Effect of Material Nonlinearity on Deflection of Beams and Frames

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Abstract

Background/Objectives: To examine the effect of material nonlinearity of mild steel on the total deflection of beams and frames. Method of Analysis: In the linear analysis, value of Young's modulus is same in all over the analysis, hence the load-deflection plot is also linear, whereas in material nonlinear analysis of structures the elastic modulus is constant upto yield stress and it decreases thereafter. In the present study linear and material nonlinear analysis of beams and frames are carried out using ANSYS mechanical APDL software. The linear deflection of the structures is also computed using a finite element based MATLAB code. Material nonlinearity is incorporated using bilinear stress-strain curves with tangent modulus E/65. E is the elastic modulus of the material. It has been found that the tangent modulus of stress-stress curve of mild steel is more resemble to E/3, so this value is also considered separately for the nonlinear analysis. In this paper cantilever beam with an end load and a two storey building frame with horizontal load is considered with above tangent modulus. The loads are applied incrementally, deflection and stress at each load increment is computed. Findings: In linear analysis the full strength of the material is not utilized and it is assumed that material will fail after reaching the yield stress. In material nonlinear analysis the strain hardening property is considered by taking the tangent modulus after the yield stress. The linear analysis is giving linear variation of deflection and stress with respect to load, but for material nonlinear (bilinear) the deflection and stress will be same as that of linear upto yield point and after that the deflection is found more and stress value found less than that of linear values. Application/Improvements: For the economical usage of materials nonlinear analysis is preferred over the linear analysis, because it is giving the actual behavior of structures and we are utilizing the maximum capacity of the material.

Keywords: Bilinear Stress-strain Curve, Finite Element Method, Linear Analysis, Load-Deflection Behavior, Nonlinear Analysis

1. Introduction

A linear finite element analysis is used when the stress is directly proportional to the strain, which means it follows Hook's law of elasticity. Linear analysis is carried out by using equation $\{F\} = [K]\{\Delta\}$, where F is the force matrix and Δ is the deflection and K is the stiffness matrix. It means that the correlation of force and displacement is linear. But for materials like steel the stress-strain behaviour is linear upto some point (yield point), thereafter it behaves non-linear nature. In order to obtain the accurate and realistic nature of material behaviour, nonlinear analysis is preferred over linear analysis. Non-linear analysis is not easy for manual computation, so finite element based computer software's like ANSYS is used for nonlinear analysis.

Linear finite element formulation is based on two assumptions

1. Stress-strain relation is linear throughout the analysis.

2. Strain displacement relation is linear

There are mainly two important types of nonlinearity related to structures. They are material nonlinearity and geometrical nonlinearity. The above two assumptions are for neglecting these two nonlinearities and for making analysis simple. The first assumption, stress-strain relation is linear throughout the analysis is neglecting the effect of material nonlinearity and the second assumption that strain displacement relation is linear is neglecting the geometrical nonlinearity.

In material nonlinear analysis the structure will not recover its real shape after the removal of load. Material nonlinearity is related with the inelastic characteristics of materials like steels. Inelastic behavior is characterized by a force-deformation relationship. The general forcedeformation relationship shows that once a structure accomplishes its yield strength, supplementary loading will cause response to deviate from the preliminary tangent stiffness. Nonlinear behaviour may then increase (hardening) to an ultimate point before decreasing (softening) to a remaining strength value. Material nonlinearity is due to nonlinear constitutive matrix [D]. For most of the materials nonlinear stress strain curves are obtained experimentally by conducting uniaxial tension/ compression tests. These results are enough to carry out nonlinear analysis of homogeneous materials like steel.

Material nonlinearity is incorporated using bilinear stress-strain curves with tangent modulus E/65. E is the elastic young's modulus of the material. The value of strain-hardening rate was attained through a huge number of elastic-plastic large deflection investigation made by ¹. Geometric and material non-linear analysis technique for frames by means of a solution procedure of diminishing the remaining displacements was presented by ². This nonlinear solution procedure was assumed to be optimum in Newton Raphson method because it tracks the easiest way for achieving the convergence. He introduced the idea of the operative tangent stiffness matrix and it was found to be well-organized, logical and simple in handling the nonlinear analysis of structures³ performed an analytical simplified technique for creation of the average stress-average strain connection of flawed steel plates by considering the effect of both material and geometric nonlinearities. Idealized bilinear stress-strain model was used for the analysis⁴ gives the results of an examination of the post-buckling characteristic and ultimate strength behaviour of flawed pitted steel plates used in ship and other marine structures. Ideal bilinear stress-strain curve model was used for the analysis of their structures.

Nonlinear analysis of torsion effect in RC structural members after getting the preliminary crack was performed by ^{5,6} developed an innovative finite element layered model used for beam-column elements in RC frame structures by means of an automatic incremental procedure. Both material and geometric nonlinearities in frame structures was considered by the suggested model⁷ studied about the performance of deep beam for numerous span/depth ratio by using finite element based ANSYS under 2-point loading of 50 KN and also studied about the stress distribution of deep beam. Finite element formulation for material and geometric nonlinear model for outwardly prestressed beams was demonstrated by ⁸. The numerically replicated behaviour was confirmed by evaluation with experimental tests existing for steel concrete composite beams and concrete beams.

