# Dynamic Modeling of Scorbot-ER Vu Plus Industrial Robot Manipulator using LabVIEW 

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#### Abstract

Dynamics deals with not only forces but also torques necessary to lead to the motion of a system of bodies. The robot manipulators can also be approached from two different points of view: Dynamical analysis and dynamical synthesis. Dynamical analysis handles the equations of motion derivation of a specified manipulator. Two types of dynamical analysis problems exist: Direct dynamics and inverse dynamics. The computational efficiency of direct dynamics is not much as critical because it is utilized chiefly for computer simulations of a robot manipulator. Moreover, a competent inverse dynamic model turns into tremendously significant for real-time, model dependent control of a robot manipulator. A set of torque and/or force functions have to be functional at the actuated joints in the idea of generating that motion. These actuating torque and/or force functions rely not only on the spatial and attributes of the given path but also on the mass properties of the links, the pay load, the outwardly applied forces and so on. This paper deals with the dynamic analysis of a SCORBOT-ER V plus robot arm that is involved in performing victorious robotic manipulation task in its 3D workspace. This research considered mathematical dynamic model based on Lagrangian equations of motion and the result is compared with LabVIEW Software.


Keywords: Direct Dynamics, Inverse Dynamics, Lagrangien Equation, LabVIEW, Robot Manipulator

## 1. Introduction

The speedy modernization of the manufacturing equipment's makes that the industrial systems turn to rise up much intricate and highly complicated. In specific, the robots usage of robots in current manufacturing industries has been extremely shoot up throughout these years. Robot technology was implemented in numerous aspects such as agriculture, medical, family service and industry etc. During motion, a manipulator must accelerate, move at constant speed and decelerate. This time varying position and orientation of the manipulator is termed as its dynamic behavior. Robot arm dynamics are handled by the mathematical formulations of the
equations of robot arm motion. These equations describe the dynamic behavior of the manipulator and correlate the robot motion to the forces/torques involved. Such equations are helpful for Computer simulation of the robot motion, appropriate control equations design for a robot arm and assessment of the kinematic design and a robot arm structure (Figure 1). Robot manipulators dynamics is extremely nonlinear that complicates their efficient control. Classical control methods have been known much; on the other hand, strong nonlinearities end up in their inadequacy. However, superior results have been achieved by nonlinear controllers but the nonlinear analysis and design is much systematic and apparent similar to the linear case.

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## 2. Previous Research

Dynamic analysis of manipulation systems has fascinated comparatively not as much of concentration till now and is partially reasonable by the fact that utmost cooperative manipulation tasks are much slower for rendering negligible dynamic effects. Even though dynamics might not play a leading role in the functioning of slow supportive manipulation tasks, only a complete dynamical model could detail and explain the structural properties of complex manipulation systems is true. Hence dynamic manipulation was well thought-out to examine grasp constancy for studying the dualities among parallel and series manipulators ${ }^{1,2}$. Characteristics and properties of the dynamic equations of motion that is useful for control purposes.

Algorithms have been brought up for the highly widespread calculations for analysing, control and simulation of robot. The typical technique to express the equations of motion has been dependent based on a Lagrangian formulation of the trouble. Algorithms built up using Lagrangian dynamics and wasmodified to control during real-time ${ }^{3,4}$.

The objective of manipulator control is for maintaining the dynamic retort of a (computer-based) manipulator in accordance with particular system functioning and preferred objective. Each link of the manipulator is treated as a rigid body and the dynamic equations of motion could be determined utilizing the specified geometrical and inertial parameters of all the links ${ }^{5,6}$. As we have already seen in kinematics, in dynamics also there exits two classes of problems, namely, forward and inverse problems. In the first type of problem given the joint torques/ forces the joint accelerations are determined which are then integrated to obtain joint velocities and coordinates ${ }^{7,8,9}$. In the inverse problem, on the other hand given joint coordinates and their first and second order time-derivatives the required joint torques/forces are determined. Time varying torques is applied at the joints to balance out the internal and external forces. Inertial, Coriolis and frictional forces are some of internal forces ${ }^{10,11}$.

Lagrange-Euler equations, that briefs the mechanical system evolution subject to holonomic constraints. In the idea of determining the Lagrange-Euler equations in a particular position Lagrangian for the system has to be formed, that is the dissimilarity among the kinetic energy and the potential energy ${ }^{12,13}$. The Lagrange-Euler
equations possess numerous significant properties which can be oppressed for designing and analyzing the feedback control algorithms. Within these are open bounds on the inertia matrix, linearity in the inertia parameters, and hence named skew symmetry and passivity properties ${ }^{14,155}$.

In this paper, the mathematical model for dynamic behavior of the manipulator is developed. The mathematical equation often referred as manipulator dynamics, are set of equations of motion which describe the dynamic answer of the manipulator in order to input actuator torques. The manipulator dynamic model is helpful to compute torque and forces necessary for implementation of distinctive work cycle that seems to be the imperative information to design links, drives, actuators and joints. The dynamic equation of motion for five axis robot can be rather complex. Hence dynamic equation of motion for five axes Scorbot-ER Vu plus is derived using Lagrange-Euler formulation that is dependent on the idea of generalized coordinates, force and energy.


Figure 1. Manipulator Arm.

## 3. Lagrange-Euler Formulation

Complex dynamic system could be modeled in a comparatively effortless, stylish manner utilizing a technique called the Lagrangien formulation. LagrangeEuler formulation is computationally inefficient so is difficult to use for real time control purposes. However, this formulation provides explicit state equations which could be involved for analyzing and designing
superior joint-variables space control strategies. An attempt to make it computationally more efficient has been successful only at the cost of losing its structure convenient for control strategy. Lagrangien formulation is based on the motion of generalized forces and energy. Let T and U symbolize the potential and kinetic energy of the arm correspondingly. Then Lagrangien function has been defined as the dissimilarity among potential and kinetic energy as follows.
$\mathrm{L}(\mathrm{q}, \mathrm{q})-\Delta \mathrm{T}(\mathrm{q}, \mathrm{q})-\mathrm{U}(\mathrm{q})$
Where,
T (q, q) - Kinetic Energy, U (q) - Potential Energy
Kinetic energy relies not only on position but also on arm velocity, while potential energy relies only on arm position. The common equation of motion of a robotic arm could be formulated in terms of the Lagrangein function as follows:
$\partial / \partial t L(q, q)-\partial / \partial q i L(q, q)=F i$

Dynamic equation of motion of $n$ degrees of freedom is mentioned as follows,
$\tau_{\mathrm{i}}=\sum_{t_{i}=j=1}^{n} M_{i j}(q)\left(\left[(q]_{j}\right)^{-}\right)+, \sum_{j, k=1}^{n} h_{i j k}\left(\dot{q}_{j}\right) \dot{q}_{k}+G_{i}$ for $j=1,2, \ldots n$

Where
$\tau_{\mathrm{i}}$ - Generalized force applied at joint i due to motion of link.
$M_{i j} M_{i j}, \boldsymbol{h}_{i j k} \boldsymbol{h}_{i j k}, \mathrm{G}_{\mathrm{i}}-$ Coefficient functions of joint displacements $q_{1}, q_{2}, \ldots \ldots, q_{n}$. When external forces works on the robot system, the left-hand side of the equation has to be altered correspondingly.

