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Dominating and Locating Sets in the Multiplication of a Graph

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Abstract

A set S of vertices of a graph G is a dominating set of G if every vertex in V(G)\S is adjacent to some vertex in S, and S is a total dominating set of G if every vertex of G is adjacent to at least one vertex of S. An ordered set W of vertices of a connected graph G is a locating set for G if distinct vertices have distinct codes with respect to W. In this paper, we study the domination and location in the multiplication of a graph. We find the necessary and sufficient conditions for the dominating and locating sets in the multiplication of a graph to exist. We also determine bounds or the exact domination and location numbers of this graph.

Keywords: Dominating Set, Locating Code, Locating Set, Multiplication of Graph, Total Dominating Set

1. Introduction

For a vertex v of a graph G, recall that a *neighbor* of v is a vertex adjacent to v in G. Also, the *neighborhood* (or *open neighborhood*) N(v) of v is the set of neighbors of v. The *degree* deg(v) of v is the cardinality of N(v) and the *maximum degree* of G is $\Delta G=\max\{\deg(v): v\in V(G)\}$ The *closed neighborhood* N[v] is defined as $N[v]=N(v)\cup\{v\}$.

A vertex v in a graph G is said to *dominate* itself and each of its neighbors, that is, v dominates the vertices in its closed neighborhood N[v]. Therefore, v dominates 1+deg(v) vertices of G. A set S of vertices of G is a *dominating set* of G if every vertex of G is dominated by some vertex in S. Equivalently, a set S of vertices of G is a dominating set of G if every vertex in V(G)\S is adjacent to some vertex in S. A *minimum dominating set* in G is a dominating set of minimum cardinality. The cardinality of a minimum dominating set is called the *domination number* of G and is denoted by $\gamma(G)$. A set S of vertices of G is a *total dominating set* of G if every vertex of G is adjacent to at least one vertex of S. A *minimum total dominating set* in G is a total dominating set of minimum cardinality. The cardinality of a minimum total dominating set is called the *total domination number* of G and is denoted by $\gamma_t(G)$.

Let G be a connected graph. For an ordered set $W=\{w_1, w_2, ..., w_k\}$ of vertices of G and a vertex v of G, the *locating code* (or simply the *code*) of v with respect to W is the k-vector

$$c_W(v)=(d(v, w_1), d(v, w_2), ..., d(v, w_k)).$$

The set W is a *locating set* for G if distinct vertices have distinct codes. A locating set containing a minimum number of vertices is a *minimum locating set* for G. The *location number* loc(G) is the number of vertices in a minimum locating set for G. Some of these concepts may be found in[1] and are investigated in[2–6]. Recently, the bound of the medium domination number of Jahangir graph $J_{m,n}$ was studied by Ramachandran and Parvathi[7], and the perfect vertex (edge) domination was studied in fuzzy graphs by Ramya and Lavanya[8].

For a graph G and a mapping ξ from V(G) into the positive integers, the ξ-*multiplication* of G, denoted by G^ξ, is a graph whose vertices are all ordered pairs (v, j), where $v\in V(G)$ and $1\le j\le\xi(v)$, and two vertices (u, i) and (v, j) are

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joined by an edge in G^{ξ} if and only if u and v are adjacent in G. This concept was introduced by Bezegová and Ivančo[9]. Observe that if G is a graph, then the ξ-multiplication G^{ξ} has order $|V(G^{\xi})|= \Sigma_{v\in V(G)}\xi(v)\}$ and size $|E(G^{\xi})|= \frac{1}{2}\ \Sigma_{v\in V(G)}\Sigma_{u\in N(G)}\xi(v)\}\xi(u)\}$. The degree of a vertex (v, j) of G^{ξ} is equal to and the maximum degree of G^{ξ} is $\Delta(G^{\xi})=\max\{\Sigma_{u\in N(v))}\xi(u):v\in V(G)\}$.

In this study we will hereinafter use the auxiliary results from these studies.

