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Non-similar Boundary Layers over a Wedge due to Thermophoresis and Viscosity Effects

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Abstract

Background/Objectives: The objective of the present work is to investigate the effects of the thermophoresis and the fluid viscosity on a forced convection heat and mass transfer on non-similar flow past a permeable wedge embedded in a porous medium. **Methods of Analysis:** The analysis is conducted by using a local non-similarity scheme of the second level truncation in the presence of thermophoresis and variable viscosity. The governing equations of the non-similar convection are presented in the dimensionless form by applying Falkner-Skan variable to obtain the set of ordinary differential equations. Runge-Kutta-Gill in conjunction with shooting method is applied to find a skin friction, rate of heat and mass transfer. The initial value problems are solved numerically using Runge Kutta-Fehlberg methods and presented graphically. **Findings:** The thermophoresis has significant effect to speed up the process for decreasing concentration significantly, while the effects of viscosity is to increase or decrease the velocity of fluid depend on the value of viscosity parameter, positive or negative, respectively. Validation is conducted by comparing the current works with previous works and found that the results are in excellent agreement. The conclusion is drawn that the non-similar velocity and the concentration of fluid are significantly affected by thermophoresis and viscosity. **Application/Improvements:** The results of an analysis on the effects of thermophoresis and viscosity on boundary layer flow over solid surfaces can be applied to design an instrument to remove pollutants from environment

Keywords: Boundary Layers, Non-Similar, Permeable Wedge, Thermophoresis, Viscosity

1. Introduction

The study of convective heat and mass transfer over solid surfaces has attracted attentions during recent years due to its various applications, mainly in the heating and cooling processes, in designing equipment for removing pollutant and in an aircraft design. Thernophoresis particle deposition is a phenomena in which a particle moves from the hotter surface towards the cooler one due to temperature different. This force plays an important role on mass transport in boundary layer flow.

The analysis of thermophoretic over boundary layer flow was first conducted by Hales et al.[2] by solving the simultaneous governing equations of aerosol and steam transfer over a vertical flat surface. Furthermore, many researchers were interested to investigate the thermophoretic effects on a velocity, thermal and mass boundary layers of Newtonian fluid in various geometry and fluids. Chamkha and Pop[3] investigated the thermophoretic effect on free convection heat transfer over a vertical surface embedded in porous medium. Mahdy and Hady[4] investigated the effects of particles deposition due thermophoretic force in a magnetohydrodynamics flow of non-Newtonian fluid under influence of magnetic force field. Duwairi and Damseh[5] studied the influence of thermophoretic force on mixed convective heat transfer mode with non-isothermal wall temperature and non-uniform wall concentration. Jayanthi and Kumari[6] used the non-Dary model to investigate the effect of time dependent viscosity on fluid flow saturated porous medium over a vertical surface under free or mixed convection modes.

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Recently, Animasaun[7] examines the influence of thermophoretic force, fluid viscosity and thermal conductivity of an incompressible Casson magneto hydrodynamics flow along a vertical porous plate by taking into account a viscous dissipation, nth order chemical reaction and suction.

The effect of thermophoretic over a wedge and inclined surface was reported by some authors. Kandasamy et al.[8] studied the thermophoretic phenomenon, surface and volume chemical reaction effects on convective boundary layers over a permeable wedge suction. Alam et al.[9] observed the influence of thermophoresis and suction parameters on an electrically conducting fluid flow past a semi-infinite inclined permeable surface under thermal surface radiation effect. The mode of the convection is a mixed convective heat and mass transfer.

The stagnation point flow of two-dimensional is a classical problem in fluid dynamics. It was first explored by Hiemenz. These kinds of flows are occurred in many situations, such as flows past the front parts of rockets, aircraft, and aerodynamics vehicles. Some of published works which are much related with our current discussion on Hiemenz flows are given by Jian et al.[10], who investigated the deposition effect of aerosol particle onto a stretching surface from porous medium. Radiah et al.[11] and Siva Raman et al.[12] analyzed the enhancement of heat transfer on nanofluid due to solar energy radiation past a porous wedge in the presence of suction/injection. In these kinds of problems, the boundary layer equations are reduced into ordinary differential equations by applying the well-known Falkner-Skan[13] transformation.