### 3.1 Inertia Tensor

Inertia Tensor $\mathrm{I}_{\mathrm{i}}=$

$$
I_{i}=\left[\begin{array}{cccc}
-I_{X X}+I_{Y Y}+\frac{I_{Z Z}}{2} & I_{X Y} & I_{X Z} & M_{i} \bar{x}  \tag{4}\\
I_{X Y} & I_{X X}+I_{Y Y}+\frac{I_{Z Z}}{2} & I_{Y Z} & M_{i} \bar{y} \\
I_{X Z} & I_{Y Z} & I_{X X}+I_{Y Y}-\frac{I_{Z Z}}{2} & M_{i} \bar{z} \\
M_{i} \bar{x} & M_{i} \bar{y} & M_{i} \bar{z} & M_{i}
\end{array}\right]
$$

Where
$(\bar{x}, \bar{y}, \bar{z})$ are the coordinates of the centre of mass of the ith link in its own coordinate frame.

### 3.2 Inertia Loading of Actuator

$\mathrm{M}_{\mathrm{ij}}=\sum_{P=\max (i, j, k)}^{n} T_{r}\left[d_{p k} I_{p} d_{p j} T\right]$ (or) $\sum_{P=\max (i, j, k)}^{n} T_{r}\left[u_{p k} I_{p} u_{p j} T\right]$

### 3.3 Coriolis Force Coefficient

$\mathrm{h}_{\mathrm{ij} \mathrm{k}}=\sum_{P=\max (i, j, k)}^{n} T_{r}\left[\frac{\partial(d p j)}{\partial q j} I_{p} d_{p i} T\right](\text { or })_{P=\max (i, j, k)}^{n} T_{r}\left[\frac{\partial(U p j)}{\partial q k} I_{p} u_{p i} T\right]$

Where 'Tr' represents trace operator

### 3.4 Loading due to Gravity

$$
\begin{equation*}
G_{i}=\sum_{p=i}^{n} M_{p} \mathrm{~g} \mathrm{~d}_{\mathrm{pi}} \mathrm{p}_{\mathrm{r}_{\mathrm{p}}} \text { (or) } \sum_{p=i}^{n} M_{p} \mathrm{~g}_{\mathrm{pi}} \mathrm{p}_{\mathrm{r}_{\mathrm{p}}} \tag{7}
\end{equation*}
$$



Figure 2. DH Conventions for frame assigning.
Table 1. D-H Parameter for Scorbot ER-Vu Plus Robot

| Joint $\mathbf{i}$ | $\boldsymbol{a}_{\mathrm{i}}$ | $\mathbf{a}_{\mathbf{i}}$ | $\mathbf{d}_{\mathrm{i}}$ | $\theta_{\mathrm{i}}$ |
| :--- | :---: | :---: | :---: | :---: |
| 1 | $-\pi / 2$ | $\mathrm{a}_{1}=10$ | $\mathrm{~d}_{1}=5$ | $\theta_{1}=30^{\circ}$ |
| 2 | 0 | $\mathrm{a}_{2}=15$ | $\mathrm{~d}_{2}=0$ | $\theta_{2}=45^{\circ}$ |
| 3 | 0 | $\mathrm{a}_{3}=20$ | $\mathrm{~d}_{3}=0$ | $\theta_{3}=60^{\circ}$ |
| 4 | $-\pi / 2$ | $\mathrm{a}_{4}=0$ | $\mathrm{~d}_{4}=0$ | $\theta_{4}=50^{\circ}$ |
| 5 | 0 | $\mathrm{a}_{5}=0$ | $\mathrm{~d}_{5}=5$ | $\theta_{5}=70^{\circ}$ |

$\alpha_{i}=$ Rotation angle about $x_{i}$ axis
$\mathrm{a}_{\mathrm{i}}=$ Translation distance along $\mathrm{x}_{\mathrm{i}}$ axis
$d_{i}=$ Translation distance along $z_{i-1}$ axis
$\theta_{\mathrm{i}}=$ Rotation angle about $\mathrm{z}_{\mathrm{i}-1}$ axis

Table 2. Data regarding Scorbot ER-Vu Plus Robot

| Link | $\mathbf{1}$ | 2 | 3 | 4 | 5 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{I}_{\mathrm{XX}}$ | 866.66 | 799.976 | 799.978 | 666.66 | 8796.069 |
| $\mathrm{I}_{\mathrm{YY}}$ | 1616.66 | 17499.667 | 12499.66 | 666.66 | 8796.069 |
| $\mathrm{I}_{\mathrm{ZZ}}$ | 1950 | 17499.667 | 12499.66 | 266.66 | 186.667 |
| $\mathrm{I}_{\mathrm{XY}}$ | 674.99 | 0 | 0 | 0 | 0 |
| $\mathrm{I}_{\mathrm{XZ}}$ | 0 | 0 | 0 | 0 | 0 |
| $\mathrm{I}_{\mathrm{YZ}}$ | 0 | 0 | 0 | 0 | 0 |
| $\bar{x}$ | -22.5 | -110.0 | -110.0 | 0 | 0 |
| $\bar{y}$ | 0 | 0 | 0 | 0 | 0 |
| $\bar{z}$ | 0 | 0 | 0 | 0 | -65.0 |
| Mass,m | 2 | 3 | 3 | 1 | 0.7 |

Table 2. Data regarding Scorbot ER-Vu Plus Robot
$\mathrm{I}_{\mathrm{XX},} \mathrm{I}_{\mathrm{YY},} \mathrm{I}_{\mathrm{ZZ}}, \mathrm{I}_{\mathrm{XY},} \mathrm{I}_{\mathrm{XZ},} \mathrm{I}_{\mathrm{YZ}}=$ Moment of Inertia for various axis
$\bar{x}, \bar{y}, \bar{z}=$ Centre of Gravity for different axis
Velocity (V) $\Theta=[20,20,20,20,20]^{\mathrm{T}}$
Acceleration $\ddot{\Theta}=[10,10,10,1010]^{\mathrm{T}}$
Location of the centre of mass of link ' 1 ' from the origin of the base co-ordinate frame, $\left({ }^{\mathrm{i}} \mathrm{r}_{\mathrm{i}}\right)=[4,5.18,7.23$, 7.73, 7.06] ${ }^{\mathrm{T}}$