Analytical model for the steel frame for conducting the nonlinear dynamic analysis was explained by 9. Kinematic strain hardening characteristic was modeled by using the bounding surface idea. The analysis was executed by means of the super minicomputer and computer graphics. Material nonlinear analysis of reinforced concrete beams by taking effect of the tension softening branch and the effect of bond slip was studied by ¹⁰. Moment-curvature characteristics of reinforced concrete sections formerly created by the means of section analysis was used for the analysis¹¹ developed a computer program for the analysis of the structural problems by taking the account for the effect of nonlinear situations of material behaviors under increasing loads. The second order effects were also considered for the analysis. The matrix replacement method was used for the development of the computer program. The method used for analysis takes the effects of axial forces on the stiffness of the structural member by means of the stability functions and the effects of plastic hinges by methodically varying the stiffness matrix in each existence of the plastic hinges. A non-linear analysis of 3-Dimensional steel frames was established by ¹². The analysis was considered for both geometric and material nonlinearities. Material nonlinearity takes the steady yielding related with member forces and geometric nonlinearity comprises the second order effects. The material nonlinearity at a section of the structure was measured by means of the thought of P–M hinges¹³ presents a nonlinear finite element computer program, ANSYS version 12.0 developed for the analysis of steel-concrete composite beam. A 3-Dimensional finite element model was established and the analytical outcomes of load-deflection response was compared with available experimental tests. Parametric studies were carried out to investigate the effect of some important material and geometrical parameters. In the present research paper material nonlinearity effect on the total deflection and stress of building structures

were discussed. The methodology process is explained in the coming sections.

2. Methodology

The material nonlinearity is considered for the analysis of beams and frame structures. For checking the effect of material nonlinearity on deflection and stress of structures, load is given gradually and at every load addition, deflection and stresses are noted. Initially linear analysis of beams and frames are carried out using FEM based MATLAB code. Cantilever beam with tip point load is formulated by considering two nodes at each end of the beam and two storey frame is formulated by taking nine nodes as shown in Figures 1 and 2 respectively. Material properties used for nonlinear analysis of beams and frames are shown in Table 1.

| Tabl | e 1. | Materi | al pro | perties | used | for | nonl | inear |
|-------|------|---------|--------|---------|------|-----|------|-------|
| analy | ysis | of bear | ns an | d frame | s | | | |

| Material properties | Values |
|-------------------------------|---|
| Elastic modulus (E) | 2.1 x 10 ⁵ N/mm ² |
| Yield stress | 250 N/mm ² |
| Poisson's ratio | 0.3 |
| Tangent modulus (Et) for E/65 | 3231 N/mm ² |
| Tangent modulus (Et) for E/3 | 7 x 10 ⁴ N/mm ² |



Figure 1. Cantilever beam with point load.

Nonlinear analysis of beams and frames are carried out using finite element based software ANSYS mechanical APDL. For nonlinear analysis, bilinear stress-strain curve with young's modulus E upto yield point and after that straight line with tangent modulus E/65 and E/3 is considered as shown in Figures 3 and 4 respectively. Material model adopted for the present study is defined as nonlinear » inelastic » rate independent » von-mises » isotropic hardening plasticity » bilinear stress strain (Given in Ansys software). The load is applied step by step and deflection and stresses are noted at each load step. For nonlinear analysis we are considering constant young's modulus upto yield stress and after that tangent modulus is considered, while for linear analysis same young's modulus is used throughout the analysis.











Figure 4. Bilinear model of stress–strain curve with tangent modulus E/3.

| Load in | Deflection Under the Load (mm) | | | | Maximum Stress in Beam (N/mm ²) | | | |
|---------|--------------------------------|---------------------------|-----------------------------------|------------------------------------|---|---------------------------|-----------------------------------|--|
| KN | Nonlinear analysis E/65 | Nonlinear analysis E/3 | Linear analysis using Ansys | Linear analysis using matlab | Nonlinear analysis E/65 | Nonlinear analysis E/3 | Linear analysis using Ansys | |
| 1 | 0.10 | 0.10 | 0.10 | 0.10 | 8.31 | 8.31 | 8.31 | |
| 5 | 0.50 | 0.50 | 0.51 | 0.50 | 41.55 | 41.50 | 41.50 | |
| 10 | 1.01 | 1.01 | 1.01 | 0.99 | 83.10 | 83.10 | 83.10 | |
| 20 | 2.02 | 2.02 | 2.02 | 1.98 | 166.18 | 166.18 | 166.18 | |
| 30 | 3.03 | 3.03 | 3.03 | 2.98 | 249.27 | 249.27 | 249.27 | |
| 40 | 4.60 | 4.12 | 4.04 | 3.97 | 332.36 | 297.38 | 297.37 | |
| 50 | 15.52 | 5.89 | 5.05 | 4.96 | 415.45 | 318.48 | 310.05 | |
| 60 | 47.32 | 8.06 | 6.06 | 5.95 | 498.54 | 376.27 | 348.14 | |

