

Underwater Target Tracking using Unscented Kalman Filter

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Abstract

For bearing and frequency measurements based on Kalman Filters in Sonar noisy situations often create problem. In our paper TMA is used based on Unscented Kalman Filter (UKF) to track target in noisy measurements. Target trajectory is explored through using this technique. The major benefit of this technique is that it provides tactical superiors over the classical bearings techniques overall, observer maneuver is rendered unnecessary. Target motion parameters can be extended for further improvement of their techniques.

Keywords: Kalman Filter, Passive Target Tracking, Target Motion Analysis, Target Motion Parameters, Unscented Kalman Filter

1. Introduction

This project concerns with an underwater Target Motion Analysis (TMA) in the passive mode¹. Here the target is assumed to be moving with a uniform velocity. The sound signals from the target are obtained at the ownship's sensor. These sound signals are processed by the own ship in order to find out the Target Motion Parameters (TMP) which are bearing, range, speed and course of the target².

Passive target tracking (PTT) is a way to find the path of the target exclusively from measurements (signals) originating from the target. Anybody under the water will generate acoustic turbulence or signal from self-generated noise or while in motion. These noise signals from the target are observed by the ownship's solar sensor. The information that is acquired at the own ship sensor is the noisy bearings and frequency measurements of the target. TMA for DBT, so far carried out does not provide accurate solution³. If the measurements are linear then the estimation of parameters can be done by Kalman filter. But in real world, no linear systems will exist. Though the Extended Kalman filter is used for nonlinear

application in contrast only achieves first order accuracy. So, EKF is not used for high nonlinearity systems.

So, in this paper the Unscented Kalman Filter is used to track the target by estimating the Target Motion Parameters from the noisy bearing and frequency measurements. Thus, no bias will be produced. Here, assuming that the measurements are available from the towed array of the own ship⁵. Hydrophones of the sonar are towed at the back of the ownship to decrease the effect of noise generated by itself. The zero mean Gaussian noise is assumed to be present in the measurements and the noise in the frequency measurement is uncorrelated with that of bearing measurement.

2. Mathematical Modelling

2.1 Motion of the Ownship(O)

The ownship motion is carried out as shown in the Figure 1. Assuming, the ownship (observer) is moving with a velocity v_o , separation distance between x-axis and the ownship is x_o , separation distance from y-axis to the ownship is y_o and Ownship Course (OCR) is the angle made by the ownship w.r.t. true north⁶.

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$$x_0 = \sin(ocr) * v_0 \tag{1}$$

$$y_0 = \cos(ocr) * v_0 \tag{2}$$

After every second, the change in the ownship's X and Y component are added to the previous ownship's position.

For t=1 sec

$$dx_0 = v_0 * \sin(ocr) * t \tag{3}$$

$$dy_0 = v_0 * \cos(ocr) * t \tag{4}$$

Where, dx_0 - alteration in the X component of the ownship,

dy_0 - alteration in Y component of the ownship in 1 sec,

v_0 is the ownship's velocity,

Ocr is the ownship's course,

(x_0, y_0) is the ownship's coordinates.

So,

$$x_t = x_0 + dx_0 \tag{5}$$

$$y_t = y_0 + dy_0$$

2.2. Initial Position of the Target

The target is denoted by T and is assumed to be moving with a unvarying velocity as follows;

$$X_t = R \sin(B) \tag{6}$$

$$Y_t = R \cos(B) \tag{7}$$

Where, (X_t, Y_t) is the location of the target with respect to ownship as origin.

2.3. Modeling for States

The measured data consists of two sets.