Remark 1.1[1] *If G is a graph of order n, then $1\leq\gamma(G)\leq n$.*

Remark 1.2[1] *Let G be a graph of order n. Then $\gamma(G)=1$ if and only if G contains a vertex v of degree n-1.*

Remark 1.3[1] *Let G be a graph of order n. Then $\gamma(G)=n$ if and only if $G\simeq\overline{K}_n$.*

Remark 1.4[1] *If G is a graph of order n containing no isolated vertices, then $2\leq\gamma_t(G)\leq n$.*

Remark 1.5[1] *Let G be a graph of order n containing no isolated vertices. Then $\gamma_t(G)=2$ if and only if G is a star or a double star.*

Remark 1.6[1] *Let G be a graph of order n containing no isolated vertices. Then $\gamma_t(G)=n$ if and only if $G\simeq kK_2$ for a positive integer k.*

Remark 1.7[1] *If G is a connected graph of order $n\geq2$, then $1\leq loc(G)\leq n-1$.*

Remark 1.8[1] *A connected graph G of order $n\geq2$ has location number 1 if and only if $G\simeq P_n$.*

Remark 1.9[1] *A connected graph G of order $n\geq2$ has location number n-1 if and only if $G\simeq\overline{K}_n$.*

2. The Domination Number in the Multiplication of a Graph

$G\simeq\overline{K}_n$

In this section we consider any graph G and we denote the set of all dominating sets of G by D(G). By Remake 1.1, we then have the following result.

Proposition 2.1 *Let G be a graph and let ξ be a mapping from V(G) into positive integers. Then $1\leq\gamma(G^{\xi})\leq|V(G^{\xi})|$.*

Theorem 2.1 *Let G be a graph of order n and let ξ be a mapping from V(G) into positive integers. Then $\gamma(G^{\xi})=1$ if and only if G contains a vertex v of degree n-1 and $\xi(v)=1$.*

Proof. Let v be a vertex of degree n-1 and let ξ(v)=1. It follows that a vertex (v, 1) has degree $|V(G^{\xi})|$-1. By Remark 1.2, $\gamma(G^{\xi})=1$. For the converse, suppose that $\gamma(G^{\xi})=1$. By Remark 1.2, G^{ξ} contains a vertex (v, j) of degree $|V(G^{\xi})|$-1 for some $v\in V(G)$ and $1\leq j\leq\xi(v)$. Assume that ξ(v)1, then there exists a vertex (v, i) adjacent to (v, j) for some $1\leq i\neq j\leq\xi(v)$. It contradicts the fact that (v, i) and (v, j) are not adjacent in G^{ξ}. Thus, ξ(v)=1. Since (v, 1) is adjacent to every vertex in $V(G^{\xi})\backslash\{(v, 1)\}$, v is adjacent to every vertex in $V(G)\backslash\{v\}$. Therefore, v has degree n-1.

Theorem 2.2 *Let G be a graph of order n and let ξ be a mapping from V(G) into positive integers. Then $\gamma(G^{\xi})=|V(G^{\xi})|$ if and only if $G\simeq\overline{K}_n$.*

Proof. Let $G\simeq\overline{K}_n$. It follows that $G^{\xi}\simeq\overline{K}_{|V(G^{\xi})|}$. By Remark 1.3, $\gamma(G^{\xi})=|V(G^{\xi})|$. For the converse, suppose that $\gamma(G^{\xi})=|V(G^{\xi})|$. By Remark 1.3, $G^{\xi}\simeq\overline{K}_{|V(G^{\xi})|}$. Since G^{ξ} contains all isolated vertices, G also contains all isolated vertices. Thus, $G\simeq\overline{K}_n$.

In the next two results we find the necessary and sufficient conditions for the dominating sets in the multiplication of a graph to exist.

Theorem 2.3 *Let G be a graph and let ξ be a mapping from V(G) into positive integers. If $T\subseteq V(G^{\xi})$ is a dominating set of G^{ξ}, then $S=\{v: (v, j)\in T\}$ is a dominating set of G.*

Proof. We show that $S=\{v: (v, j)\in T\}$ is a dominating set of G. Since every vertex in $V(G^{\xi})\backslash T$ is adjacent to some vertex in T, every vertex in $V(G)\backslash S$ is adjacent to some vertex in S. Thus, S is a dominating set of G.

