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Graph labeling in competition graph

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Abstract

In this paper we present an algorithm and prove the existence of graph labelings such as Z_3- magic, E-cordial, total E-cordial, Product cordial, total product cordial, Product E-cordial, total product E-cordial labelings for the competition graph of the Cayley digraphs associated with Z_n.

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Introduction

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Labeled graphs serve as useful models for broad range of applications such as coding theory, X-ray crystallography, radar, astronomy, circuit design, communication networks and data base management and models for constraint programming over finite domain. The concept of graph labeling was introduced by Rosa in 1967. Hence in the intervening years various labeling of graphs such as graceful labeling, harmonious labeling, magic labeling, antimagic labeling, bimagic labeling, prime labeling, cordial labeling, total cordial labeling, k-graceful labeling and odd graceful labeling etc., have been studied in over 1100 papers (Gallian, 2009).

Cahit (1987) has introduced cordial labeling. Cahit (1990) proved that every tree is cordial; K_n is cordial if and only if $n \leq 3$, $K_{m,n}$ is cordial for all m and n. Friendship graph $C_3^{(t)}$ is cordial if and only if t ⍰ 2 (mod 2) and all fans are cordial. Andar *et al.*, (2005) proved that the t-ply graph $P_t(u,v)$ is cordial except when it is Eulerian and the number of edges is congruent to 2(mod 4). Youssef (2006) proved that every Skolem-graceful graph is cordial.

A new labeling called E-cordial was introduced by Yilmaz and Cahit (1997). They proved the following graphs are E-cordial: trees with n vertices if and only if $n \neq 2 \pmod 4$; K_n if and only if $n \neq 2 \pmod 4$; $K_{m,n}$ if and only if $m+n \neq 2 \pmod 4$; C_n if and only if $n \neq 2 \pmod 4$; regular graphs of degree 1 on 2n vertices if and only if n is even; friendship graphs $C_3^{(n)}$ for all n; fans F_n if and only if $n \neq 1 \pmod 4$; and wheels W_n if and only if $n \neq 1$ (mod 4). They also observe that with $n \equiv 2 \pmod 4$ vertices cannot be E-cordial. More over the graph labelings on digraphs has been extensively studied in literature (Thirusangu *et al.*, 2009, 2011).

In 1878, Cayley constructed a graph with a generating set which is now popularly known as Cayley graphs. A directed graph or digraph is a finite set of points called vertices and a set of arrows called arcs connecting some vertices. The Cayley graphs and Cayley digraphs are excellent models for interconnection networks (Annexstein *et al.*,1990; Heydemann,1997). Many well-known interconnection networks are Cayley digraphs. For example hypercube, butterfly, and cube-connected cycle's networks are Cayley graphs. The Cayley digraph of a group provides a method of visualizing the group and its properties. The properties such as commutativity and the multiplication table of a group can be recovered from a Cayley digraph.

The original concept of an A-magic graph is due to dedlack, who defined it to be a graph with real-valued edge labeling such that distinct edges have distinct non-negative labels which satisfies the condition that the sum of the labels of the edges incident to a particular vertex is the same for all vertices. It is easy to verify whether a graph is Z_3-magic or not. Bala *et al.* (2011) proved that the Competition graph of Cayley digraph associated with diheadral group D_n admits Z_3 magic, Cordial, total cordial, E-cordial, total E-cordial, Product cordial, total product cordial, Product E-cordial and total product E-cordial labelings.

In this paper we prove the existence of graph labelings such as Z_3 magic, E-cordial, total E-cordial, Product cordial, total product cordial, Product E-cordial, total product E-cordial for the competition graph of the Cayley digraphs associated with Z_n.

2. Preliminaries

In this section we give the basic notation relevant to this paper. Let G = G(V,E) be a finite, simple and undirected graph with p vertices and q edges. By a labeling we mean a one-to-one mapping that carries a set of graph elements onto a set of numbers called labels (usually the set of integers). In this paper we deal with the labeling with domain either the set of all vertices or the set of all edges or the set of all vertices and edges. We call these labelings as the vertex labeling or the edge labeling or the total labeling respectively.

Definition 2.1: Let G be a graph with vertex set V and edge set E and let f be function from E to {0,1}. Define f* on V by $f^*(v) = \sum\{f(uv)/uv \in E\} \pmod 2$. The function f is called E-cordial labeling of G if the number of vertices labeled 0 and the number of vertices labeled 1 differ atmost by 1, and number of edges labeled 0 and the number of edges labeled 1 differ at most by 1.A graph that admits E-cordial labeling is called E-cordial.

Definition 2.2: A function f from the vertex set V→ {0,1} such that each edge uv assign the label f(u)×f(v) is said to be a product cordial labeling if the number of vertices labeled 0 and the number of vertices labeled 1 differ at most by 1, and number of edges labeled 0 and the number of edges labeled 1 differ at most by 1.