1. Frequency and 2. Bearing measurements.

$M(i)$ is the measurement vector and is given by

$$M(i) = \begin{bmatrix} B_{meas}(i) \\ F_{meas}(i) \end{bmatrix} \tag{8}$$

The measured bearing is given as

$$B_{meas}(i) = B_A(i) + y_B(i) \tag{9}$$

Where, the measured bearing is denoted as $B_{max}(i)$, relative to the Y-axis at i^{th} instant ($i = 1, 2, \dots, n$),

$B_A(i)$ is the actual bearing and

$Y_B(i)$ zero mean and variance $\sigma_B^2(i)$ Gaussian Random Variable (GRV). $B_A(i)$ is given by

$$\tan B_A(i) = \frac{R_{xc}(i)}{R_{yc}(i)} \tag{10}$$

Generally, more than one tonals are found in the sonar broadband noises. The ownship's measured frequency is given by

$$f_{meas}^{(j)}(i) = f_{sour}^{(j)}(i) \left(1 + \frac{\dot{R}_{xc}(i) \sin B_A(i) + \dot{R}_{yc}(i) \cos B_A(i)}{C} \right) + \gamma_{freq}^{(j)}(i) \tag{11}$$

Where,

The j^{th} measured frequency obtained at the ownship at i^{th} instant is $f_{meas}^{(j)}(i)$,

The j^{th} unknown source frequency is $f_{sour}^{(j)}$, which is constant,

The speed of propagation of the signal is C ,

$\gamma_{freq}^{(j)}(i)$ is the zero mean Gaussian random measurement error of the frequency with variance σ_{freq}^2

\dot{R}_{xc} and \dot{R}_{yc} are components of relative velocity between the target and the ownship.

Constant state vector formulation¹ (due to the increase in number of tonals the state vector's size does not increase) are obtained as follows.

Let

$$\begin{aligned} F_{meas}(i) &= \sum_{j=1}^n f_{meas}^{(j)}(i) \\ F_{sour}(i) &= \sum_{j=1}^n f_{sour}^{(j)}(i) \\ \gamma'_{freq}(i) &= \sum_{j=1}^n \gamma_{freq}^{(j)}(i) \end{aligned} \tag{12}$$

$$F_{meas}(i) = F_{sour}(i) \left(1 + \frac{\dot{R}_{xc}(i) \sin B_A(i) + \dot{R}_{yc}(i) \cos B_A(i)}{C} \right) + \gamma'_{freq}(i) \tag{13}$$

In general the main tracking problem, to estimate the state vector $Z_s(i)$, from a set of measurements $M(i) = [M(1) M(2) \dots M(i)]^T$. $Z_s(i)$ is defined to have the specified state vector

$$z_s(i) = [\dot{x} \quad \dot{y} \quad R_{xc} \quad R_{yc} \quad F_{sour} \quad \omega_x \quad \omega_y \quad \omega_f \quad \sigma_b \quad \sigma_{freq}]^T \tag{14}$$

Where (R_{xc}, R_{yc}) denote the relative range components between the target and the ownship. F_{sour} is source frequency and superscript T denotes transpose. $\omega_x(i)$, $\omega_y(i)$ and ω_f are the $\omega_f(i)$ disturbances in x component of acceleration, y component of acceleration and frequency measurement respectively. The ownship state is similarly defined as

$$z_0 = [x_0 \quad y_0 \quad \dot{x}_0 \quad \dot{y}_0]^T \tag{15}$$

Let us assume that the noise in the bearing and frequency measurements are uncorrelated. The state dynamic equation of the target is given by

where $\Psi(i + 1, i)$ is the transition matrix and $d(i + 1)$ is the deterministic vector. The transition matrix can be written as

$$\Psi(i + 1, i) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & t & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & t & 0 & 0 & 0 \\ t & 0 & 1 & 0 & 0 & \frac{t^2}{2} & 0 & 0 & 0 & 0 \\ 0 & t & 0 & 1 & 0 & 0 & \frac{t^2}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (17)$$

where the sample time is specified as ‘t’ and $d(i + 1)$ is given by

$$d(i + 1) = [0 \ 0 \ -(x(i + 1) - x(i)) \ -(y(i + 1) - y(i)) \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T \quad (18)$$