Theorem 2.4 *Let G be a graph and let ξ be a mapping from V(G) into positive integers. If $S\in D(G)$, then $T=\{(v, j): v\in S, 1\leq j\leq\xi(v)\}$ is a dominating set of G^{ξ}.*

Proof. We show that $T=\{(v, j): v\in S, 1\leq j\leq\xi(v)\}$ is a dominating set of G^{ξ}. If $x\in T$, then x is dominated by itself. Thus, we can may assume that $x\notin T$ and so $x\in V(G^{\xi})\backslash T$. We can write x in the form x=(u, i) for some $u\in V(G)\backslash S$ and $1\leq i\leq\xi(u)$. We show that x is adjacent to some vertex in T. Since S is a dominating set of G, u is adjacent to some vertex v in S. Thus, x=(u, i) is adjacent to some vertex (v, j) in T. Therefore, T is a dominating set of G^{ξ}.

Notice that if S is a minimum dominating set of a graph G, then $T=\{(v, j): v\in S, 1\leq j\leq\xi(v)\}$ is not necessary to be a minimum dominating set of G^{ξ}. We consider a star G on vertices a, b, c and d with the mapping ξ from V(G) into positive integers defined by ξ(a)=4 and ξ(b)=ξ(c)=ξ(d)=1. It is clear that {a} is a minimum dominating set of G (see Figure 1), but {(a, 1), (a, 2), (a, 3), (a, 4)} is not a minimum dominating set of G^{ξ} (see Figure 2).

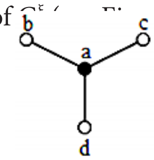

Figure 1. A minimum dominating set of a star G.

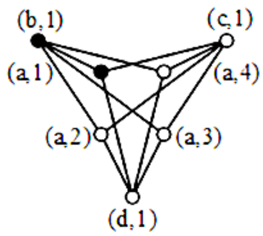

Figure 2. A minimum dominating set of a graph G^{ξ}.

Applying Theorem 2.4, we can determine an upper bound for the domination number in the multiplication of a graph as follows.

Corollary 2.1 *Let G be a graph and let ξ be a mapping from V(G) into positive integers. Then* $\gamma(G^{\xi} \leq \min\{\Sigma_{v\in S}\xi(v):S\in D(G)\}$.

Proof. Let U be a minimum dominating set of G^{ξ}. By Theorem 2.4, we obtain that for any $S\in D(G)$, the set $T=\{(v, j): v \in S, 1\leq j\leq \xi(v)\}$ is a dominating set of G^{ξ}. Therefore, $|U|\leq|T|= \Sigma_{v\in S}\xi(v)$ and it follows that $\gamma(G^{\xi})=|U|\leq\min\{\Sigma_{v\in S}\xi(v):S\in D(G)\}$.

3. The Total Domination Number in the Multiplication of a Graph

In this section we consider any graph G containing no isolated vertices and we denote the set of all total dominating sets of G by $D_t(G)$. By Remark 1.4, we then have the following result.

Proposition 3.1 *Let G be a graph containing no isolated vertices and let ξ be a mapping from V(G) into positive integers. Then* $2\leq\gamma_t(G^{\xi})\leq|V(G^{\xi})|$.

Theorem 3.1 *Let G be a graph containing no isolated vertices and let ξ be a mapping from V(G) into positive integers. Then* $\gamma_t(G^{\xi})=2$ *if and only if G is a star or a double star.*

Proof. If G is a star with a center vertex u, then all leaves v are adjacent to a vertex u in G. It follows that all vertices (v, j) are adjacent to all vertices (u, i) in G^{ξ}, where $1\leq i\leq\xi(u)$ and $1\leq j\leq\xi(v)$. Choose $T=\{(u, 1), (v, 1)\}$ for any leaf v. It is clear that T is a total dominating set of G^{ξ}. Thus, $\gamma_t(G^{\xi})=2$. If G is a double star with center vertices u and v, then u and v are adjacent to each other and all leaves w are adjacent to a vertex u or v in G. It follows that all vertices (u, i) and (v, j) are adjacent to each other and all vertices (w, k) are adjacent to all vertices (u, i) or (v, j) in G^{ξ}, where $1\leq i\leq\xi(u)$, $1\leq j\leq\xi(v)$ and $1\leq k\leq\xi(w)$. Choose $T=\{(u, 1), (v, 1)\}$. It is easy to see that T is a total dominating set of G^{ξ}. Thus, $\gamma_t(G^{\xi})=2$. For the converse, suppose that $\gamma_t(G^{\xi})=2$. Say that $T=\{(u, i), (v, j)\}$ is a total dominating set of G^{ξ} for some $u, v \in V(G)$, $1\leq i\leq\xi(u)$ and $1\leq j\leq\xi(v)$. Thus, all vertices of G^{ξ} are adjacent to at least one vertex of (u, i) and (v, j). It follows that all vertices of G are adjacent to at least one vertex of u and v. If they are adjacent to either u or v, then G is a star. If they are adjacent to both u and v, then G is a double star.