### 3.5 Transformation Matrices

If a joint i is revolute, the general form of ${ }^{-1} \mathrm{~A}_{\mathrm{i}}$ is as follows, ${ }^{i-1} A_{i}=\left[\begin{array}{cccc}\operatorname{Cos} \theta i & -\operatorname{Cos} \alpha i \operatorname{Sin} \theta i & \operatorname{Sin\alpha } i-\operatorname{Sin} \theta i & \text { aiCos} \theta i \\ \operatorname{Sin} \theta i & \operatorname{Cos} \alpha i \operatorname{Cos} \theta i & -\operatorname{Sin\alpha i\operatorname {Cos}\theta i} & \text { aiSin} \theta i \\ 0 & \operatorname{Sin} \alpha i & \operatorname{Cos} \alpha i & d i \\ 0 & 0 & 0 & 1\end{array}\right]$

To derive the equations of motion applicable to revolute joints, the variable $q_{i}$ is used to represent the generalized coordinate of joint i which is $\theta i$.

Q matrix if the $\mathrm{i}^{\text {th }}$ joint is revolute
$Q_{i}=\left[\begin{array}{cccc}0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0\end{array}\right]$
$U_{p j k}=\frac{\partial(U p j)}{\partial q j}\left\{\begin{array}{c}d_{i j}=U_{i j}={ }^{0} A_{j-1} Q_{j}^{j-1} A_{i} ; \text { for } j \leq I 0 ; \text { for } j>i \\ ={ }^{0} A_{j-1} Q_{j}^{j-1} A_{k-1} Q_{k}^{k-1} A_{p} \quad ; \quad \text { for } j \leq k \leq p\end{array}\right.$
$d_{p \mathrm{p} k}=\frac{\partial(\mathrm{dpj})}{\partial \mathrm{qj}}\left\{\begin{array}{c}==^{0} \mathrm{~A}_{\mathrm{k}-1} \mathrm{Q}_{\mathrm{k}}{ }^{k-1} \mathrm{~A}_{\mathrm{j}-1} \mathrm{Q}_{\mathrm{j}}^{j-1} \mathrm{~A}_{\mathrm{p}} \quad ; \quad \text { for } \mathrm{k} \leq \mathrm{j} \leq \mathrm{p} \\ 0 ; \quad \text { for } \mathrm{k}>\mathrm{p} \text { (or) } \mathrm{j}>\mathrm{p}\end{array}\right.$

Where $\mathrm{p}=\max (\mathrm{i}, \mathrm{j}, \mathrm{k})$

## 4. Dynamic Model of Scorbot-Er V plus Robot

$$
\begin{equation*}
\mathrm{T}=\mathrm{M} \Theta \ddot{\theta}+\mathrm{H}+\mathrm{G} \tag{10}
\end{equation*}
$$

$$
M=\left[\begin{array}{lllll}
M_{11} & M_{12} & M_{13} & M_{14} & M_{15}  \tag{11}\\
M_{12} & M_{22} & M_{23} & M_{24} & M_{25} \\
M_{13} & M_{23} & M_{33} & M_{34} & M_{35} \\
M_{14} & M_{24} & M_{34} & M_{44} & M_{45} \\
M_{15} & M_{25} & M_{35} & M_{45} & M_{55}
\end{array}\right]
$$

$$
H=\left[\begin{array}{l}
H_{1}  \tag{12}\\
H_{2} \\
H_{3} \\
H_{4} \\
H_{5}
\end{array}\right]
$$

$$
G=\left[\begin{array}{l}
G_{1}  \tag{13}\\
G_{2} \\
G_{3} \\
G_{4} \\
G_{5}
\end{array}\right]
$$

## 5. Transformation Matrices

The kinematic model with frame assignment is shown in Figure 2, as per the Denavit-Hardenberg (D-H) notations. Kinematic parameters as per this model have been mentioned in Table 1. From the D-H Parameter for SCORBOT ER-Vu Plus table values; we can find the various transformation matrices.
${ }^{0} A_{0}={ }^{1} A_{1}={ }^{2} A_{2}={ }^{3} A_{3}={ }^{4} A_{4}={ }^{5} A_{5}=\left[\begin{array}{cccc}1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1\end{array}\right]$ for $\Theta=0^{\circ}, \alpha_{i}=0, a_{i}=0, d_{i}=0$
${ }^{0} \mathrm{~A}_{1}=\left[\begin{array}{cccc}0.866 & 0 & -0.5 & 8.660 \\ 0.5 & 0 & 0.866 & 5 \\ 0 & -1 & 0 & 5 \\ 0 & 0 & 0 & 1\end{array}\right]$ for $\Theta_{1}=30^{\circ}, \alpha_{1}=\pi / 2, \mathrm{a}_{1}=10, \mathrm{~d}_{1}=5$
${ }^{1} \mathrm{~A}_{2}=\left[\begin{array}{cccc}0.707 & -0.707 & 0 & 10.606 \\ 0.707 & 0.707 & 0 & 10.606 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1\end{array}\right]$ for $\Theta_{2}=45^{\circ}, \alpha_{2}=0, \mathrm{a}_{2}=15, \mathrm{~d}_{2}=0$
${ }^{2} \mathrm{~A}_{3}=\left[\begin{array}{cccc}0.5 & -8.666 & 0 & 10 \\ 0.866 & 0.5 & 0 & 10 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1\end{array}\right]$ for $\Theta_{3}=60^{\circ}, \alpha_{3}=0, \mathrm{a}_{3}=20, \mathrm{~d}_{3}=0$
${ }^{3} \mathrm{~A}_{4}=\left[\begin{array}{cccc}0.642 & 0 & -0.766 & 0 \\ 0.766 & 0 & 0.642 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1\end{array}\right]$ for $\Theta_{4}=50^{\circ}, \alpha_{4}=-\pi / 2, \mathrm{a}_{4}=0, \mathrm{~d}_{4}=0$
${ }^{4} \mathrm{~A}_{5}=\left[\begin{array}{cccc}0.342 & -0.939 & 0 & 0 \\ 0.939 & 0.342 & 0 & 0 \\ 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 1\end{array}\right]$ for $\Theta_{5}=70^{\circ}, \alpha_{5}=0, \mathrm{a}_{5}=0, \mathrm{~d}_{5}=5$
$\mathrm{Q}_{1}=\mathrm{Q}_{2}=\mathrm{Q}_{3}=\mathrm{Q}_{4}=\mathrm{Q}_{5}=\left[\begin{array}{cccc}0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0\end{array}\right]$
Similarly we can find the other transformation matrices are,
${ }^{0} \mathrm{~A}_{2}=\left[\begin{array}{cccc}0.612 & -0.612 & -0.5 & 17.844 \\ 0.353 & -0.353 & 0.866 & 10.303 \\ -0.707 & -0.707 & 0 & -5.606 \\ 0 & 0 & 0 & 1\end{array}\right]$
${ }^{0} \mathrm{~A}_{3}=\left[\begin{array}{cccc}-0.223 & -0.835 & -0.5 & 13.370 \\ -0.129 & -0.482 & 0.866 & 7.722 \\ -0.965 & 0.258 & 0 & -24.914 \\ 0 & 0 & 0 & 1\end{array}\right]$