2.4. Algorithm for Unscented Kalman Filter

Consider a random variable z (dimension L_1) propagating through a nonlinear function, $y = O(z)$. Assume z has mean \bar{z} and covariance P_z . To compute the statistics of y a matrix χ of $2L_1 + 1$ sigma vectors χ_j (with corresponding weights W_j), is formed according by the following equation (7):

$$\begin{aligned} \chi_0 &= \bar{z} \\ \chi_j &= \bar{z} + \left(\sqrt{(L_1 + \delta) P_z} \right)_j, \quad j = 1, \dots, L_1 \\ \chi_j &= \bar{z} - \left(\sqrt{(L_1 + \delta) P_z} \right)_{j-L_1}, \quad j = L_1 + 1, \dots, 2L_1 \\ W_0^{(m)} &= \delta / (L_1 + \delta) \\ W_0^{(c)} &= \delta / (L_1 + \delta) + (1 - \rho^2 + \xi) \\ W_j^{(m)} = W_j^{(c)} &= 1/2(L_1 + \delta), \quad j = 1, \dots, 2L_1 \end{aligned} \quad (19)$$

Where $\lambda = \vartheta^2 (L_1 + k) - L_1$ is a scaling parameter. The spread of the sigma points is determined by ϑ around \bar{z} and is generally set to a very small positive value, secondary scaling parameter ‘k’ which is generally set to 0 and ξ is used to incorporate prior information of the distribution of z ($\xi = 2$ which is optimal for Gaussian distributions). $\left(\sqrt{(L_1 + \delta) P_z} \right)_j$ is the j^{th} row of the matrix square root. $W_0^{(m)}$, $W_0^{(c)}$, $W^{(m)}$ and $W^{(c)}$ are the weights of initialized state vector of the target, covariance matrix of initial state vector of the target, sigma point vector of the target’s state and covariance matrix of sigma point vector of the target’s state respectively⁸. Propagate these sigma vectors through the non linear function

$$y_j = O(\chi_j), \quad j = 1, \dots, 2L_1 \quad (20)$$

Weighted sample covariance and mean of the posterior sigma points are used to approximate the mean and covariance⁹.

In UKF, the random variable of the state is redefined as the combination of the original state along with noise variables. Sigma point selection scheme in the unscented transformation is used to obtain the new improved random variable of the state. These are used in the calculation of sigma matrices correspondingly¹⁰.

The standard UKF implementation consists of the following steps:

- The first step is to generate $2n+1$ sigma points.
- Then calculate the weights of each sigma point.
- Sigma point propagation: Next each point through nonlinear function is instantiated, and then compute the transformed sigma points.
- Prediction of state and measurement: Calculation of weighted mean and covariance is done for the transformed points.
- Updating of State and covariance: Updating of the state and covariance by using the measurement is done.
- Repeat the same procedure for estimating the next state.

3. Simulation and Results

This algorithm is implemented for different scenario. Considering, simulation period as 1800s and the measurement interval as 1s. Due to the zero mean Gaussian noise the raw frequency and bearing measurements corrupts up to a very small value. All the angles are calculated with respect to Y-axis, 0-360° and clockwise positive.

Here in this paper, the result of one scenario is implemented by using values in the Table-1. The results of one scenario are shown in figures. The position of the target along with the ownship are shown in the Figure-3. Plots of the error in Range, Speed, Bearing and Course estimates are shown below in Figure-4, 5, 6, 7 respectively. Plot of Time versus Target’s Velocity is given in the Figure-7. Plot of Time versus Target’s Course is shown in Figure-9. In underwater applications the acceptable errors in estimated speed, course and range are less than or equal to 19%, 4.9° and 9.9% respectively. As per the required accuracies the entire solution of the estimated

parameters speed, course and range are obtained at 75th, 60th and 180th s for the scenario values given above.