Theorem 3.2 *Let G be a graph containing no isolated vertices and let ξ be a mapping from V(G) into positive integers. Then* $\gamma_t(G^{\xi})=|V(G^{\xi})|$ *if and only if* $G^{\xi}\cong kK_2$ *for a positive integer k and ξ(v)=1 for all* $v\in V(G)$.

Proof. Let $G\cong kK_2$ for a positive integer k and let ξ(v)=1 for all $v\in V(G)$. It follows that $G^{\xi}\cong G\cong kK_2$. By Remark 1.6, $\gamma_t(G^{\xi})=|V(G^{\xi})|$. For the converse, suppose that $\gamma_t(G^{\xi})=|V(G^{\xi})|$. By Remark 1.6, $G^{\xi}kK_2$ for some positive integer k. Assume, to the contrary, that $\xi(v) \neq 1$ for some vertex $v\in V(G)$. Thus, there exist vertices (v, i) and (v, j), for some $1\leq i\leq\xi(v)$, such that they are adjacent to each other or adjacent to distinct vertices. It contradicts the fact that (v, i) and (v, j) are not adjacent to each other and adjacent to the same vertices in G^{ξ}. Thus, ξ(v)=1 for all $v\in V(G)$ and so $G\cong kK_2$.

In the next two results we find the necessary and sufficient conditions for the total dominating sets in the multiplication of a graph to exist.

Theorem 3.3 *Let G be a graph containing no isolated vertices and let ξ be a mapping from V(G) into positive integers. If* $T\subseteq V(G^{\xi})$ *is a total dominating set of* G^{ξ}, *then* $S=\{v: (v, j)\in T\}$ *is a total dominating set of G.*

Proof. We show that S={v: (v, j) ∈ T} is a total dominating set in G. Since every vertex of G^{ξ} is adjacent to at least one vertex of T, every vertex of G is adjacent to at least one vertex of S. Thus, S is a total dominating set in G.

Theorem 3.4 *Let G be a graph containing no isolated vertices and let ξ be a mapping from V(G) into positive integers. If* $S \in D_t(G)$*, then* $T=\{(v, j): v \in S, 1\le j\le \xi(v)\}$ *is a total dominating set of* G^{ξ}*.*

Proof. We show that $T=\{(v, j): v \in S, 1\le j\le \xi(v)\}$ is a total dominating set of G^{ξ}. If $x \in V(G^{\xi})$, then we can write x in the form x=(u, i) for some u ∈ V(G) and 1≤i≤ξ(u). We show that x is adjacent to at least one vertex in T. Since S is a total dominating set of G, u is adjacent to at least one vertex v in S. This means that x=(u, i) is adjacent to at least one vertex (v, j) in T. Therefore, T is a total dominating set of G^{ξ}.

It is clear that if S is a minimum total dominating set of a graph G, then $T=\{(v, j): v \in S, 1\le j\le \xi(v)\}$ is not necessary to be a minimum total dominating set of G^{ξ}. We consider a graph $2K_2$ on vertices a, b, c and d with the mapping ξ from V(G) into positive integers defined by ξ(a)=ξ(b)=ξ(c)=ξ(d)=2. It is clear that the vertex set of $2K_2$ is a unique minimum total dominating set of $2K_2$ (see Figure 3), but the vertex set of $(2K_2)^{\xi}$ is not a minimum total dominating set of $(2K_2)^{\xi}$ (see Figure 4).