$$
\begin{aligned}
& { }^{0} \mathrm{~A}_{4}=\left[\begin{array}{cccc}
-0.782 & 0.5 & -0.365 & 13.370 \\
-0.452 & -0.866 & -0.210 & 7.722 \\
-0.421 & 0 & 0.904 & -24.914 \\
0 & 0 & 0 & 1
\end{array}\right] \\
& { }^{0} \mathrm{~A}_{5}
\end{aligned}{ }^{1} \mathrm{~A}^{1} \mathrm{~A}_{3}=\left[\begin{array}{ccccc}
0.202 & 0.905 & -0.365 & 11.545 \\
-0.967 & -0.128 & -0.210 & 6.672 \\
-0.143 & 0.395 & 0.904 & -20.394 \\
0 & 0 & 0 & 1
\end{array}\right]
$$

## 6. Inertia Tensor

Rotational motions have been detailed by Euler`s equations. Similarly for translational motions, summing "Inertia Torque" to the static balance of moments give way to the dynamic equations. Here, the single body mass properties corresponding to rotations about the centroid has been detailed. The mass properties have been symbolized by an inertia tensor, or an inertia matrix that is a $4 \times 4$ symmetry matrix.

From Table 2, substitute all values in Equation 4, we can find $\mathrm{I}_{1}, \mathrm{I}_{2}, \mathrm{I}_{3}, \mathrm{I}_{4}$, and $\mathrm{I}_{5}$
$I_{1}=\left[\begin{array}{cccc}1350 & 675 & 0 & -45 \\ 675 & 600 & 0 & 0 \\ 0 & 0 & 266.7 & 0 \\ -45 & 0 & 0 & 2\end{array}\right]$
$\mathrm{I}_{2}=\left[\begin{array}{cccc}17100 & 0 & 0 & -330 \\ 0 & 400 & 0 & 0 \\ 0 & 0 & 400 & 0 \\ -330 & 0 & 0 & 3\end{array}\right]$
$I_{3}=\left[\begin{array}{cccc}12100 & 0 & 0 & -330 \\ 0 & 400 & 0 & 0 \\ 0 & 0 & 400 & 0 \\ -330 & 0 & 0 & 3\end{array}\right]$
$\mathrm{I}_{4}=\left[\begin{array}{cccc}133.35 & 0 & 0 & 0 \\ 0 & 133.33 & 0 & 0 \\ 0 & 0 & 533.33 & 0 \\ 0 & 0 & 0 & 1\end{array}\right]$
$I_{5}=\left[\begin{array}{cccc}93.3 & 0 & 0 & 0 \\ 0 & 93.3 & 0 & 0 \\ 0 & 0 & 870.27 & -45.5 \\ 0 & 0 & -45.5 & 0.7\end{array}\right]$
From equation (8), substitute $i=1, j=1$, we can find,
$\mathrm{d}_{11}=\mathrm{U}_{11}={ }^{0} \mathrm{~A}_{0} \mathrm{Q}_{1}{ }^{0} \mathrm{~A}_{1}$
When $(\mathrm{i}=1 ; \mathrm{j}=1), \mathrm{d}_{11}=\left[\begin{array}{cccc}-0.5 & 0 & -8.666 & -5 \\ 0.866 & 0 & -0.5 & 8.660 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0\end{array}\right]$
When $\mathrm{i}=1 ; \mathrm{j}=2 \mathrm{~d}_{12}=0(\therefore j>i)$
When $\mathrm{i}=1 ; \mathrm{j}=3 \mathrm{~d}_{13}=0(\therefore j>i)$
When $\mathrm{i}=1 ; \mathrm{j}=4 \mathrm{~d}_{14}=0(\therefore j>i)$
When $\mathrm{i}=1 ; \mathrm{j}=5 \mathrm{~d}_{15}=0(\therefore j>i)$
When $i=2 ; j=1$
$\mathrm{d}_{21}={ }_{0} \mathrm{~A}^{0} \mathrm{Q}_{1}{ }^{0} \mathrm{~A}_{2}=\left[\begin{array}{cccc}-0.353 & 0.353 & -0.866 & -10.303 \\ 0.612 & -0.612 & -0.5 & 17.884 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0\end{array}\right]$
When $\mathrm{i}=2 ; \mathrm{j}=2$
$\mathrm{d}_{22}={ }^{0} \mathrm{~A}_{1} \quad \mathrm{Q}_{2}{ }^{1} \mathrm{~A}_{2}=\left[\begin{array}{cccc}-0.612 & -0.612 & 0 & -9.814 \\ -0.353 & -0.353 & 0 & -5.303 \\ -0.707 & 0.707 & 0 & -10.606 \\ 0 & 0 & 0 & 0\end{array}\right]$

When $\mathrm{i}=2 ; \mathrm{j}=3, \quad \mathrm{~d}_{23}=0 \quad(\therefore j>i)$
When $\mathrm{i}=2 ; \mathrm{j}=4, \quad \mathrm{~d}_{24}=0 \quad(\therefore j>i)$,
When $\mathrm{i}=2 ; \mathrm{j}=5, \quad \mathrm{~d}_{25}=0 \quad(\therefore j>i)$,
When $i=3 ; j=1$,

$$
\mathrm{d}_{31}={ }^{0} \mathrm{~A}_{0} \mathrm{Q}_{1}{ }^{0} \mathrm{~A}_{3}=\left[\begin{array}{cccc}
0.129 & 0.482 & -0.866 & -7.722 \\
-0.223 & -0.835 & -0.5 & 13.370 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{array}\right]
$$