Definition 2.3: A function f from the vertex set V→ {0,1} such that each edge uv assign the label f(u)×f(v) is said to be a total product cordial labeling if the number of vertices and edges labeled with 0 and the number of vertices and edges labeled with 1 differ at most by 1. A graph with total product cordial labeling is called total product cordial graph.

Definition 2.4: A (p,q)-digraph G = (V,E) is defined by a set V of vertices such that |V|=p and a set E of arcs or directed edges with |E|=q. The set E is a subset of elements (u,v) of V × V. The out-degree (or in-degree of a vertex u of a digraph G is the number of arcs (u,v)(or(v,u)) of G and is denoted by d^+ (u) (or d^- (u)). A digraph is said to be regular if d^+ (u) = d^- (v) for every vertex u of G.

3. Main results

In this section we present an algorithm and prove the existence of graph labeling such as Z_3-magic, E-cordial, total E-cordial, Product cordial, total product cordial, Product E-cordial, total Product E-cordial for the Competition graphs of Cayley digraphs associated with Z_n.

Definition 3.1: Let G(V,E) be a (p,q) digraph. It is said to admit E-cordial labeling if there exists a function f from E onto the set {0,1} such that the induced map f* on V is defined as $f^*(v_i) ==\sum\{f(v_i v_j)/ v_i v_j \in E\}(\text{mod } 2)$ satisfies the property that the number of arcs labeled 0 and the number of arcs labeled 1 differ at most by 1, and number of vertices labeled 0 and the number of vertices labeled 1 differ at most by 1.

Definition 3.2: Let G(V,E) be a (p,q) digraph. It is said to admit total E-cordial labeling if there exists a function f from E onto the set {0,1} such that the induced map f* on V is defined as $f^*(v_i) = \sum\{f(v_i v_j)/ v_i v_j \in E\}(\text{mod } 2)$ satisfies the property that the number of vertices and arcs labeled with 0 and the number of vertices and arcs labeled with 1 differ atmost by 1.

Definition 3.3: Let G(V,E) be a (p,q) digraph. It is said to admit product E-cordial labeling if there exists a function f from E onto the set {0,1} such that the induced map f* on V is defined as $f^*(v_i) = \Pi\{f(v_i v_j)/ v_i v_j \in E\}(\text{mod } 2)$ satisfies the property that if the number of vertices labeled 0 and the number of vertices labeled 1 differ at most by 1, and number of arcs labeled 0 and the number of arcs labeled 1 differ at most by 1.

Definition 3.4: Let G(V,E) be a (p,q) digraph. It is said to admit total product E-cordial labeling if there exists a function f from E onto the set {0,1} such that the induced map f* on V is defined as $f^*(v_i) = \Pi\ \{f(v_i v_j) / v_i v_j \in E\}(\text{mod } 2)$ satisfies the property that that the number of vertices and arcs labeled with 0 and the number of vertices and arcs labeled with 1 differ at most by 1.

Definition 3.5: Let G(V,E) be a (p,q) digraph. It is said to admit Z_3-magic labeling if there exists a function f from E onto the set { 1,2} such that the induced map f* on V defined by $f^*(v_i) =\{ \sum f(e) \ (\text{mod } 3) = m$, a constant and $e = (v_i v_j) \in E \}$

Definition 3.6: Let G be a finite group and S be a generating subset of G. The Cayley digraph Cay(G,S) is the digraph whose vertices are the elements of G, and there is an edge from g to gs whenever $g \in G$ and $s \in S$. If $S = S^{-1}$ then there is an edge from g to gs if and only if there is an arc from gs to g

Definition 3.7: For any natural number n, we use Z_n to denote the additive cyclic group of integers modulo n. Consider the group (Z_{2k} , A) where A is a generating set {a,b,b+k} with the property that
gcd (a,b,k) = 1 and either
gcd (a-b,k) ≠ 1 or
gcd (a,2k) = 1 or
gcd (b,k) = 1 or
both a and k are even or
a is odd and either b or k is odd.

Definition 3.8: The structure of the Cayley digraph Cay (Z_n , (a,b,b+k)) for the group Z_n is defined as follows. By using step 1 of the above algorithm, the Cayley digraph for the group Z_n, Cay(Z_n, (a,b,b+k)) has n vertices and 3n arcs. Let us denote the vertex set of Cay(Z_n,(a,b,b+k)) as V ={v_0, v_1, v_2, v_3....v_{n-1}} corresponding to the elements {0,1,2,3.....,n-1} of the group Z_n respectively. For each $i \in Z_n$, $j \in A$ define a directed edge v_i to $v_{(i+j)\text{mod } n}$ as follows:
For all $0 \le i \le n-1$
$E_a =\{(v_i , v_{(a+i)\text{mod } n}) \mid 0 \le i \le n-1 \}$
$E_b =\{(v_i , v_{(b+i)\text{mod } n}) \mid 0 \le i \le n-1 \}$
$E_c =\{(v_i , v_{(b+k+i)\text{mod } n}) \mid 0 \le i \le n-1 \}$
Thus, the edges of Cay(Z_n,(a,b,b+k)) is the set E(E_a,E_b,E_c) = { e_1, e_2, e_3....e_{3n}}.