All analyses in the published works mentioned above were done by using a similarity transformation in which the flow fields are assumed to be similar. In case of the flow is non-similar, one had to solve nonlinear partial differential equations. The non-similarity of governing equations may results from a variety of causes, such as non-isothermal wall temperature, non-uniform wall concentration, non-uniform magnetic field along the wall, and time dependent viscosity. Because of the mathematical complexity, most researchers avoid to use the non-similarity model on boundary-layers equations. This is a reason why most of published works on governing equations of boundary layers are related to similarity flow. This is to simplicity. This article is intended to investigate theoretically the convective boundary layers along a Newtonian, incompressible, viscous fluid on non-similar Hiemenz flow past a permeable wedge embedded in a non-isothermal porous wedge due to thermophoretic force of particle deposition and thermal variable viscosity in the presence of suction / injection with variable stream conditions.

Nomenclature

- C : species concentration of fluid.
- c_p : specific heat capacity at constant pressure
- D : coefficient of diffusion
- g : gravitational force
- K : permeability of a porous medium
- k_e : porous medium effective thermal conductivity
- T : fluid temperature.
- u, v : components of velocity in x and y direction
- a : thermal diffusivity
- m : dynamic viscosity
- r : fluid density
- n : kinematic viscosity

Subscripts

- ∞ : stream conditions
- w : wall conditions

2. Mathematical Formulation

Let us consider a Hiemenz stagnation flow coupled with forced convective heat and mass transfer past a non-isothermal heated permeable wedge embedded in a highly porous media. The flow is assumed to be a steady, two-dimensional and laminar. The fluid is a Newtonian, viscous and incompressible. The non-Darcy model is used to characterize the flow due to flow in highly porous medium. The density variation due to temperature is considered in the momentum equation and the species concentration in stream conditions is infinitesimal small. The viscosity is assumed to be varies inverse linearly with temperature. The velocity at free stream condition is assumed to be the power of the longitudinal distance from the leading edge. The temperature and concentration at the surface vary along the surface of the wedge.

Let the x-axis be put along the wedge wall from stagnation point and y-axis normal to it as shown in Figure 1. Slot suction or injection of fluid is placed at the wedge surface. The inertia force and viscosity are taken into account in the momentum equation due to the flow in highly porous medium. Under these assumptions, the system of equations governing flow fields (continuity of

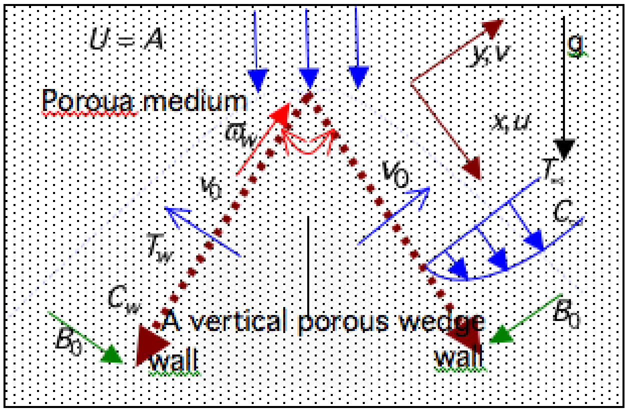

Figure 1. Flow analyses and coordinate system.

mass, velocity, thermal and concentration boundary layers) under Boussinesq's approximation are given by

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho}\frac{\partial}{\partial y}\left(\frac{\mu\partial u}{\partial y}\right)+U\frac{dU}{dx}-\frac{\nu(u-U)}{K}-\frac{F(u^2-U^2)}{\sqrt{K}} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\mu}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-\frac{\partial(V_T C)}{\partial y} \tag{4}$$

with the conditions at the boundary

$$\text{at } y=0: u=0, v=-v_0, T_w=T_\infty+bx^n, C_w=C_\infty+cx^n \tag{5}$$

$$\text{at } y\to\infty: u=U(x)=Ax^m,\ T=T_\infty,\ C=C_\infty \tag{6}$$

where $V_T=-k^*(\nu/T)(\partial T/\partial y)$ is the velocity of thermophoretic, with k^* is the constant coefficient of thermophoretic in the resistance of second order. When Forchheimer number $F=0$, Equation (2) is reduced to the Darcy model. The term four and five on the right-hand side of Equation (2) represent the first-order and the second-order Darcy resistance, respectively.