When $i=3 ; j=2$,

$$
\mathrm{d}_{32}={ }^{0} \mathrm{~A}_{1} \mathrm{Q}_{2}{ }^{1} \mathrm{~A}_{3}=\left[\begin{array}{cccc}
-0.835 & 0.223 & 0 & -25.911 \\
-0.482 & 0.129 & 0 & -14.960 \\
0.258 & 0.965 & 0 & -5.430 \\
0 & 0 & 0 & 0
\end{array}\right]
$$

When $i=3 ; j=3$,

$$
\mathrm{d}_{33}={ }^{0} \mathrm{~A}_{2} \quad \mathrm{Q}_{3}{ }^{2} \mathrm{~A}_{3}=\left[\begin{array}{cccc}
-0.835 & 0.223 & 0 & -16.719 \\
-0.482 & 0.129 & 0 & -9.643 \\
0.258 & 0.965 & 0 & 5.175 \\
0 & 0 & 0 & 0
\end{array}\right]
$$

When $\mathrm{i}=3 ; \mathrm{j}=4, \mathrm{~d}_{34}=0(\therefore j>i)$
When $\mathrm{i}=3 ; \mathrm{j}=5, \mathrm{~d}_{35}=0(\therefore j>i)$
When $\mathrm{i}=4 ; \mathrm{j}=1$,

$$
\mathrm{d}_{41}={ }^{0} \mathrm{~A}_{0} \quad \mathrm{Q}_{1}{ }^{0} \mathrm{~A}_{4}=\left[\begin{array}{cccc}
0.452 & 0.866 & 0.210 & -7.722 \\
-0.782 & 0.5 & -0.365 & 13.370 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{array}\right]
$$

Similarly we can find the other $d$ values are,
$\begin{aligned} \mathrm{d}_{42} & =\left[\begin{array}{cccc}-0.364 & 0 & 0.782 & -25.911 \\ -0.210 & 0 & 0.452 & -14.960 \\ 0.904 & 0 & 0.421 & -5.430 \\ 0 & 0 & 0 & 0\end{array}\right] \\ \mathrm{d}_{43} & =\left[\begin{array}{cccc}-0.364 & 0 & 0.782 & -16.719 \\ -0.210 & 0 & 0.451 & -9.643 \\ 0.904 & 0 & 0.421 & 5.175 \\ 0 & 0 & 0 & 0\end{array}\right]\end{aligned}$
$\mathrm{d}_{44}=\left[\begin{array}{cccc}-0.365 & 0 & 0.782 & 0 \\ -0.210 & 0 & 0.452 & 0 \\ 0.904 & 0 & 0.421 & 0 \\ 0 & 0 & 0 & 0\end{array}\right]$
When $\mathrm{i}=4, \mathrm{j}=5, \quad \mathrm{~d}_{45}=0 \quad(\therefore j>i)$
When $\mathrm{i}=5 ; \mathrm{j}=1$

$$
\begin{aligned}
& \mathrm{d}_{51}={ }^{0} \mathrm{~A}_{0} \mathrm{Q}_{1}{ }^{0} \mathrm{~A}_{5}=\left[\begin{array}{cccc}
0.967 & -0.128 & 0.210 & -6.672 \\
0.202 & 0.905 & -0.365 & 11.545 \\
0.904 & 0 & 0.421 & -5.430 \\
0 & 0 & 0 & 0
\end{array}\right] \\
& \mathrm{d}_{52}=\left[\begin{array}{cccc}
-0.123 & 0.342 & 0.782 & -21.997 \\
-0.071 & 0.197 & 0.452 & -12.700 \\
0.309 & -0.849 & 0.421 & -3.325 \\
0 & 0 & 0 & 0
\end{array}\right] \\
& \mathrm{d}_{53}=\left[\begin{array}{cccc}
-0.124 & 0.342 & 0.783 & -12.821 \\
-0.074 & 0.197 & 0.451 & -7.395 \\
0.308 & -0.849 & 0.421 & 2.10 \\
0 & 0 & 0 & 0
\end{array}\right] \\
& \mathrm{d}_{54}=\left[\begin{array}{cccc}
-0.124 & 0.342 & 0.782 & 3.91 \\
-0.071 & 0.197 & 0.452 & 2.26 \\
0.308 & -0.849 & 0.421 & 2.10 \\
0 & 0 & 0 & 0
\end{array}\right] \\
& \mathrm{d}_{55}=\left[\begin{array}{cccc}
0.905 & -0.202 & 0 & 0 \\
0.130 & 0.968 & 0 & 0 \\
0.395 & 0.143 & 0 & 0 \\
0 & 0 & 0 & 0
\end{array}\right]
\end{aligned}
$$

## 7. Inertia Loading of Actuator

The matrix M comprises all the mass properties of the entire mechanism of robot, as reflected to the joint axes, and has been expressed as the Multi-Body Inertia Matrix. Monitor the dissimilarity among the multi-body inertia matrix and the $4 \times 4$ inertia matrices of the separate links. An aggregate inertia matrix is the former one comprising the components as latter. The multi-body inertia matrix, although, holds properties identical to those of separate matrices of inertia. The multi-body inertia matrix is known to be a symmetric matrix. The quadratic form combined with the multi-body inertia matrix symbolizes kinetic energy, hence does the individual inertia matrix. Kinetic energy is forever severely positive if not the system is at rest. On the other hand, the multi-body inertia matrix deals Jacobian matrices that differ with linkage configuration. As a result the multi-body inertia matrix is configuration-dependent and stands for the instantaneous composite mass properties of the entire linkage at the existing linkage configuration.

