Indian Journal of Science and Technology, *Vol 9(11), DOI: 10.17485/ijst/2016/v9i11/89401, March 2016*
ISSN (Print) : 0974-6846
ISSN (Online) : 0974-5645

Non-Dimensional Equation of Resistance Coefficient with Reynolds Number of Porous Medium Flow

M. Thiruvengadam[1*], G. N. Pradeep Kumar[2], D. Sivakumar[1] and C. Mohan[1]

[1]Vel Tech High Tech Dr. Rangarajan Dr. Sakunthala Engineering College, Chennai - 600062, Tamil Nadu, India; m_thriuvengadam@hotmail.com, shri.sivakumar1974@gmail.com, pselvar@yahoo.com
[2]S.V.U. College of Engineering, Tirupati - 517502, Andra Pradesh, India; saignp@gmail.com

Abstract

Objectives: To review the Darcy's equation and ascertain the reliability of the present experimental investigations with that of the past study. To present a non-dimensional form of relation between resistant to flow with fluid and particle parameters. To analyze the relation between Reynolds numbers with resistant coefficient (lambda) in ground water flow. **Methodology:** In order to achieve these objectives, experimental program planned, designed and carried out. Experiments conducted on porous medium of large spread of sizes of different materials in parallel flow permeameter for all regimes of flow. **Findings:** Experimental results compared with Darcy's equation and the validity of this equation is verified. A new form of Reynolds number is derived taking hydraulic mean radius as characteristic length and seepage velocity as characteristic velocity absorbing void ratio, volume diameter, and kinematic viscosity. Another non-dimensional form of resistant co-efficient is also derived and used to get unique relation between Reynolds numbers with resistance coefficient. **Applications:** Observed experimental data applied in Darcy's equations, and verified in its applicability. The derived equations can be applied in porous medium flow, such that velocity of flow can be determined, from which discharge through porous medium can be estimated.

Keywords: Darcy, Friction Factor, Hydraulic Gradient, Porous Medium Flow, Reynolds Number, Velocity

1. Introduction

Water plays a vital part in all forms of life on the earth. The resources of water is plenty of which groundwater is important one[1]. In view of the significant contribution made by groundwater resources to water supply, any fact contributing to a greater understanding of the problems relating to groundwater flow, either directly or indirectly, is of prime concern[2]. The subject of seepage flow is, at first sight appears conceptually simple; however, the character of flow of a fluid filament through the tortuous passages of a granular medium is complicated which is shown in Figure 1[3]. A steadily increasing interest created to study the laws governing the flow of fluids through beds of granular media. Numerous theoretical studies, modeling approaches, laboratory and field tests, and mathematical models developed to establish the true relationship between different porous medium variables.

1.1 Importance of Seepage Flow

The laws governing the flow of fluids in porous medium, describes the theory of seepage. At the time of design of hydraulic structures, the estimation of seepage through hydraulic structures is prime requirement[4,5]. The dimensions of the hydraulic structures are designed using quantity of seepage[6]. Scientific treatment of irrigation structures, flow around the well, sub-surface drainage and soil erosion is prime concern in porous medium flow[7]. Thus, the seepage flow problems play an important role in seepage flow analysis. Darcy described the seepage flow analysis in 1856 by his experiments[8,9]. Further, examination of literature indicates that a little agreement between the results of different researchers are of limited use. The subject is further complicated as different researchers adopted various methods of expressing their results of research. In addition, majority of the results are found to lack describing complete range of seepage flow.

**Author for correspondence*

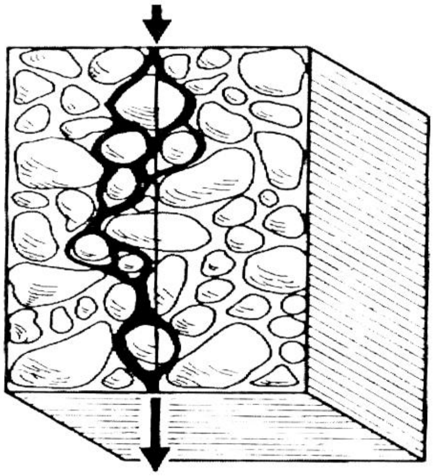

Figure 1. Porous Medium Flow.