Throughout this paper we consider the generating set of Z_n as {a,b,c} where c =b+k.

Definition 3.9: Let V = {v_0, v_1, v_2, v_3....v_{n-1} } and E (E_a , E_b, E_c) = { e_1, e_2, e_3....e_{3n}} be the vertex and arc sets of Cayley digraphs of Z_n. The competition graph of Cay(Z_n,(a,b,c)) denoted by ComCay(Z_n, (a,b,c)) is a digraph consisting of same set of vertices and if for any path $v_i e_r v_j e_s v_k$ where $v_i,v_j,v_k \in V$ & $e_r ,e_s \in E$, draw a new edge $v_i v_k$.

Definition 3.10: The structure of competition graph ComCay(Z_n, (a,b,c)) is defined as follows. From the construction of competition graph using definition 3.7, the ComCay(Z_n, (a,b,c)) has n vertices and 7n arcs. Let us denote the vertex set of ComCay(Z_n, (a,b,c)) as V={v_0, v_1, v_2, v_3....v_{n-1}}. Denote the arc set of ComCay(Z_n, (a,b,c)) as E(E_{aa}, E_{ac}, E_{bb}, E_{bc}, E_{cc}, E_{ca}, E_{cb}) = { e_1, e_2, e_3....e_{7n}}, *w*here E_{aa}, E_{ac}, E_{bb}, E_{cc}, E_{bc}, E_{ca} and E_{cb} are the sets of all arcs obtained through (a,a), (a,c), (b,b), (b,c), (c,c), (c,a) and (c,b) respectively.

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Thus the arc set of the ComCay(Z_n, (a,b,c)) is as follows:
For all $1 \le i \le n-1$

(i) $v_i v_{(i+2) \bmod n} \in E_{aa}$

(ii) $v_i v_{(i+(n/2)) \bmod n} \in E_{ac}$

(iii) $v_i v_{(i+n-2) \bmod n} \in E_{bb}$

(iv) $v_i v_{(i+(n/2)-2) \bmod n} \in E_{bc}$

(v) $v_i v_{(i+n-2) \bmod n} \in E_{cc}$

(vi) $v_i v_{(i+(n/2)) \bmod n} \in E_{ca}$

(vii) $v_i v_{(i+(n/2)-2) \bmod n} \in E_{cb}$

From the arc set of the ComCay(Z_n, (a,b,c)), it is clear that $E_{ac} = E_{ca}$; $E_{bc} = E_{cb}$; $E_{bb} = E_{cc}$.
Therefore, throughout this paper we consider the vertex set of ComCay(Z_n, (a,b,c)) as V={$v_0, v_1, v_2, v_3 \ldots v_{n-1}$}and the arc set as E($E_{aa}, E_{bb}, E_{ca}, E_{cb}$) = { $e_1, e_2, e_3 \ldots e_{4n}$}
Now we present an algorithm to get Z_3-magic, E-cordial, Product cordial and Product E-cordial labeling for the competition graph ComCay(Z_n, (a,b,c)).

Algorithm:

Input: Z_n with the generating set (a,b,c).

Step 1: Using definition 3.7 , Construct Cayley digraphs Cay(Z_n,(a,b,c))

Step 2: Using definition 3.9 & 3.10, Construct competition graph ComCay(Z_n,(a, b, c)).

Step 3: Denote the vertex set of ComCay(Z_n,(a, b, c)) as V = { $v_0, v_1, v_2, v_3 \ldots v_{n-1}$ }
and the the arc set as E($E_{aa}, E_{bb}, E_{ca}, E_{cb}$) = { $e_1, e_2, e_3 \ldots e_{4n}$ } where E_{aa} , E_{bb}, E_{ca}, E_{cb} are the set of arcs obtained through (a,a), (b,b), (c,a) and (c,b) respectively.

Step 4: (for Z_3- magic labeling)
Define f on E as follows:

For all $0 \le i \le n-1$, $f(v_iv_j) = \begin{cases} 2, & \text{where } v_iv_j \in E_{ca,} \ \& \ E_{cb} \\ 1, & \text{where } v_iv_j \in E_{aa,} \ \& \ E_{bb} \end{cases}$

Step 5: (for E-cordial labeling)
Define f on E as follows:

(i) For all $v_iv_j \in E_{ca}$, $f(v_iv_{i+(n/2)}) = 1$, $0 \le i \le (n/2)-1$
$f(v_iv_{i-(n/2)}) = 0$, $(n/2) \le i \le n-1$

(ii) For all $v_iv_j \in E_{cb}$, and $0 \le i \le n-1$
$f(v_iv_{(i+(n/2)-2) \bmod n}) = \begin{cases} 1, & i \equiv 1 \pmod 2 \\ 0, & i \equiv 0 \pmod 2 \end{cases}$