Substitute $\mathrm{i}=1, \mathrm{j}=1$ and $\mathrm{d}, \mathrm{I}$ values in Equation (5), we can find, $\mathrm{M}_{11}=T_{r}\left(\mathrm{~d}_{11} \mathrm{I}_{1} \mathrm{~d}_{11}{ }^{\mathrm{T}}\right)+T_{r}\left(\mathrm{~d}_{21} \mathrm{I}_{2} \mathrm{~d}_{21}{ }^{\mathrm{T}}\right)+T_{r}\left(\mathrm{~d}_{31} \mathrm{I}_{3} \mathrm{~d}_{31}{ }^{\mathrm{T}}\right)+$ $T_{r}\left(\mathrm{~d}_{41} \mathrm{I}_{4} \mathrm{~d}_{41}{ }^{\mathrm{T}}\right)+T_{r}\left(\mathrm{~d}_{51} \mathrm{I}_{5} \mathrm{~d}_{51}{ }^{\mathrm{T}}\right)=9611.78$

Substitute $\mathrm{i}=1, \mathrm{j}=2$ and d , I values in Equation (5), we can find,
$\mathrm{M}_{12}=\mathrm{M}_{21}=T_{r}\left(\mathrm{~d}_{22} \mathrm{I}_{2} \mathrm{~d}_{21}{ }^{\mathrm{T}}\right)+T_{r}\left(\mathrm{~d}_{32} \mathrm{I}_{3} \mathrm{~d}_{31}{ }^{\mathrm{T}}\right)+T_{r}\left(\mathrm{~d}_{42} \mathrm{I}_{4} \mathrm{~d}_{41}{ }^{\mathrm{T}}\right)+$ $T_{r}\left(\mathrm{~d}_{52} \mathrm{I}_{5} \mathrm{~d}_{51}{ }^{\mathrm{T}}\right)=-10.363$

Substitute $i=1, j=3$ and $d, I$ values in Equation (5), we can find,
$\mathrm{M}_{13}=\mathrm{M}_{31}=T_{r}\left(\mathrm{~d}_{33} \mathrm{I}_{3} \mathrm{~d}_{31}{ }^{\mathrm{T}}\right)+T_{r}\left(\mathrm{~d}_{43} \mathrm{I}_{4} \mathrm{~d}_{41}{ }^{\mathrm{T}}\right)+T_{r}\left(\mathrm{~d}_{53} \mathrm{I}_{5} \mathrm{~d}_{51}{ }^{\mathrm{T}}\right)$ $=-2.526$

Substitute $\mathrm{i}=1, \mathrm{j}=4$ and $\mathrm{d}, \mathrm{I}$ values in Equation (5), we can find,
$\mathrm{M}_{14}=\mathrm{M}_{41}=T_{r}\left(\mathrm{~d}_{44} \mathrm{I}_{4} \mathrm{~d}_{41}{ }^{\mathrm{T}}\right)+T_{r}\left(\mathrm{~d}_{54} \mathrm{I}_{5} \mathrm{~d}_{51}{ }^{\mathrm{T}}\right)=-6.956$
Substitute $i=1, j=5$ and $d, I$ values in Equation (5), we can find,
$\mathrm{M}_{15}=\mathrm{M}_{51}=T_{r}\left(\mathrm{~d}_{55} \mathrm{I}_{5} \mathrm{~d}_{51}{ }^{T}\right)=168.846$
Similarly we can find the other M values are,
$\mathrm{M}_{22}=16371.40, \mathrm{M}_{23}=\mathrm{M}_{32}=10234.62, \mathrm{M}_{24}=\mathrm{M}_{42}=$ 10199.04, $\mathrm{M}_{25}=\mathrm{M}_{52}=0.172$,
$\mathrm{M}_{33}=11417.81, \mathrm{M}_{34}=\mathrm{M}_{43}=9607.34, \mathrm{M}_{35}=\mathrm{M}_{53}=0.039$, $M_{44}=8961.92$,
$\mathrm{M}_{45}=\mathrm{M}_{54}=0.039, \mathrm{M}_{55}=185.68$
Substitute all M values in equation (11), we obtain(14)
$\mathrm{M}=10^{4}\left[\begin{array}{ccccc}0.9611 & -0.0010 & -0.0002 & -0.0006 & 0.0168 \\ -0.0010 & 1.6371 & 1.0234 & 1.0199 & 0 \\ -0.0002 & 1.0234 & 1.1417 & 0.9607 & 0 \\ -0.0006 & 1.0199 & 0.9607 & 0.8961 & 0 \\ 0.0168 & 0 & 0 & 0 & 0.0185\end{array}\right]$

## 8. Coriolis Force Coefficient

The coefficient $\mathrm{th}_{\mathrm{ijk}}$ is related to the velocity of the joint variables. The last two indices, jk , are related to the velocities of joints j and k , whose dynamic interplay induces a reaction torque (or force) at joint i. Thus, the first index $i$ is always related to the joint where the velocity-induced reaction torques (or forces) are "felt". In particular, when $\mathrm{j}=\mathrm{k}, \mathrm{h}_{\mathrm{ijj}}$ is related to the centrifugal force generated by the angular velocity of joint $j$ and "felt" at joint i , while for $\mathrm{j} \neq k, \mathrm{~h}_{\mathrm{ijk}}$ is related to the Coriolis force generated by the velocities of joints $j$ and $k$ and "felt" at joint i.

Substitute all $\mathrm{i}, \mathrm{j}, \mathrm{d}$, I values in equation (6), we can find all ' h ' values are,

$$
\begin{aligned}
& h_{112}=h_{121}=-2986.75, h_{155}=0, h_{115}=h_{151}=0, h_{114}=h_{141} \\
&=-3846.14, h_{113}=h_{131}=751.04, h_{125}=h_{152}=39.2, h_{135}=h_{153} \\
&=39.20, h_{145}=h_{154}=39.26, h_{211}=-2188.35, h_{233}=2927.82,
\end{aligned}
$$

$\mathrm{h}_{255}=0.052 \mathrm{~h}_{215}=\mathrm{h}_{251}=-39.07, \mathrm{~h}_{223}=\mathrm{h}_{232}=2845.26, \mathrm{~h}_{225}=$ $0.031, \mathrm{~h}_{322}=-2844.34, \mathrm{~h}_{311}=-2.358, \mathrm{~h}_{315}=\mathrm{h}_{351}=-39.16, \mathrm{~h}_{411}=$ $-6.956, \mathrm{~h}_{415}=\mathrm{h}_{451}=39.08, \mathrm{~h}_{422}=-332.37, \mathrm{~h}_{423}=-546.63, \mathrm{~h}_{433}=$ $-460.40 \mathrm{~h}_{511}=168.24, \mathrm{~h}_{514}=\mathrm{h}_{541}=39.09, \mathrm{~h}_{513}=\mathrm{h}_{531}=39.18$, $h_{512}=h_{521}=39.09, h_{542}=h_{524}=-0.037 h_{523}=h_{532}=-0.077$,
$h_{522}=-0.048, h_{534}=h_{543}=0, h_{533}=-27.70, h_{544}=-0.043$
Let
$\mathrm{H}_{1}=\left(\mathrm{h}_{155} \mathrm{xV}_{5}{ }^{2}\right)+\left(\mathrm{h}_{112} \mathrm{xV}_{1} \mathrm{xV} V_{2}\right)+\left(\mathrm{h}_{121} \mathrm{xV}_{2} \mathrm{xV} \mathrm{D}_{1}\right)+$ $\left(h_{115} \mathrm{xV}_{1} \mathrm{xV}_{5}\right)+\left(\mathrm{h}_{114} \mathrm{xV}_{1} \mathrm{xV} \mathrm{V}_{4}\right)+\left(\mathrm{h}_{113} \mathrm{~V}_{1} \mathrm{xV}_{3)}+\left(\mathrm{h}_{125} \mathrm{xV}_{2} \mathrm{xV}_{5}\right)+\right.$ $\left(h_{131} x V_{3} x V_{1}\right)+\left(h_{135} \mathrm{xV}_{3} x V_{5}\right)+\left(\mathrm{h}_{141} \mathrm{xV}_{4} x \mathrm{~V}_{1}\right)+\left(\mathrm{h}_{145} \mathrm{xV}_{4} \mathrm{xV}_{5}\right)$ $+\left(\mathrm{h}_{151} \mathrm{x} \mathrm{V}_{5} \mathrm{xV} \mathrm{I}_{1}\right)+\left(\mathrm{h}_{152} \mathrm{xV}_{5} \mathrm{xV}_{2}\right)+\left(\mathrm{h}_{153} \mathrm{xV}_{5} \mathrm{xV}_{3}\right)+\left(\mathrm{h}_{154} \mathrm{xV} \mathrm{V}_{5} \mathrm{x}\right.$ $\mathrm{V}_{4}$ )