1.2 Darcy's law

Darcy obtained the relationship between the bulk velocities (V_b) and hydraulic gradient (i), and shown in Figure 2[10]. A linear relationship (V_b) is obtained between bulk velocity (V_b) and hydraulic gradient (i) as well-known Darcy's law, which is expressed as

$$V_b = \frac{Q}{A} = -k\frac{\partial h}{\partial x} = -ki$$

Where, V_b is the bulk velocity of flow, Q is the discharge, A is the bulk cross sectional area, k is the Darcy's coefficient of permeability and $\frac{\partial h}{\partial x} = i$ = Hydraulic gradient.

In Darcy's graph, each diameter of the particle forms a separate straight line and shown in Figure 2. From the Figure 2, it may be observed that each size of the particle has different 'k' values and with same size with different porosities will have different 'k' values. All these equations, formed without considering seepage velocity, porosity, and viscosity.

It is generally accepted that Darcy's law is valid for low velocities. In many of flow situations velocity of flow is high. At higher Reynolds number, the linear relationship between V_b and ino longer holds good and exhibit non-linear relationship[11-13].

The main objectives of the present investigation are to verify and ascertain the reliability of the present experimental investigation with that of the past study, to present a non-dimensional form of relation between resistant to flow with fluid and particle parameters, and to analyze the relation between Reynolds numbers with resistant coefficient (lambda). In order to achieve these objectives, experimental program planned, designed and carried out.

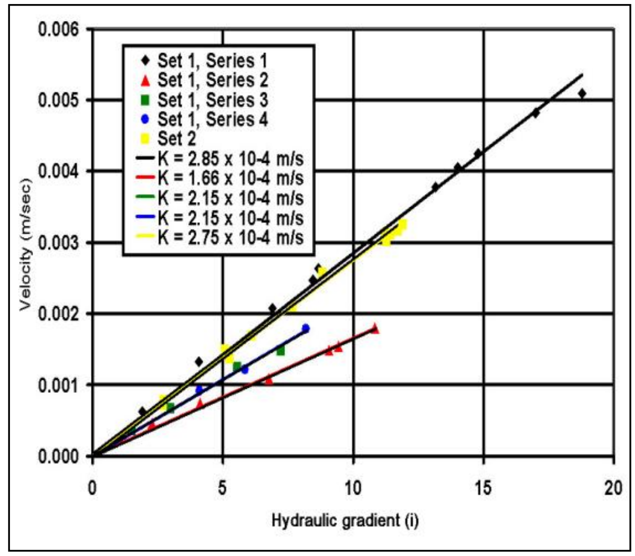

Figure 2. Darcy's observation between bulk velocity and hydraulic gradient (i).

2. Materials and Methods

2.1 Porous Medium

Porous medium contains inter connected pore space, which allows the fluid to flow through it. A porous medium characterized by a variety of properties, like surface area of particles, volume diameter, porosity, pore diameter, bulk area, flow area, particle surface area of fluid contact, hydraulic mean radius, etc. The problem of porous medium flow may be complicated due to the changes in physical properties of the medium itself. In the present study, fifteen sizes of porous medium are used and it consists of five sizes of sand particles, seven sizes of gravel and three sizes of glass spheres. The present experimentation consists of nine hundred and two observations in three types of materials. Gravel and sand sieved through a set of I.S.I. sieves and it ensured uniform size of particle to avoid the particle gradation effect. The volume mean diameter of the medium, is the diameter of the sphere having a volume equal to that of an irregular shaped particle, is used as the diameter of the particle d_p, and its porosity 'n' are listed in Table 1.

2.2 Permeameter

In general, cylindrical permeameters are used for the experimentation on seepage flow[14,15]. The dimensional

Table 1. Porous Medium Properties

Sl. No.	Description of the media	Volume Diameter 'dp' (mm)	Porosity 'n' (%)	k m/sec
1	Gravel	5.8	49.0	0.300
2	Gravel	7.8	51.0	0.380
3	Gravel	9.0	46.0	0.430
4	Gravel	12.3	46.5	0.650
5	Gravel	14.6	44.0	0.850
6	Gravel	17.5	42.0	1.200
7	Gravel	20.0	43.0	1.400
8	River sand	1.7	34.0	0.050
9	River sand	2.4	36.9	0.080
10	River sand	3.3	35.6	0.097
11	River sand	4.2	35.6	0.150
12	River sand	6.7	34.0	0.250
13	Glass spheres	16.7	59.0	2.500
14	Glass spheres	20.0	56.0	3.200
15	Glass spheres	35	54.0	5.750

details of the permeameter used in the present study shown in Figure 3. It consists of a vertical circular G.I. column of 6.2 m high and 0.15 m diameter. The earlier investigators computed the hydraulic gradient for a single length of travel with one set of head loss readings between two piezometers. In the present study, three different lengths of travel with three sets of piezometric points used to compute the hydraulic gradients. This avoids the error due to non-uniformity in packing, if any.