(iii) For all $v_iv_j \in E_{aa}$ and $0 \le i \le n-1$
$f(v_iv_{(i+2) \bmod n}) = \begin{cases} 1, & i \equiv 0 \pmod 2 \\ 0, & i \equiv 1 \pmod 2 \end{cases}$

(iv) For all $v_iv_j \in E_{bb}$ and $2 \le i \le n-1$
$f(v_iv_{i-2}) = \begin{cases} 1, & i \equiv 0 \pmod 2 \\ 0, & i \equiv 1 \pmod 2 \end{cases}$
$f(v_0v_{n-2}) = 1$, $f(v_1v_{n-1}) = 0$

Step 6: (for product cordial labeling)
Define f on V as follows: For all $0 \le i \le n-1$, $v_i \in V$
$f(v_i) = \begin{cases} 0, & i \equiv 1 \pmod 2 \\ 1, & i \equiv 0 \pmod 2 \end{cases}$

Step 7: (for product E-cordial labeling)
(i) For all $v_iv_j \in E_{ca}$,
For $0 \le i \le (n/2)-1$, $f(v_iv_{i+(n/2)}) = \begin{cases} 0, & i \equiv 1 \pmod 2 \\ 1, & i \equiv 0 \pmod 2 \end{cases}$
For $(n/2) \le i \le n-1$, $f(v_iv_{i-(n/2)}) =$ 0, $i \equiv 1 \pmod 2$
1, i ⍰ 0 (mod 2)

(ii) For all $v_iv_j \in E_{cb}$, and $0 \le i \le n-1$
$f(v_iv_{(i+(n/2)-2) \bmod n}) = \begin{cases} 1, & i \equiv 0 \pmod 2 \\ 0, & i \equiv 1 \pmod 2 \end{cases}$

(iii) For all $v_iv_j \in E_{aa}$ and $0 \le i \le n-1$
$f(v_iv_{(i+2) \bmod n}) = \begin{cases} 1, & i \equiv 0 \pmod 2 \\ 0, & i \equiv 1 \pmod 2 \end{cases}$

(iv) For all $v_iv_j \in E_{bb}$ and $2 \le i \le n-1$
$f(v_iv_{i-2}) = \begin{cases} 1, & i \equiv 0 \pmod 2 \\ 0, & i \equiv 1 \pmod 2 \end{cases}$
$f(v_0v_{n-2}) = 1$, $f(v_1v_{n-1}) = 0$

Output: Z_3-magic, E-cordial, Product cordial and Product E-cordial labeling for
the competition graph ComCay(Z_n, (a,b,c)).

Theorem 3.1: The competition graph ComCay(Z_n,(a,b,c)) admits Z_3 magic labeling.

Proof: From the construction of Competition graph ComCay(Z_n,(a,b,c)), we have n vertices and 4n arcs. Denote the vertex set and arc set using step 3 of algorithm.
To prove ComCay(Z_n,(a,b,c)) admits Z_3-magic labeling, we have to show that there exists a function f from E onto the set Z_3- {0} such that the induced map f* on V defined by $f^*(v_i) = \{\sum f(e) \pmod 3 = m$, a constant | $e = (v_iv_j) \in E\}$.
i.e., for any $v_i \in V$, the sum of the labels of the arcs incident at v_i is a constant. Consider an arbitrary vertex $v_i \in V$ of ComCay(Z_n, (a,b,c)). Using step 4 of algorithm, define a map f: E→ Z_3- {0} such that

$f(v_iv_j) = \begin{cases} 2, & \text{where } v_iv_j \in E_{ca,} \ \& \ E_{cb} \\ 1, & \text{where } v_iv_j \in E_{aa,} \ \& \ E_{bb} \end{cases}$

Hence for the induced map $f^* : V \to Z_3$, for all $v_i \in V$, $f^*(v_i) = \sum f(v_iv_j) \pmod 3 = (1 + 1 + 2 + 2) \pmod 3 = 0 \pmod 3$ where $v_iv_j \in E_{aa}$, E_{bb}, E_{ca} & E_{cb} . Thus $f^*(v_i) = 0 \pmod 3$ which is a constant for all i.
Hence Competition graph ComCay(Z_n,(a,b,c)) admits Z_3 - magic labeling.

Example 3.1: Competition graph of the Cayley digraph associated with Z_{12} and its Z_3- *magic labeling* is shown in Fig. 1.

Theorem 3.2: *The competition graph ComCay(Z_n,(a,b,c)) is E-Cordial*.