Table 1. Simulation Table

velocity of the target	-	14 knots
velocity of the ownship	-	14 knots
Course of the target	-	300 (deg)
Course of the ownship	-	325 (deg)
Range	-	3800 (mts)
Bearing	-	5 (deg)
Sigmab	-	0.17(deg)
Sigmaf	-	0.33(deg)
Frequency	-	800 Hz

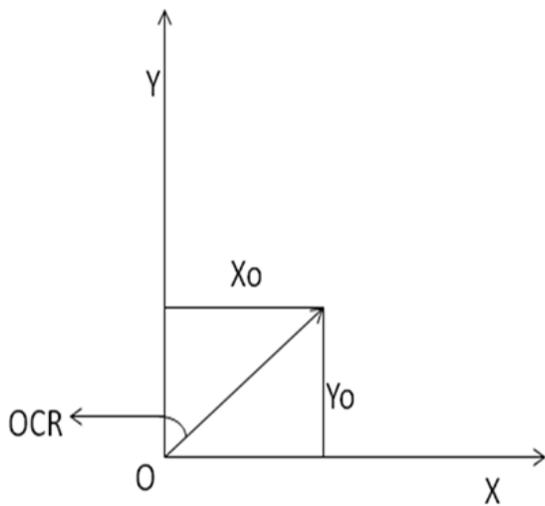
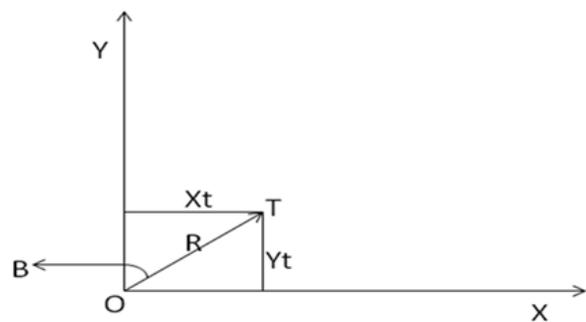


Figure 1. Own Ship's Motion in the Coordinate Axis.



Where,
 O is the observer or ownship,
 T is the target,
 R is range,
 B is bearing i.e. angle between ownship (O) and target (T)

Figure 2. Target's Position in the Coordinate Axis.

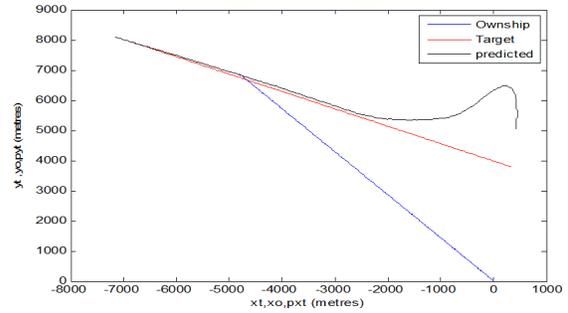


Figure 3. Plot of target Verses Ownship's Position.

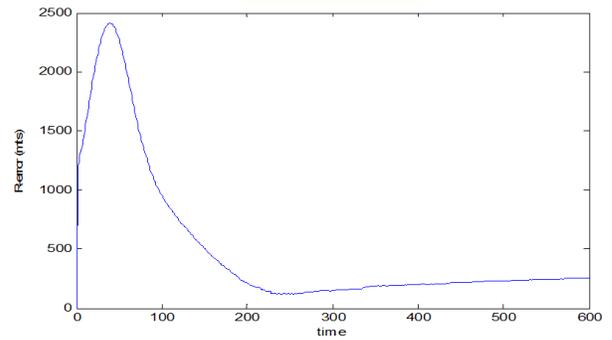


Figure 4. Plot of Error in Range Estimate.

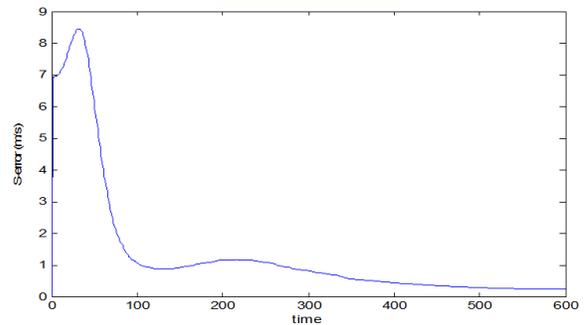


Figure 5. Plot of Error in Speed Estimate.