In this investigation, water used as a fluid medium. Water pumped from sump to the balancing tank, located at the entrance to the permeameter. A perforated horizontal pipe fixed at the end of the inlet pipe to ensure the water does not fall in the form of a thick jet. Further, an aluminum screen with 85% perforations placed at the entrance of permeameter to facilitate relatively turbulent-free entry of water into the permeameter. Constant water level in the permeameter is maintained in the header tank to establish steady state flow. Wide range of discharges used with both low and high rates of flow. At regular intervals, the temperatures of the outflow is noted, during every run from which viscosity is determined. From the observed experimental values the bulk velocity (V_b), seepage velocity (Vs), kinematic viscosity (υ), void ratio (e), porosity (n), hydraulic mean radius (m), hydraulic gradient (i) are calculated. Laminar range of bulk velocities (v^1) (0.00025 to 0.004 m/sec) and corresponding hydraulic gradients are selected from present experimental data for gravel and the results are between bulk velocities and hydraulic gradients and are shown in Figure 4.

The results plotted (Figure 4) between hydraulic gradient and bulk velocity of laminar range of flow, follows the trends of the Darcy's pattern (Figure 2), which conforms and proves that certain range of present data follows the Darcy's law and the linear relationship between V_b and 'i' is found to be valid[12-14]. It may be observed from Figure 4 that each diameter of porous medium forms an individual straight line irrespective of its particle and fluid properties[9]. It is also observed from Figure 4 that when the size of medium increases, the fluid conveying

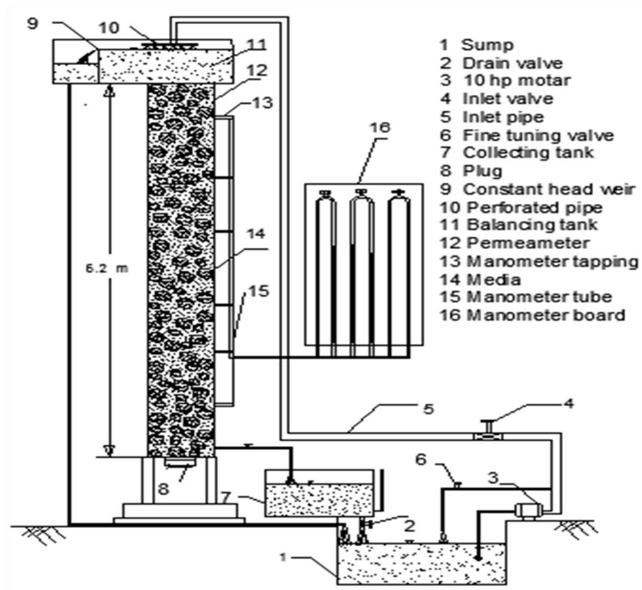

Figure 3. Experimental Set Up.

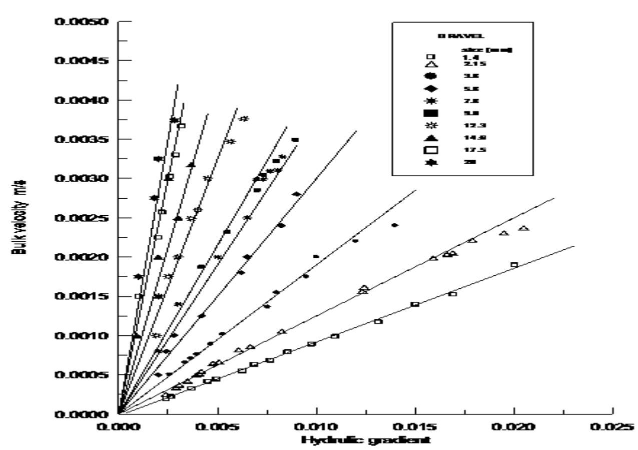

Figure 4. Variation of Bulk velocity with hydraulic Gradient (i) for gravel.

capacity (hydraulic conductivity) of medium increases. The coefficient of permeability (k) is conforming to the earlier findings that given in Table 1. The coefficient of permeability (k) values given in Table 1 are coinciding with others data, however, it is not coinciding with the Darcy's value, since Darcy's calculation for hydraulic gradient is questionable.