Proof: From the construction of Competition graph ComCay(Z_n,(a,b,c)), we have n vertices and 4n arcs. To prove Comcay(Z_n,(a,b,c)) admits E-Cordial labeling, we have to show that there exists a function f: E → {0,1} such that the induced function f* on V is defined as $f^*(v_i) = \sum\{f(v_iv_j) \ / \ v_iv_j \in E (E_{aa,} E_{ab} , E_{ca,} , E_{cb})\} \pmod 2$ satisfies the property that the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1 and the number of arcs labeled 0 and the number of arcs

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labeled 1 differ by at most 1. Consider the arbitrary vertex $v_i \in V$. Using step 5 of algorithm we define a map $f:E \rightarrow \{0,1\}$ as follows:

(i) For all $v_iv_j \in E_{ca}$, $f(v_iv_{i+(n/2)}) = 1$, $0 \le i \le (n/2)-1$
$f(v_iv_{i-(n/2)}) = 0$, $(n/2) \le i \le n-1$

(ii) For all $v_iv_j \in E_{cb}$, and $0 \le i \le n-1$
$$f(v_iv_{(i+(n/2)-2) \bmod n}) = \begin{cases} 1, & i \equiv 1 \pmod 2 \\ 0, & i \equiv 0 \pmod 2 \end{cases}$$

(iii) For all $v_iv_j \in E_{aa}$ and $0 \le i \le n-1$
$$f(v_iv_{(i+2) \bmod n}) = \begin{cases} 1, & i \equiv 0 \pmod 2 \\ 0, & i \equiv 1 \pmod 2 \end{cases}$$

(iv) For all $v_iv_j \in E_{bb}$ and $2 \le i \le n-1$
$$f(v_iv_{i-2}) = \begin{cases} 1, & i \equiv 0 \pmod 2 \\ 0, & i \equiv 1 \pmod 2 \end{cases}$$
$f(v_0v_{n-2}) = 1$, $f(v_1v_{n-1}) = 0$

From these definitions of the labeling functions, we have the total number of arcs labeled 0 is (n/2)+ (n/2) + (n/2) + (n/2) = 2n and the total number of arcs labeled 1 is (n/2)+ (n/2) + (n/2) + (n/2) = 2n. Thus the number of arcs labeled 0 and the number of arcs labeled 1 differ by at most 1.In order to get the labels for the vertices , define the induced map $f^* : V \rightarrow \{0,1\}$ such that

(i) For all $0 \le i \le (n/2)-1$ & $v_iv_j \in E$ $(E_{aa}, E_{bb}, E_{ca}, E_{cb})$
$$f^*(v_i) = \sum f(v_iv_j) = \begin{cases} 0+0+0+1 = 1, & i \equiv 1 \pmod 2 \\ 1+1+0+0 = 0, & i \equiv 0 \pmod 2 \end{cases}$$

(ii) For all $n/2 \le i \le n-1$ & $v_iv_j \in E$ $(E_{aa}, E_{bb}, E_{ca}, E_{cb})$
$$f^*(v_i) = \sum f(v_iv_j) = \begin{cases} 0+0+1+1 = 0, & i \equiv 1 \pmod 2 \\ 1+1+1+0 = 1, & i \equiv 0 \pmod 2 \end{cases}$$

Under this map, the number of vertices labeled 1 is (n/4)+(n/4)=n/2 and the number of vertices labeled 0 is (n/4) +(n/4)= n/2. Thus the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1. Hence the competition graph ComCay(Z_n,(a,b,c)) admits E-cordial labeling.

Example 3.2: Competition graph of the Cayley digraph associated with Z_{12} and its E-cordial labeling is shown in Fig. 2.

Theorem 3.3: The competition graph ComCay(Z_n,(a,b,c)) admits total E-cordial labeling.

*Proof***:** To prove Comcay(Z_n, (a,b,c)) admits total E-cordial labeling, we have to show that there exists a function $f: E \rightarrow \{0,1\}$ such that the induced function f^* on V is defined as $f^*(v_i) = \sum\{f(v_iv_j) \;/\; v_iv_j \in E\ (E_{aa}, E_{bb}, E_{ca}, E_{cb})\} \pmod 2$ satisfies the property that the number of zeroes on the vertices and arcs taken together differ by atmost 1 with the number of one's on vertices and arcs taken together.

By the above theorem, using the map f on E and there by the induced map f* on V, we have the number of arcs labeled 0 is 2n and the number of vertices labeled 0 is n/2. Also, the number of arcs labeled by 1 is 2n and the number of vertices labeled by 1 is n/2. Thus, the total number of one's on vertices and arcs taken together is (n/2)+2n =5n/2 and the total number of zeroes on vertices and arcs taken together is (n/2)+2n =5n/2 . Thus the number of zeroes on the vertices and arcs taken together differ by at most 1 with the number of one's on vertices and arcs taken together.

Hence The competition graph ComCay(Z_n,(a,b,c)) admits total E-cordial labeling.

Theorem 3.4: The competition graph ComCay(Z_n,(a,b,c)) is Product Cordial.