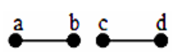

Figure 3. A minimum total dominating set of a graph $2K_2$.

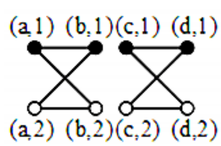

Figure 4. A minimum total dominating set of a graph $(2K_2)^{\xi}$.

According to Theorem 3.4, we can determine an upper bound for the total domination number in the multiplication of a graph as below.

Corollary 3.1 *Let G be a graph containing no isolated vertices and let ξ be a mapping from V(G) into positive integers. Then* $\gamma_t(G^{\xi})\le \min\{\Sigma_{v\in S}\xi(v): S \in D_t(G)\}$.

Proof. Let U be a minimum dominating set of G^{ξ}. By Theorem 3.4, we obtain that for any $S \in D_t(G)$, the set $T=\{(v, j): v \in S, 1\le j\le \xi(v)\}$ is a total dominating set of G^{ξ}. Therefore, $|U|\le|T|=\Sigma_{v\in S}\xi(v)$ and it follows that $\gamma_t(G^{\xi})=|U| \le \min\{\Sigma_{v\in S}\xi(v): S \in D_t(G)\}$.

4. The Location Number in the Multiplication of a Graph

In this section we consider the connected graph G of order n≥2 and we denote the set of all locating sets of G by L(G). By Remark 1.7, we obtain the following result.

Proposition 4.1 *Let G be a connected graph of order n≥2 and let ξ be a mapping from V(G) into positive integers. Then* $1\le loc(G^{\xi})\le |V(G^{\xi})|-1$.

Theorem 4.1 *Let G be a connected graph of order n≥2 and let ξ be a mapping from V(G) into positive integers. Then* $loc(G^{\xi})=1$ *if and only if* $G\cong P_n$ *and ξ(v)=1 for all v ∈ V(G).*

Proof. Let $G \cong P_n$ and let ξ(v)=1 for all vV(G). Then $G^{\xi} \cong G \cong P_n$. By Remark 1.8, $loc(G^{\xi})=1$. For the converse, assume that G^{ξ} has a location number 1. By Remark 1.8, $G^{\xi}\cong P_{|V(G\xi)|}$. Assume, to the contrary, that ξ(v) ≠ 1 for some vertex v ∈ V(G). Thus, there exist vertices (v, i) and (v, j), for some 1≤i≠j≤ξ(v), such that they are adjacent to each other or adjacent to different vertices. It contradicts the fact that (v, i) and (v, j) are not adjacent to each other and adjacent to the same vertices in G^{ξ}. Thus, ξ(v)=1 for all v ∈ V(G) and so $G \cong P_{|V(G\xi)|}$. Since $|V(G^{\xi})| = \Sigma_{v \in V(G)}\xi(v) = n$, $G \cong P_n$.

Theorem 4.2 *Let G be a connected graph of order n≥2 and let ξ be a mapping from V(G) into positive integers. Then* $loc(G^{\xi})=|V(G^{\xi})|-1$ *if and only if* $G \cong K_n$ *and ξ(v)=1 for all v ∈ V(G).*

Proof. Let $G \cong K_n$ and ξ(v)=1 for all v ∈ V(G). Then $G^{\xi} \cong G \cong K_n$ and $|V(G^{\xi})|=n$. By Remark 1.9, $loc(G^{\xi})=|V(G^{\xi})|-1$. For the converse, suppose that $loc(G^{\xi})=|V(G^{\xi})|-1$. By Remark 1.9, $G^{\xi}\cong K_{|V(G\xi)|}$. Assume, to the contrary, that ξ(v) ≠1 for some vertex v ∈ V(G). Thus, there exist vertices (v, i) and (v, j) adjacent to each other for some 1≤i ≠ j≤ξ(v). It contradicts the fact that (v, i) and (v, j) are not adjacent to each other in G^{ξ}. Thus, ξ(v)=1 for all vertices v ∈ V(G) and so $G^{\xi}\cong K_{|V(G\xi)|}$. Since $|V(G^{\xi})|= \Sigma_{v \in V(G)}\xi(v) = n$, $G \cong K_n$

In the next two results we find the necessary and sufficient conditions for the locating sets in the multiplication of a graph to exist.

