its sub bands in it. The SPF function uses the statistical information to modulate the signs of the pressure forces inside and outside the region of interest so that the contour shrinks when it is outside the object or expands when inside the object. We derive c_1, c_2 which are the mean intensities inside and outside the contour respectively as given below.

$$C_1(\phi) = \frac{\int_{\Omega} I_{ill}(x) \cdot H(\phi) dx}{\int_{\Omega} H(\phi) dx} \tag{10}$$

$$C_2(\phi) = \frac{\int_{\Omega} I_{ill}(x) \cdot (1 - H(\phi)) dx}{\int_{\Omega} (1 - H(\phi)) dx} \tag{11}$$

Real and imaginary Filter coefficients of dual tree

complex wavelet transform are given by Equation 12. Where j represents the scale

$$\begin{aligned} w\{j\}\{p\}\{k\}\{d\} \\ w\{j\}\{q\}\{k\}\{d\} \end{aligned} \tag{12}$$

The wavelet coefficients w are stored as a cell array. For $j = 1..J$, $p = 1..2$, $k = 1..2$, $d = 1..3$, $w\{j\}\{p\}\{k\}\{d\}$ are the wavelet coefficients produced at scale j and orientation (k, d) . With $p = 1$ we get the real part, with $p = 2$ we get the imaginary part.

$$spf(I(x)) = \frac{I(x) - \frac{c_1 + c_2}{2}}{\max\left(I(x) - \frac{c_1 + c_2}{2}\right)}, \quad x \in \Omega \tag{13}$$

Using Equation 12 and 13 in Equation 5 we get the level set formulation of our model as follows

$$\frac{\partial \phi}{\partial t} = spf |\nabla \phi| \left(\operatorname{div} \left(\frac{\nabla \phi}{|\nabla \phi|} \right) + \alpha \right) + (I_{ri} + I_{ii}) \cdot \nabla \phi \tag{14}$$

I_{ri} and I_{ii} are the real and imaginary orientations of j^{th} level. The above Equation 14 is used for contour evolution where α is mue or the balloon force which

basically use for evolution of the contour either shrinks or expands. ϕ is a dirac function.

Table 1. Comparative analysis

Image	Conventional levelset method			Proposed method	
	μ	Iterations	Evolution time(sec)	Iterations	Evolution time(sec)
Image 1	30	100	12.1452	60	12.5241
	40	100	11.5245	60	10.5482
Image 2	30	100	14.2252	60	14.2214
	40	100	14.0147	60	13.2542
Image 3	30	100	10.0325	60	09.2541
	40	100	09.2214	60	08.2287

5. Results and Discussions

The results are compared and analyzed with the normal level set and proposed method. In Figure 6, original input images are shown in first column and the second & third columns give the results of previous and proposed works. If a comparative analysis is made in the segmented images shown below of the second row and third row, We can

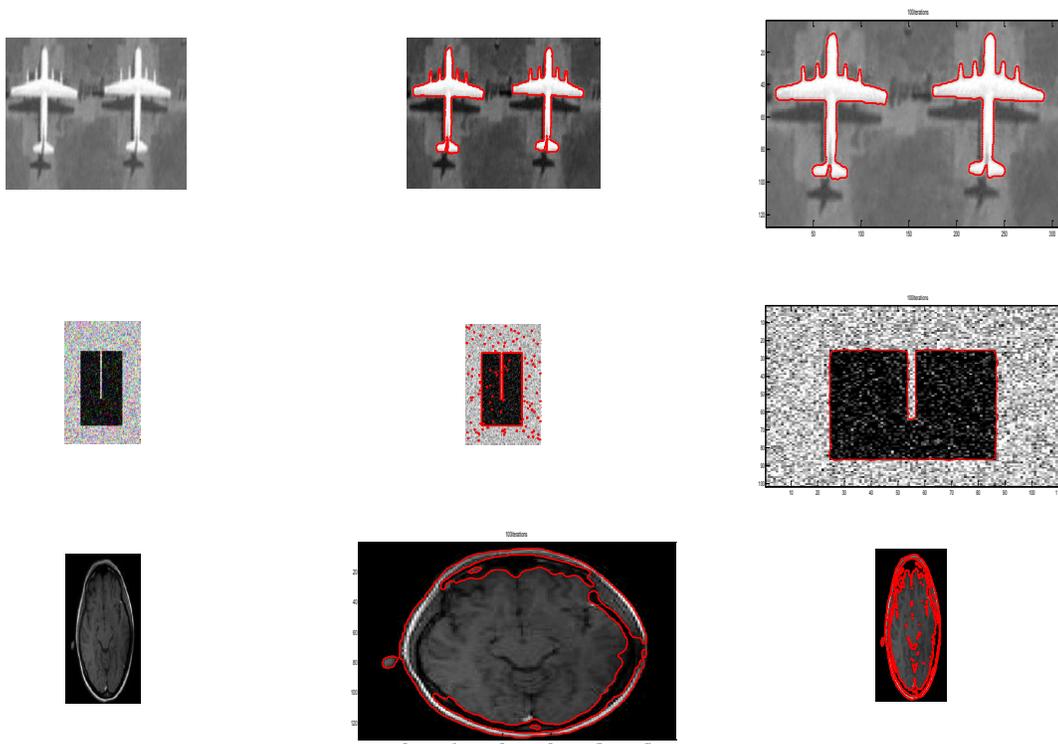


Figure 6. First column shows input image, second using level set and third column.

clearly observe that the segmented images which are the results of segmentation using our proposed model shown in the third row give us a complete segmentation, whereas the second row images which are the results of segmentation using previous work the contour generated is unable to segment properly. This experimentation is done on Matlab 12 with 3 Ghz processor and a RAM of 4 GB. The no. of iterations and evolution times are calculated for the same μ values. Results shows the regions are segmented properly with less number of iterations and approximately in same execution time even with wavelet filter designing process. In this paper, we proposed Level Set Evolution by considering directionality of complex wavelets. This image segmentation model has been successfully implemented on the images and results reveal the speed and accuracy of convergence.

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