Substitute all $h$ and $V$ (from Table 2) values in above H formula, we obtain,
$H_{1}=-4771340.8$
Similarly Substitute all h and V values, we can find other H values are

$$
\begin{aligned}
& \mathrm{H}_{2}=\left(\mathrm{h}_{211} \mathrm{xV}_{1}{ }^{2}\right)+\left(\mathrm{h}_{233} \mathrm{xV}_{3}{ }^{2}\right)+\left(\mathrm{h}_{255} \mathrm{xV}_{5}{ }^{2}\right)+\left(\mathrm{h}_{215} \mathrm{xV}_{1} \mathrm{xV}_{5}\right) \\
& +\left(\mathrm{h}_{223} \mathrm{xV}_{2} \mathrm{xV}_{3}\right)+\left(\mathrm{h}_{225} \mathrm{xV}_{2} \mathrm{xV}_{5}\right)+\left(\mathrm{h}_{232} \mathrm{xV}_{3} \mathrm{xV}_{2}\right)+ \\
& \left(\mathrm{h}_{251} \mathrm{xV}_{5} \mathrm{xV}_{1}\right)=2540778.4 \\
& \mathrm{H}_{3}=\left(\mathrm{h}_{322} \mathrm{xV}_{2}{ }^{2}\right)+\left(\mathrm{h}_{311} \mathrm{xV}_{1}{ }^{2}\right)+\left(\mathrm{h}_{315} \mathrm{xV}_{1} \mathrm{xV}_{5}\right)+ \\
& \left(\mathrm{h}_{351} \mathrm{xV}_{5} \mathrm{xV}_{1}\right)=-1170012 \\
& \mathrm{H}_{4}=\left(\mathrm{h}_{411} \mathrm{xV}_{1}{ }^{2}\right)+\left(\mathrm{h}_{415} \mathrm{xV}_{1} \mathrm{xV}_{5}\right)+\left(\mathrm{h}_{422} \mathrm{xV}_{2}{ }^{2}\right)+ \\
& \left(\mathrm{h}_{423} \mathrm{xV}_{2} \mathrm{xV}_{3}\right)+\left(\mathrm{h}_{433} \mathrm{xV}_{3}{ }^{2}\right)+\left(\mathrm{h}_{451} \mathrm{xV}_{5} \mathrm{xV}_{1}\right)=-507280 \\
& \mathrm{H}_{5}=\left(\mathrm{h}_{511} \mathrm{x} \mathrm{~V}_{1}{ }^{2}\right)+\left(\mathrm{h}_{514} \mathrm{xV}_{1} \mathrm{xV}_{4}\right)+\left(\mathrm{h}_{513} \mathrm{xVV}_{1} \mathrm{xVV}_{3}\right) \\
& +\left(\mathrm{h}_{512} \mathrm{xV}_{1} \mathrm{xV}_{2}\right)+\left(\mathrm{h}_{531} \mathrm{x} \mathrm{~V}_{3} \mathrm{xV}_{1}\right)+\left(\mathrm{h}_{524} \mathrm{xV}_{2} \mathrm{xV}_{4}\right)+ \\
& \left(\mathrm{h}_{523} \mathrm{xV}_{2} \mathrm{xV}_{3}\right)+\left(\mathrm{h}_{522} \mathrm{xV}_{2}{ }^{2}\right)+\left(\mathrm{h}_{521} \mathrm{xV}_{2} \mathrm{xV}_{1}\right)+\left(\mathrm{h}_{534} \mathrm{xV}_{3} \mathrm{xV}_{4}\right)+ \\
& \left(\mathrm{h}_{533} \mathrm{xV}_{3}^{2}\right)+\left(\mathrm{h}_{532} \mathrm{xV}_{3} \mathrm{xV}_{2}\right)+\left(\mathrm{h}_{541} \mathrm{xV}_{4} \mathrm{xV}_{1}\right)+\left(\mathrm{h}_{543} \mathrm{xV}_{4} \mathrm{xV}_{3}\right)+ \\
& \left(\mathrm{h}_{544} \mathrm{xV}_{4}^{2}\right)+\left(\mathrm{h}_{542} \mathrm{xV}_{4} \mathrm{xV}_{2}\right)=149978.8
\end{aligned}
$$

Substitute all H values in equation (12), we obtain

$$
H=\left[\begin{array}{c}
-4771340.8  \tag{15}\\
2540778.4 \\
-1170012.0 \\
-507280.0 \\
149978.8
\end{array}\right]
$$

## 9. Loading due to Gravity

$G_{i}$, represents the moments created by the mass $m$ concerning its separate joint axes. The moment relies on the configuration of arm and when it is entirely comprehensive along the x -axis, the gravity moments goes up to maximum.