Proof.: From the construction of Competition graph ComCay(Z_n,(a,b,c)) using algorithm, we have n vertices and 4n arcs. To prove ComCay(Z_n,(a,b,c)) admits Product Cordial labeling, we have to show that there exists a function $f: V \rightarrow \{0,1\}$ such that the induced function $f^*(v_iv_j) = \{(f(v_i) \times f(v_j) \mid v_iv_j \in E\ (E_{aa}, E_{bb}, E_{ca}, E_{cb})\}$ satisfies the property that the number of vertices labeled 0 and the number of vertices labeled 1 differ atmost by 1 and number of arcs labeled 0 and the number of arcs labeled 1 differ by atmost 1. Consider the arbitrary vertex $v_i \in V$. Using step 6 of algorithm we define a map $f:V \rightarrow \{0,1\}$ as follows.

For all $0 \le i \le n-1$ & $v_i \in V$
$$f(v_i) = \begin{cases} 0, & i \equiv 1 \pmod 2 \\ 1, & i \equiv 0 \pmod 2 \end{cases}$$

From this definition of the labeling functions, we have the total number of vertices labeled 0 is n/2 and the number of vertices labeled 1 is n/2. Hence the number of vertices labeled 0 and the number of vertices labeled 1 differ by atmost 1. In order to get the labels for the arcs,define the induced map $f^*:E \rightarrow \{0,1\}$ such that $f^*(v_iv_j) = \{f(v_i) \times f(v_j) \mid v_iv_j \in E\ (E_{aa}, E_{bb}, E_{ca}, E_{cb})\}$

Now for all $v_iv_j \in E_{aa}, E_{bb}, E_{ca}$ & E_{cb}
$$f^*(v_iv_j) = f(v_i) \times f(v_j) = \begin{cases} 1, & i \,\&\, j \equiv 0 \pmod 2 \\ 0, & i \,\&\, j \equiv 1 \pmod 2 \end{cases}$$

Under this map the number of arcs labeled 0 is (n/2) +(n/2)+ (n/2)+ (n/2) = 2n and the number of arcs labeled 1 is (n/2) +(n/2)+ (n/2)+ (n/2) = 2n . Thus the number of arcs labeled 0 and the number of arcs labeled 1 differ by atmost 1.

Hence ComCay(Z_n, (a,b,c)) admits Product Cordial labeling.

Example 3.3: Competition graph of the Cayley digraphs associated with Z_{12} and its Product cordial labeling is shown in Fig.3.

Theorem 3.5*:* The competition graph ComCay(Z_n,(a,b,c)) admits total product cordial labeling.

Proof.: From the construction of Competition graph ComCay(Z_n,(a,b,c)) using algorithm, we have n vertices and 4n arcs.

To prove ComCay(Z_n, (a,b,c)) admits total product cordial labeling, we have to show that there exists a function $f: V \rightarrow \{0,1\}$ such that the induced function $f^*(v_iv_j) = \{(f(v_i) \times f(v_j) \mid v_iv_j \in E\ (E_{aa}, E_{bb}, E_{ca}, E_{cb})\}$ satisfies the property that the number of zeroes on the vertices and arcs taken together differ by at most 1 with the number of one's on vertices and arcs taken together.

Fig.1. Z_3 - magic labeling for ComCay(Z_{12}, (7,5,11))

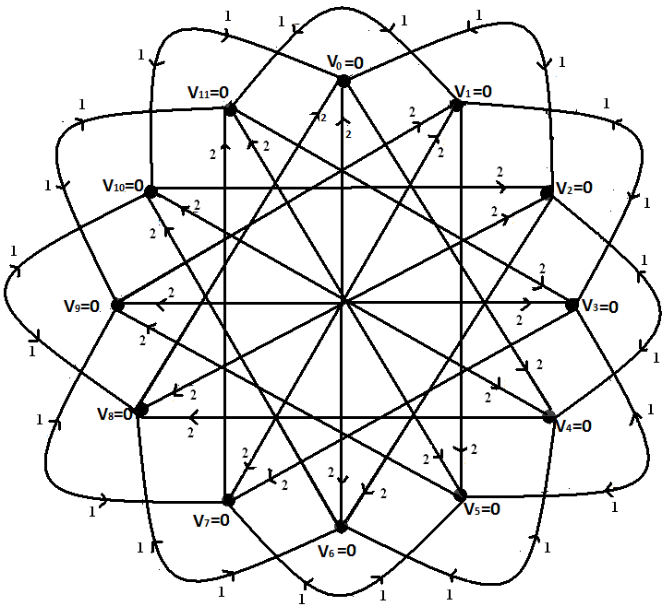


Fig.2. E-cordial labeling for ComCay(Z_{12}, (7,5,11))