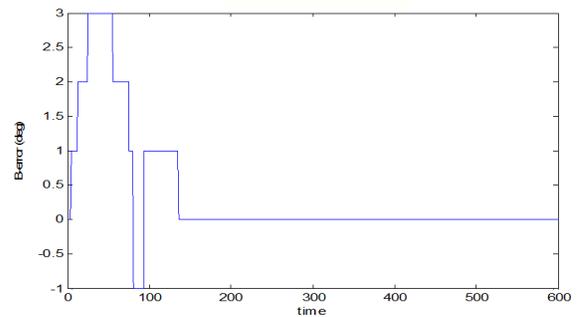


Figure 6. Plot of Error in Bearing Estimate.

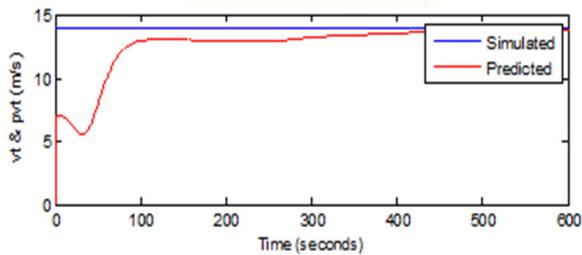


Figure 8. Plot of Time versus Target's Velocity.

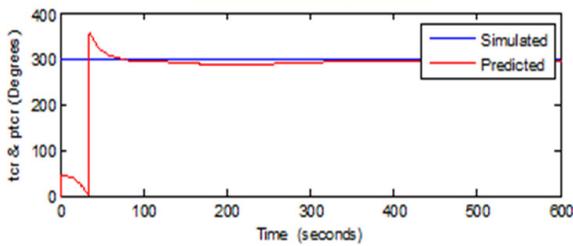


Figure 9. Plot of Time versus Target's Course.

4. Conclusion

Now a days there are many methods for obtaining the target trajectory by using sonar in passive mode which is in the ownship without manoeuvring. But accuracy is not achieved in the estimated target motion parameters. So, one can use DBT for obtaining the target motion parameters, observer maneuver is rendered unnecessary. This method is very easy to implement and adopt in underwater application for passive target tracking. Here Unscented Kalman filter is proposed to estimate target motion parameters and makes the ownship maneuver unnecessary.

5. References

1. Chan YT, Rudnicki SW. Bearings-only and Doppler-bearing tracking using instrumental variables. *IEEE Trans Aerosp Electron Syst.* 1992 Oct; 28(4): 1076–83.
2. Tao XJ, Zou CR, He ZY. Passive target tracking using maximum likelihood estimation. *IEEE Trans Aerosp Electron Syst.* 1996 Oct; 32(4):1348–54.
3. Ho KC, Chan YT. An asymptotically unbiased estimator for bearings-only and Doppler-bearing target motion analysis. *IEEE Tran on Signal Processing.* 2006 Mar; 54(3): 809–22.
4. Wan EA, Merwe RVD. The unscented Kalman filter for nonlinear estimation. *Proceedings of IEEE Symposium 2000 on Adaptive Systems for Signal Processing, Communication and Control*; Alberta, Canada. US: IEEE Press; 2000. p. 153–8.
5. Rao S K. Application of parameterized modified gain bearings-only extended Kalman filter for passive torpedo tracking communicated to IET Radar, Sonar and Navigation. 2009 Sep.
6. Poor HV. *An introduction to signal detection and estimation.* Berlin, Germany: Springer Science and Business Media; 2009.
7. Bar-Shalom Y. *Multitarget-multisensor tracking: advanced applications.* Norwood, MA: Artech House; 1990.
8. Atlas N, Gupt S. Reduction of speckle noise in ultra sound images using various filtering techniques and discrete wavelet transform: Comparative Analysis. *IJR.* 2014; 3(5):68–71.
9. Lu H, Yamawaki A, Serikawa S. Curvelet approach for deep-sea sonar image denoising. *Contrast Enhancement and Fusion. Journal of International Council on Electrical Engineering.* 2013; 3(3):250–6.
10. Padmavathi G, Subashini P, Kumar M M, Thakur S K. Comparison of filters used for underwater image pre-processing. *IJCSNS.* 2010;10(1):58–65.