Substitute all m, r, d (from Table 2) in equation (7), we obtain

$$
\mathrm{G}_{1}=-\left(\mathrm{m}_{1} \mathrm{gT} \mathrm{r}_{\mathrm{r}}\left(\mathrm{~d}_{11}\right) \mathrm{r}_{1}^{1}+\mathrm{m}_{2} \mathrm{gT}_{\mathrm{r}}\left(\mathrm{~d}_{21}\right) \mathrm{r}_{2}^{2}+\mathrm{m}_{3} g \mathrm{~T}_{\mathrm{r}}\left(\mathrm{~d}_{31}\right) \mathrm{r}_{3}^{3}\right.
$$

$$
\left.+\mathrm{m}_{4} \mathrm{gT}_{\mathrm{r}}\left(\mathrm{~d}_{41}\right) \mathrm{r}_{4}^{4}+\mathrm{m}_{5} \mathrm{gT} \mathrm{r}_{\mathrm{r}}\left(\mathrm{~d}_{51}\right) \mathrm{r}_{5}^{5}\right)=-224735.96
$$

$$
\begin{aligned}
& \mathrm{G}_{2}=-\left(\mathrm{m}_{2} \mathrm{~g}_{2} \mathrm{~T}_{\mathrm{r}}\left(\mathrm{~d}_{22}\right) \mathrm{r}_{2}{ }^{2}+\mathrm{m}_{3} \mathrm{~g}_{3} \mathrm{~T}_{\mathrm{r}}\left(\mathrm{~d}_{32}\right) \mathrm{r}_{3}^{3}+\mathrm{m}_{4} \mathrm{~g}_{4} \mathrm{~T}_{\mathrm{r}}\left(\mathrm{~d}_{42}\right)\right. \\
& \left.\mathrm{r}_{4}{ }^{4}+\mathrm{m}_{5} \mathrm{~g}_{5} \mathrm{~T}_{\mathrm{r}}\left(\mathrm{~d}_{52}\right) \mathrm{r}_{5}^{5}\right)=-53002.41 \\
& \mathrm{G}_{3}=-\left(\mathrm{m}_{3} \mathrm{gT}_{\mathrm{r}}\left(\mathrm{~d}_{33}\right) \mathrm{r}_{3}^{3}+\mathrm{m}_{4} \mathrm{gT}_{\mathrm{r}}\left(\mathrm{~d}_{43}\right) \mathrm{r}_{4}^{4}+\mathrm{m}_{5} \mathrm{gT}_{\mathrm{r}}\left(\mathrm{~d}_{53}\right) \mathrm{r}_{5}^{5}\right) \\
& =-53644.00 \\
& \mathrm{G}_{4}=-\left(\mathrm{m}_{4} g \mathrm{gT}_{\mathrm{r}}\left(\mathrm{~d}_{44}\right) \mathrm{r}_{4}^{4}+\mathrm{m}_{5} g \mathrm{~T}_{\mathrm{r}}\left(\mathrm{~d}_{54}\right) \mathrm{r}_{5}^{5}\right)=-61461.51 \\
& \mathrm{G}_{5}=-\mathrm{m}_{5} g \mathrm{gT}_{\mathrm{r}}\left(\mathrm{~d}_{55}\right) \mathrm{r}_{5}^{5}=-225594.40
\end{aligned}
$$

Substitute all G values in equation (13), we obtain
$G=\left[\begin{array}{c}-224735.96 \\ -53002.41 \\ -53644.00 \\ -61461.51 \\ -225594.40\end{array}\right]$
Substitute Equations (14), (15) and (16) in Equation (10) we obtain,
$T=10^{5}\left[\begin{array}{c}-48.9846 \\ 28.5571 \\ -9.1109 \\ -2.8113 \\ -0.7209\end{array}\right]$

## 10. Simulation of Scorbot-Er Vu Plus Robot in Labview

LabVIEW (Laboratory Virtual Instrument Engineering Workbench) is a graphical programming language which implements icons as an alternative to text lines for creating applications. In dissimilarity to text-based programming language which instructs the program execution order, LabVIEW utilizes dataflow programming, where the data flow via the nodes on the block diagram brings up the order of execution of the VIs and its functions. VIs, or virtual instruments, is LabVIEW programs which replicate physical instruments. Ever VI holds components that are three in quantity: A front panel, a block diagram and a connector panel. The control panel symbolizes the VI in the block diagrams of other, calling VIs. Controls and indicators on the front panel permits input data or extract data to an operator from a functioning virtual instrument. In LabVIEW, a user interface could be build by means of a tools and objects set. The user interface has been referred much as the front panel and it could also serve as a programmatic interface. Hence a virtual instrument could either be run as a program, holding the front panel helping as a user interface, or, on dropping as a node onto the block diagram, the front panel briefs the inputs and outputs for the given node via the connector
pane. Into a larger program this involve each VI could be effortlessly tested previous of being embedded as a subroutine. Then sum up code utilizing graphical representations of functions for controlling the front panel objects. This graphical source code has also referred to be G code or block diagram code. By some means, the block diagram looks like a flowchart. Figure 3 illustrates simulated window of LabVIEW program output.


Figure 3. LabVIEW result output view.

## 11. Result

When dynamics have been computed for the objective of performing a numerical simulation of a manipulator, much interested in solving for the joint accelerations provided the manipulator's current position velocity, and the input torques. Provided set of parameter, a program in LabVIEW has been done and comparison of its output along the theoretical outcome as follows.

## Torque

Mathematical Result
$T=10^{5}\left[\begin{array}{c}-48.9846 \\ 28.5571 \\ -9.1109 \\ -2.8113 \\ -0.7209\end{array}\right]$
LabVIEW Program Output
$T=10^{5}\left[\begin{array}{c}-49.1139 \\ 28.2544 \\ -9.0040 \\ -3.0848 \\ -1.2640\end{array}\right]$

The Final Torque values are checked against the physical positions of the robot arm in Table 3 and Figure 3.

Table 3. Difference between analytical and physical values of the robot

| Position | Analytical <br> Torque Values | LabVIEW <br> Torque Values | Difference |
| :--- | :---: | :---: | :---: |
| 1 | -48.9846 | -49.1139 | 0.1293 |
| 2 | 28.5571 | 28.2544 | 0.3027 |
| 3 | -9.1109 | -9.0040 | 0.1069 |
| 4 | -2.8113 | -3.0848 | 0.2735 |
| 5 | -0.7209 | -1.2640 | 0.5431 |

## 12. Summary and Concluding Remarks

The complete dynamic analysis for a set of dynamic parameter of 5-DOF SCORBOT-ER Vu Plus robot has been researched. Preparation of the mathematical model has been done and solved for torque and forces for endeffector on preparation of a programme in LabVIEW. Theoretical and LabVIEW outcomes are just about same. Therefore this confirms the Scorbot-ER Vu plus robot arm utility for doing successful dynamic manipulations and also be used for deriving the dynamic model of other robotic arms. This technology can also be extended to other robot control such as path planning and trajectory planning in real time mode, optimal routing in various types of environment, error tracking control and other control aspects.

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