The co-efficient of permeability (k) can be get from experimentation. In porous medium flow many number of particle, sizes are available to analyze and estimate co-efficient of permeability. Therefore, it is laborious to get co-efficient for each size. Darcy type of graphs are also not included the porosity and viscosity effect. Hence, a non-dimensional form of relationship between the hydraulic gradient and other parameters proposed to overcome above problem. Therefore, dimensionless parameters like Reynolds number and lambda[7] is proposed absorbing bulk area, seepage velocity, void ratio, porosity, volume diameter, and kinematic viscosity. Co-efficient of resistance (lambda) related with another non-dimensional parameter like Reynolds number to get unique relation between them.

3. Derivation of Non-Dimensional Form of Equation (Reynolds Number) in Porous Medium Flow

Consider a cross sectional area of porous medium flow is shown in Figure 5. Let its bulk area is A_n, Porosity is, flow area is '$A_n n$', discharge is 'Q', volume diameter of particle and 'd_n' kinematic velocity 'v'.

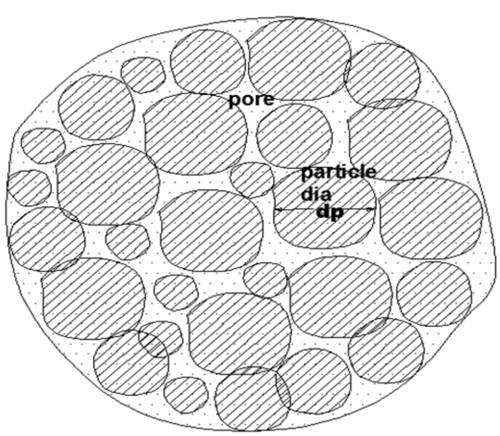

Figure 5. Cross Sectional Area of Porous Medium.

$$\text{seepage velocity} = V_s = \frac{Q}{A_b n}$$

Cross sectional area of solid particles = *Bulk area* (A_b) *flow area*

Solid area $A_b = A_b n = A_b(1-n)$

Cross sectional area of each particle = $\frac{\pi d_p^{\,2}}{4}$

Number of particles in cross sectional area = $\frac{solid\ area}{c.s\ area\ of\ each\ particle} = \frac{4A_b(1-n)}{\pi d_p^{\,2}}$

Assuming number of particle = number of pores = $= \frac{4A_b(1-n)}{\pi d_p^{\,2}}$

$$\therefore \text{Area of each pore} = \frac{flow\ area}{Number\ of\ pores}$$

$$\frac{A_b n\, \pi d_p^{\,2}}{4A_b(1-n)} = \frac{\pi d_p^{\,2}}{4}\left(\frac{n}{1-n}\right) = \frac{\pi d_p^{\,2}}{4} e$$

$$where \left(\frac{n}{1-n}\right) = e = void\ ratio$$

$$\therefore \text{Area of each pore} = \frac{\pi d_p^{\,2}}{4} e \qquad (1)$$

If diameter of each pore = d, Area of each pore = $\frac{\pi d^2}{4}$ (2)

$$\text{Equating Eq.1 and Eq.2 } \frac{\pi d^2}{4} = \frac{\pi d_p^{\,2}}{4} e$$

$$d_p^2 e = d^2 \ \therefore d = d_p\sqrt{e}$$

$$\text{Wetted perimeter of each pore} = \pi d_p\sqrt{e}$$

$$\text{Pore area} = \frac{\pi (d_p\sqrt{e})^2}{4} = \frac{\pi d_p^2 e}{4}$$

Hydraulic mean radius of each pore is $= \frac{porearea}{wettedperimeter}$

$$= \frac{\pi d_p^{\,2} e}{4\pi d_p\sqrt{e}} = \frac{d_p\sqrt{e}}{4}$$

Let Hydraulic mean radius of each pore is m_1

$$m_1 = \frac{porearea}{wettedperimeter\ of\ each\ pore} = \frac{\pi d_p^{\,2} e}{4\pi d_p\sqrt{e}} = \frac{d_p\sqrt{e}}{4}$$