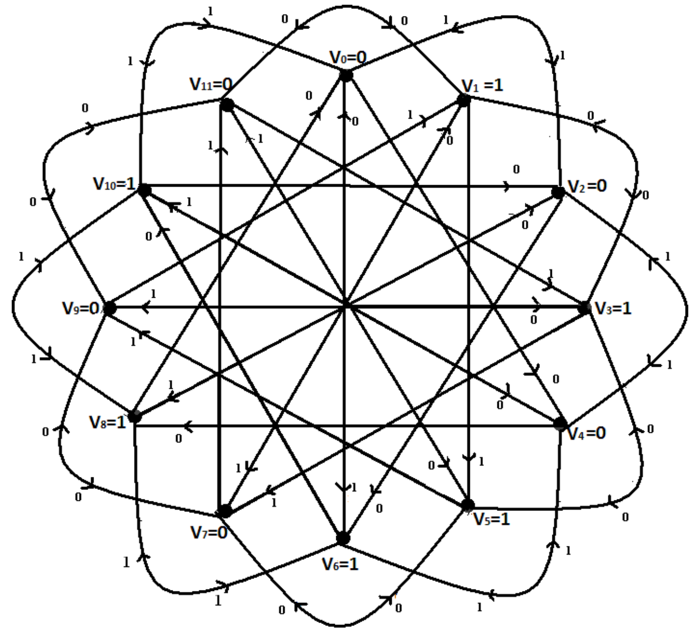


Fig. 3. Product cordial labeling for ComCay(Z_{12}, (7,5,11))

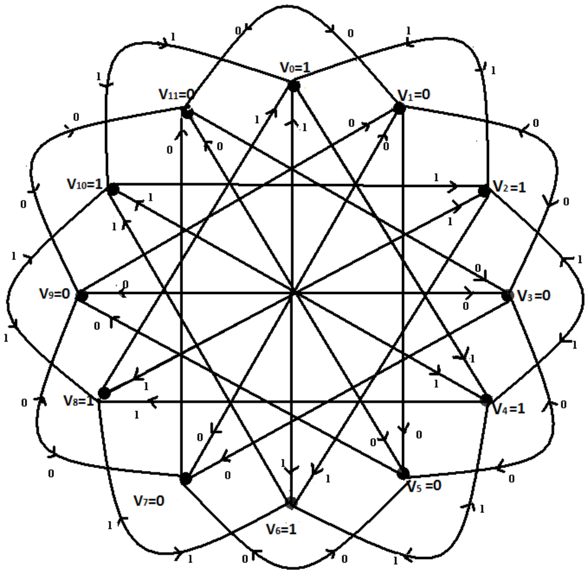
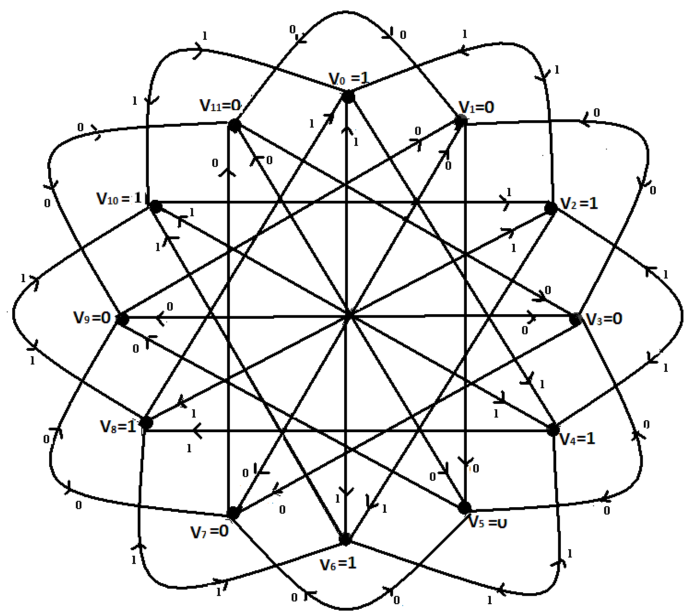


Fig.4. Product E-cordial labeling for ComCay(Z_{12}, (7,5,11))



By the above theorem, using the map f on V and there by the induced map f* on E, we have the number of arcs labeled 0 is 2n and the number of vertices labeled 0 is n/2. Thus the total number of zeroes on vertices and arcs taken together is (n/2)+2n =5n/2 Also, the number of arcs labeled by 1 is 2n and the number of vertices labeled by 1 is n/2. Thus, the total number of one's on vertices and arcs taken together is (n/2)+2n =5n/2 .Thus the number of zeroes on the vertices and arcs taken together differ by at most 1 with the number of one's on vertices and arcs taken together. Hence The competition graph ComCay(Z_n,(a,b,c)) admits total product cordial labeling

Theorem 3.6: The competition graph ComCay(Z_n,(a,b,c)) is Product E-cordial.

Proof: From the construction of Competition graph ComCay(Z_n,(a,b,c)) using algorithm, we have n vertices and 4n arcs.

To prove Comcay(Z_n, (a,b,c)) admits Product E-Cordial labeling, we have to show that there exists a function f: E → {0,1} such that the induced function f* on V is defined as $f^*(v_i) = \Pi\{f(v_iv_j) \;/\; v_iv_j \in E\ (E_{aa}, E_{bb}, E_{ca}, E_{cb})\}$ satisfies the property that the number of vertices labeled 0 and the number of vertices labeled 1 differ by atmost 1 and the number of arcs labeled 0 and the number of arcs labeled 1 differ by at most 1.