 Table 2. Stress and deflection of cantilever steel beam with point load at the tip

Table 3. Deflection and stress of two-storey steel frame with horizontal load

| Load in | Horizontal Deflection of the Frame (mm) | | | | Maximum Stress in Frame (N/mm ²) | | |
|---------|---|---------------------------|-----------------------------------|------------------------------------|--|---------------------------|-----------------------------------|
| KN | Nonlinear analysis E/65 | Nonlinear analysis E/3 | Linear analysis using Ansys | Linear analysis using matlab | Nonlinear analysis E/65 | Nonlinear analysis E/3 | Linear analysis using Ansys |
| 2 | 19.00 | 19.00 | 19.00 | 19.02 | 38.00 | 38.00 | 38.00 |
| 4 | 38.00 | 38.00 | 38.02 | 38.04 | 76.13 | 76.13 | 76.13 |
| 6 | 57.10 | 57.10 | 57.00 | 57.06 | 114.20 | 114.20 | 114.20 |
| 8 | 76.00 | 76.00 | 76.00 | 76.08 | 152.27 | 152.27 | 152.27 |
| 10 | 95.00 | 95.00 | 95.00 | 95.09 | 190.34 | 190.34 | 190.34 |
| 12 | 114.24 | 114.24 | 114.00 | 114.11 | 228.40 | 228.40 | 228.40 |
| 13 | 123.76 | 123.76 | 123.58 | 123.62 | 247.60 | 247.60 | 247.44 |
| 14 | 133.30 | 133.30 | 133.00 | 133.13 | 266.67 | 266.67 | 266.47 |
| 16 | 152.36 | 152.36 | 152.00 | 152.15 | 304.78 | 304.78 | 304.54 |
| 18 | 175.68 | 172.80 | 171.12 | 171.17 | 307.13 | 303.20 | 342.60 |
| 20 | 211.45 | 195.20 | 190.13 | 190.19 | 284.96 | 297.60 | 380.68 |
| 22 | 262.37 | 221.65 | 209.14 | 209.21 | 304.29 | 314.80 | 418.75 |
| 24 | 353.14 | 251.50 | 228.16 | 228.23 | 294.00 | 338.56 | 456.80 |
| 25 | 446.14 | 266.67 | 237.66 | 237.73 | 305.00 | 350.50 | 475.85 |

3. Results and Discussions

Linear Analysis of beam and frame is carried out using finite element based ANSYS mechanical APDL software and a MATLAB code established using finite element technique, whereas material nonlinear analysis is carried out by using Ansys APDL. The nonlinear and linear stress and deflection of cantilever beam is shown in Table 2 and building frame is shown in Table 3. The deflection of the cantilever steel beam with end point load is same for linear and nonlinear case upto a load of 30 KN and the stresses at that time is nearly 250 N/mm² which is identical to the yield stress of the material (Given in Table 1). In linear analysis of mild steel maximum level of stress before failure is its yield stress (250 N/mm²) and it is assumed that the material will fail after reaching the yield stress. For considering the strain hardening property of mild steel this study considered

two models, one with tangent modulus one by sixty-five of young's modulus and other with tangent modulus onethird of young's modulus. In first case we are not using that much reserved strength of material but second case is more resembles to the original stress strain behavior of mild steel. From the tables it is clear that bilinear curve with lesser tangent modulus is showing greater value of deflection and lesser value stress when compared to other model. In case of beam with 60 KN load gives a deflection of 48 mm and stress 498 N/mm² for model with tangent modulus E/65, whereas it is 8 mm and 375 N/mm² for model with tangent modulus E/3, which resembles more with actual behavior. Linear deflection and stresses for 60 KN load is 6 mm and 350 N/mm² which is near to the model with tangent modulus E/3. The nature of deflection and stress in frame is also similar to the beam. The loaddeflection and load-stress diagram for beam is shown in Figure 5 and Figure 6 respectively and load-deflection diagram for frame is shown in Figure 7.



Figure 5. Load vs deflection diagram for cantilever beam with end load.



Figure 6. Load vs stress diagram for cantilever beam with end load.



Figure 7. Load vs deflection diagram for frame with horizontal load.

4. Conclusions

This research article studies the effect of material nonlinearity of mild steel on total deflection and stresses of beams and frames. The studies on finite element based software Ansys and theoretical outcomes related with them lead to the succeeding conclusions:

- Strain hardening parameter (Tangent modulus) is effecting significantly on total deflection and stress. If we are considering the higher value for tangent modulus, the stress will be less and the deflection will be more.
- The linear analysis is showing linear variation of deflection and stresses with respect to applied loads.
- Nonlinear analysis the deflection and stress is found the same as that of linear upto yielding and thereafter the deflection is found more and stress is less as compared to the linear value.
- Linear deflection that we obtained from Ansys software and FEM based code by MATLAB values are almost same

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