Let hydraulic mean radius for all pores is 'm' which is taken as characteristic length

$$\therefore m = \frac{d_p\sqrt{e}}{4} x \ number \ of \ pores$$

$$m = \frac{d_p\sqrt{e}}{4}\frac{4A_b(1-n)}{\pi d_p{}^2} = \frac{A_b\sqrt{e}(1-n)}{\pi d_p}$$

Let $m1$ is characteristic length, and Reynold's number is R_e, therefore

$$R_e = \frac{seepage \ velocity(V_s)x \ charateristiclength(m_1)}{kinematic \ viscosity(v)}$$

$$Reynolds \ number = Re = \frac{V_s d_p\sqrt{e}}{4v}$$

Coefficient resistance (lambda) was discussed many investigators 6, as $lambda = \frac{igd_p}{V_b^2}$ taking particle diameter as characteristic length. Intrinsic permeability as characteristic length, which includes only particle property[7]. However, in this study hydraulic mean radius taken as characteristic length. Therefore coefficient of resistance is

$$lambda = \frac{igm}{V_b{}^2} = \frac{ig \ A_b\sqrt{e}(1-n)}{\pi V_b{}^2 d_p}$$

Where,

d_p =diameter of the particle
A_b = Bulk area
V_b = Bulk velocity
e = Void ratio
n = Porosity
i = Hydraulic gradient
g = Acceleration due to gravity

Substituting present experimental data in the derived equation of Re and lambda and the results are plotted between non-dimensional parameter lambda in y-axis and Reynolds number (Re) in x-axis and are shown in Figure 6, Figure 7 and Figure 8 for gravel, sand and glass balls respectively. A logarithmic plot of friction factor verses Reynolds number for various values of roughness depicted by Nikuradse and Moody for pipe flow. Similar type of curves obtained in the present study for porous medium flow. All seven sizes of gravel, five sizes of sand, and three sizes of glass balls aliened as single curve. An equation fitted between lambda and Reynolds numbers for gravel, sand, and glass balls. The equation obtained is

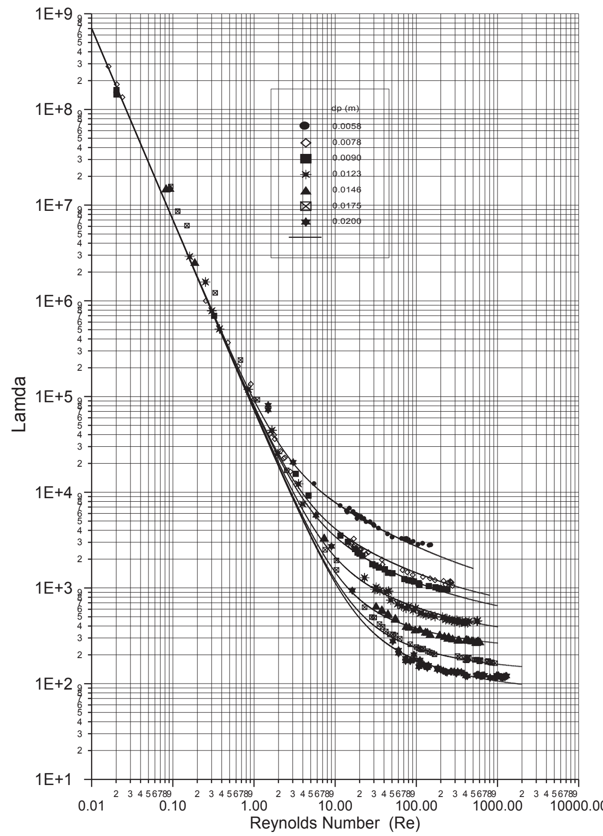

Figure 6. Variation of lambda with Reynolds number for gravel.