Consider the arbitrary vertex $v_i \in V$. Using step 7 of algorithm we define a map $f:E \rightarrow \{0,1\}$ as follows.

(i) For all $v_iv_j \in E_{ca}$,

For $0 \le i \le (n/2)-1$, $f(v_iv_{i+(n/2)}) = \begin{cases} 0, & i \equiv 1 \pmod 2 \\ 1, & i \equiv 0 \pmod 2 \end{cases}$

For $(n/2) \le i \le n-1$, $f(v_iv_{i-(n/2)}) = \begin{cases} 0, & i \equiv 1 \pmod 2 \\ 1, & i \equiv 0 \pmod 2 \end{cases}$

(ii) For all $v_iv_j \in E_{cb}$, and $0 \le i \le n-1$

$f(v_iv_{(i+(n/2)-2) \bmod n}) = \begin{cases} 1, & i \equiv 0 \pmod 2 \\ 0, & i \equiv 1 \pmod 2 \end{cases}$

(iii) For all $v_iv_j \in E_{aa}$ and $0 \le i \le n-1$

$f(v_iv_{(i+2) \bmod n}) = \begin{cases} 1, & i \equiv 0 \pmod 2 \\ 0, & i \equiv 1 \pmod 2 \end{cases}$

(iv) For all $v_iv_j \in E_{bb}$ and $2 \le i \le n-1$

$f(v_iv_{i-2}) = \begin{cases} 1, & i \equiv 0 \pmod 2 \\ 0, & i \equiv 1 \pmod 2 \end{cases}$

$f(v_0v_{n-2}) = 1$, $f(v_1v_{n-1}) = 0$

From these definitions of the labeling functions, we have the total number of arcs labeled 0 is (n/2)+ (n/2)+(n/2)+(n/2) = 2n and the number of arcs labeled 1 is (n/2)+ (n/2) + (n/2) +(n/2) = 2n. Hence the number of arcs labeled 0 and the number of arcs labeled 1differ by at most 1.

In order to get the labels for the vertices define the induced map $f^* : V \rightarrow \{0,1\}$ such that $f^*(v_i) = \Pi\{f(v_iv_j) / v_iv_j \in E\ (E_{aa}, E_{bb}, E_{ca}, E_{cb})\}$. For all $v_iv_j \in E_{aa}, E_{bb}, E_{ca}$ & E_{cb}:

$f^*(v_i) = \Pi f(v_iv_j) = \begin{cases} 0\times0\times0\times0 = 0, & i \equiv 1 \pmod 2 \\ 1\times1\times1\times1 = 1, & i \equiv 0 \pmod 2 \end{cases}$

where $v_i \in V$ & $0 \le i \le n-1$

Under this map the number of vertices labeled 0 is n/2 and the number of vertices labeled 1 is n/2 . Thus the number of vertices labeled 0 and the number of vertices labeled 1 differ by at most 1. Hence, ComCay(Z_n, (a,b,c)) admits Product E-Cordial labeling.

Example 3.4: Competition graph of the Cayley digraphs associated with Z_{12} and its Product E-cordial labeling is shown in Fig.4.

Theorem 3.7: The competition graph ComCay(Z_n,(a,b,c)) admits total product E-Cordial labeling.

Proof: From the construction of Competition graph ComCay(Z_n,(a,b,c),we have n vertices and 4n arcs.

To prove Comcay(Z_n, (a,b,c)) admits total product E-Cordial labeling, we have to show that there exists a function $f: E \rightarrow \{0,1\}$ such that the induced function f^* on V is defined as $f^*(v_i) = \Pi\{f(v_iv_j) \mid v_iv_j \in E\ (E_{aa}, E_{bb}, E_{ca}, E_{cb})\}$ which satisfies the property that the number of zeroes on the vertices and arcs taken together differ by at most 1 with the number of one's on vertices and arcs taken together.

By the above theorem, using the map f on E and there by the induced map f^* on V, we have the number of arcs labeled 0 is 2n and the number of vertices labeled 0 is n/2. Thus the total number of zeroes on vertices and arcs taken together is (n/2)+2n =5n/2 Also, the number of arcs labeled by 1 is 2n and the number of vertices labeled by 1 is n/2. Thus the total number of one's on vertices and arcs taken together is (n/2)+2n =5n/2 .Thus the number of zeroes on the vertices and arcs taken together differ by at most 1 with the number of one's on vertices and arcs taken together. Hence The competition graph ComCay(Z_n,(a,b,c)) admits total product E-cordial labeling.

Conclusion

In this paper we have presented an algorithm and proved that the competition graph of Cayley digraph associated with Z_n denoted by ComCay(Z_n, (a,b,c)) admits Z_3- magic, E-cordial, total E-cordial, Product cordial, total Product Cordial, Product E-Cordial and total Product E-cordial labelings.

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