Theorem 4.3 *Let G be a connected graph of order $n\geq 2$ and let ξ be a mapping from $V(G)$ into positive integers. If $T \subseteq V(G^{\xi})$ is a locating set of G^{ξ}, then $S=\{v: (v, j) \in T\}$ is a locating set of G.*

Proof. We show that $S=\{v: (v, j) \in T\}$ is a locating set in G. It is obvious that the vertices in S have distinct codes with respect to itself. Assume, to the contrary, that there are vertices u and w in $V(G)\backslash S$ having the same codes with respect to S. Thus, $d(u, v)=d(w, v)$ for all v in S. It follows that $d((u, i), (v, j))=d((w, k), (v, j))$ for all (v, j) in T, where $1\leq i\leq\xi(u)$, $1\leq j\leq\xi(v)$ and $1\leq k\leq\xi(w)$. It contradicts the fact that T is a locating set of G^{ξ}. Therefore, S is a locating set of G.

Theorem 4.4 *Let G be a connected graph of order $n\geq 2$ and let ξ be a mapping from $V(G)$ into positive integers. If $S \in L(G)$ and $\xi(v)=1$ for all vertices $v \in V(G)\backslash S$, then $T=\{(v, j): v \in S, 1\leq j\leq\xi(v)\}$ is a locating set of G^{ξ}.*

Proof. We show that $T=\{(v, j): v\in S, 1\leq j\leq\xi(v)\}$ is a locating set of G^{ξ}. It is obvious that the vertices in T have distinct codes with respect to itself. Assume, to the contrary, that there are two vertices $(u, 1)$ and $(w, 1)$ in $V(G^{\xi})\backslash T$ having the same codes with respect to T. Thus, $d((u, 1), (v, j))=d((w, 1), (v, j))$ for all (v, j) in T. It follows that $d(u, v)=d(w, v)$ for all v in S. It contradicts the fact that S is a locating set of G. Therefore, T is a locating set of G^{ξ}.

Observe that if S is a locating set of a graph G, then the set $T=\{(v, j): v \in S, 1\leq j\leq\xi(v)\}$ is not necessary to be a locating set of G^{ξ}. Moreover, if S is a minimum locating set of G and $\xi(v)=1$ for all $v \in V(G)\backslash S$, then the set T is not necessary to be a minimum locating set of G^{ξ}. We consider a complete graph K_3 on vertices a, b and c with the mapping ξ from V(G) into positive integers defined by $\xi(a)=2$ and $\xi(b)=\xi(c)=1$. It is clear that $\{b, c\}$ is a locating set of G, but $\{(b, 1), (c, 1)\}$ is not a locating set of G^{ξ}. Similarly, $\{a, b\}$ is a minimum locating set of G (see Figure 5), but $\{(a, 1), (a, 2), (b, 1)\}$ is not a minimum locating set of G^{ξ} (see Figure 6).

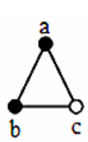

Figure 5. A minimum locating set of a graph K_3.

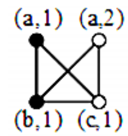

Figure 6. A minimum locating set of a graph $(K_3)^{\xi}$.

Applying Theorem 4.4, we can determine an upper bound for the location number in the multiplication of a graph.

Corollary 4.1 *Let G be a connected graph and let ξ be a mapping from $V(G)$ into positive integers such that for any $S \in L(G)$, $\xi(v)=1$ for all vertices $v \in V(G)\backslash S$. Then $loc(G^{\xi})\leq min\{\Sigma_{v\in V(G)}\xi(v) : S \in L(G)\}$.*

Proof. Let U be a minimum locating set of G^{ξ}. By Theorem 4.4, for any $S \in L(G)$ such that $\xi(v)=1$ for all $v \in V(G)\backslash S$, the set $T=\{(v, j): v \in S, 1\leq j\leq\xi(v)\}$ is a locating set of G^{ξ}. Therefore, $|U|\leq|T|= \Sigma_{v\in S}\xi(v)$ and it follows that $loc(G^{\xi})=|U|\leq min\{\Sigma_{v\in V(G)}\xi(v) : S \in L(G)\}$.

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