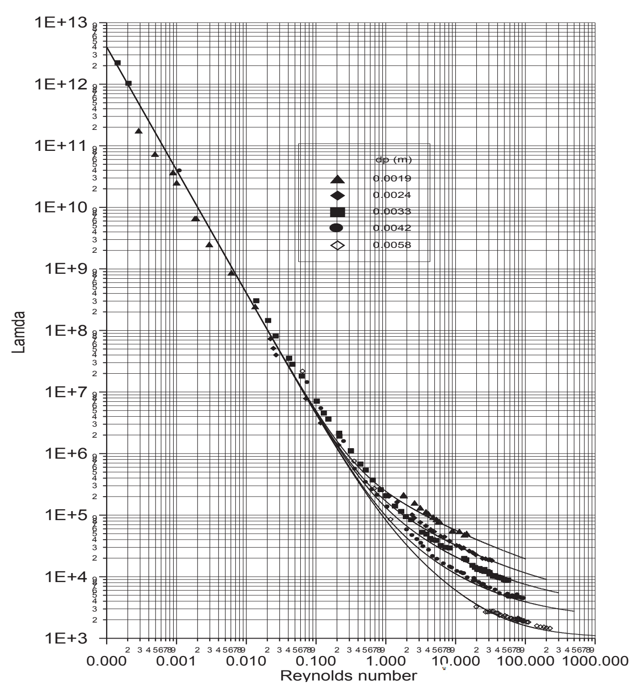

Figure 7. Variation of lambda with Reynolds number for sand.

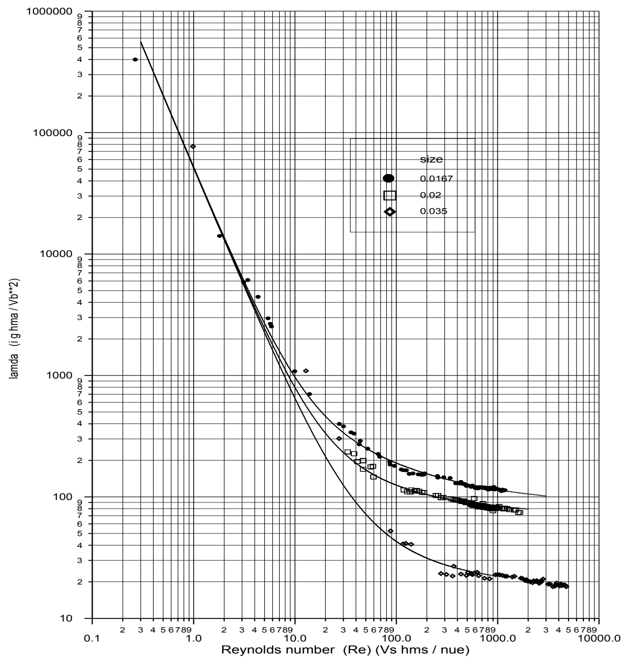

Figure 8. Variation of lambda with Reynolds number for glass balls.

Table 2. Lambda coefficients of turbulent regime (a), laminar (b), pre laminar (c), and constant (d)

gravel				
Dp(mm)	a	b	c	d
5.8	70000	1000	20000	700
7.8	70000	1000	9000	520
9.0	70000	1000	7000	430
12.3	70000	1000	3200	240
14.6	70000	1000	1400	220
17.5	70000	1000	1200	160
20.0	70000	1000	900	130
sand				
Dp(mm)	a	b	c	d
1.7	50000	40000	35000	4000
2.4	50000	40000	30000	3500
3.3	50000	40000	20000	3000
4.2	50000	40000	15000	2000
6.7	50000	40000	2000	1000
Glass balls				
Dp(mm)	a	b	c	d
16.7	50000	1000	900	85
20.0	50000	1000	400	70
35	50000	1000	100	18

$$lambda = \frac{a}{R_e^{\,2}} + \frac{b}{R_e^{\,1}} + \frac{c}{R_e^{\,0.5}} + \frac{d}{R_e^{\,0}}$$

Where,

Re – Reynolds number
a – Turbulent coefficient
b – Laminar coefficient
c – Pre laminar coefficient
d – constant

The coefficients are given in Table 2. Using this unique equation with dimensionless coefficients and substituting measured hydraulic gradient, bulk area, porosity, void ratio, volume diameter of particle, viscosity, the value of bulk velocity can be calculated by trial and error. Multiplying this bulk velocity with bulk area the discharge from any porous medium can be estimated irrespective of regimes of flow for any fluids and particles.

4. Conclusions

In order to achieve the objectives, a lengthy permeameter designed to get wide range of data. Present experimental data, verified with that of the past study. Experiments conducted in wide range of velocity and low hydraulic gradients to analyze all regimes of flow. Validity of Darcy's equation is discussed. The Darcy's coefficients (k) is obtained for gravel. Various regimes of flow that occur in a seepage flow reviewed. The concepts such as characteristic length, resistance coefficients (lambda) discussed. A non-dimensional form of relation between resistant to flow with fluid and particle parameters is derived and presented in the form of equation. Using the coefficients given in Table 2, and substituting measured hydraulic gradient, bulk area, porosity, void ratio, volume diameter of particle, and viscosity, the value of bulk velocity can be calculated by trial and error. Multiplying this bulk velocity with bulk area the discharge from any porous medium can be estimated irrespective of its regimes of flow for any fluid and any particle.

5. Acknowledgements

The financial support of the All India council for Technical Education, vide Reference No.20/AICTE/RIFD/RPS/(policy)34/20/13-14 dated July 18-2013, is appreciated. The authors expresses their sincere gratitude to the Founder & Chairman Dr. R. Rangarajan, Vice Chairman

Dr. Sakunthla Rangarajan, Chairperson & Managing Trustee Dr. Rangarajan Mahalakshmi Kishore and Vice President Sri. KVD. Kishore Kumar, Vel Tech High Tech Dr. Rangarajan Dr. Sakunthala Engineering College, Avadi, Chennai, India, for their constant engorgement and support.

6. References

1. Pranavam DTS, Rao VT, Punithavathi L, Karunanithi S, Bhaskaran A. Groundwater Pollution in the Palar Riverbed near Vellore, Tamil Nadu, India. Indian Journal of Science and Technology. 2011 Jan; 4(1):176–90.
2. Balakrishnan M, Arul Antony S, Gunasekaran S, Natarajan RK. Impact of Dyeing Industrial Effluents on the Groundwater Quality in Kancheepuram (India). Indian Journal of Science and Technology. 2008 Dec; 1(7):1–8.
3. Dupuit AJEJ. Etudes theoriques et pratiques sur le mouvement des eaux, Paris. 1863.
4. Fand RM. Resistance to the flow of fluids through simple and complex porous media whose matrices are composed of randomly packed spheres. Journal of Fluid Engineering, ASME. 1987; 109(3):263–73.
5. Kumar PGN, Venkatraman P. Non-Darcy flow converging flow through coarse granular media. IE(I) Journal. 1995; 76(5):6–11.
6. Thiruvengadam M, Kumar G. Validity of Forchheimer equation in radial flow through coarse granular media. Journal of Engineering Mechanics, ASCE. 1997; 123(7):696–705.
7. Venkatraman P, Rao RMP. Validation of Forchheimer's law for flow through porous media with converging boundaries. Journal of Hydraulic Engineering. 2000; 126(1):63–71.
8. Murali T, Thiruvengadam M, Kumar PGN. A further study on Forchheimer equation as applied to seepage flow. ISH Journal of Hydraulic Engineering. 2004; 10(2):1–13.
9. Krishna RK, Murthy SD. An experimental study on flow of water through porous media in laminar, transition and turbulent regime. International Journal of Emerging Technology and Advanced Engineering. 2013; 3(4):559–69.
10. Henry D. The public fountains of the city of Dijon experience and application principles to follow formulas to be used in the question of the distribution of water, Victor Dalmont: Paris, 1856.
11. Wojciech S, Anna T. Darcy's and Forchheimer's Laws in Practice. Part 1: The Experiment. Technical Sciences. 2014; 17(4):321–35.
12. Shijie L, Jacob HM. Non-linear flows in porous media. Journal of Non-Newtonian Fluid Mechanics. 1999; 86(1-2):229–52.
13. Thiruvengadam M, Kumar PGN, Kumar SD. Validity of Darcy and Forchheimer equations of porous medium flow, Poll Res. 2014; 33(3):511–18.
14. Ahmed N, Sunada DK. Non - Linear flow porous media. Journal of Hydraulic Divsion, ASCE. 1969; 95(6):1847–57.
15. Khalifa AM, Aberahmane OA, Wahyudi I, Thomas P. A new device for measuring permeability under high gradients and sinusoidal gradients. ASTM Geotechnical Testing Journal. 2000; 23(4):404–12.