3. Falkner-Skan Transformation

In the boundary layers problem, the behaviors of flow, heat and mass transport inside boundary layers can be predict based on the performance of fluid flow at the free stream level which is called by stream function y. For non-similar flow over a wedge, the non-similarity dimensionless stream function is given by Falkner-Skan transformation as proposed by Kumari et al[14].

$$\psi(x,\eta)=\sqrt{\frac{2U\nu x}{1+m}}f(x,\eta) \tag{7}$$

$$\eta=y\sqrt{\frac{(1+m)U}{2\nu x}} \tag{8}$$

where the stream function velocity

$$U=Ax^m, m=\beta_1/(2-\beta_1) \tag{9}$$

where $\beta_1=\Omega/\pi$ is called a parameter of Hartree pressure gradient and Ω is total angle of the wedge.
By defining the stream function $\psi(x,\eta)$ as shown in Equation (7), the continuity equation (1) is satisfied such that

$$u=\frac{\partial\psi}{\partial y}\text{and } v=-\frac{\partial\psi}{\partial x} \tag{10}$$

Furthermore, the velocity components in x and y directions can be written in the form of

$$u=Uf', v=-\left(\frac{2}{(1+m)}\frac{U\nu}{x}\right)^{1/2}\left(\frac{f}{2}+\frac{x}{2U}\frac{dU}{dx}f+(1+x)\frac{\partial f}{\partial x}\right) \tag{11}$$

By assuming the viscosity is a linear function of temperature as given by Abo-Eldahab[15]

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}\left[1+\gamma_T(T-T_\infty)\right] \tag{12}$$

where μ_∞ is the dynamic viscosity of ambient fluid and γ_T is a thermal property of the fluid, the equation (13) can be represented as follows

$$\frac{1}{\mu}=a(T-T_r) \tag{13}$$

where $a=\gamma_T/\mu_\infty$ and $T_r=T_\infty-1/\gamma_T$. The constants values of a and T_r can be calculated depend on the thermal property of the fluid and reference state.

The governing equations (2) – (4) subject to the boundary conditions (5) and (6) can be written in the dimensionless form by defining the following dimensionless variables.

$$\theta(x,\eta)=\frac{T-T_\infty}{T_w-T_\infty}(\text{dimensionless of temperature}) \tag{14}$$

$$\varphi(x,\eta)=\frac{C-C_\infty}{C_w-C_\infty}(\text{dimensionless of concentration}) \quad (15)$$

Substituting equations (7) – (15) into equations (2) – (4) and rearranging yields

$$f'''-\frac{\theta'}{\theta-\theta_r}f'+\frac{2}{1+m}\left(\frac{\mathrm{Re}_x}{\mathrm{Re}_k^2}Fn+m\right)\left(\frac{\theta-\theta_r}{\theta_r}\right)\left(f'^2-1\right)-\frac{\theta-\theta_r}{\theta_r}ff''+\frac{2\lambda}{1+m}\xi^2\frac{\theta-\theta_r}{\theta_r}(f'-1)=\frac{2x}{1+m}\frac{\theta-\theta_r}{\theta_r}\left(f''\frac{\partial f}{\partial x}-\frac{\partial f}{\partial\eta}\frac{\partial f'}{\partial x}\right) \quad (16)$$

$$\theta''-\frac{2n}{1+m}\Pr f'\theta+\Pr\left(f\theta'+\frac{2}{1+m}Ecf''^2\right)=\frac{2x}{1+m}\Pr\left(f'\frac{\partial\theta}{\partial x}-\frac{\partial f}{\partial x}\theta'\right) \quad (17)$$

$$\varphi''-\frac{2n}{1+m}Scf'\varphi-\tau Sc\left(\varphi\theta''+\theta'\varphi'\right)+Scf\varphi'=\frac{2x}{1+m}Sc\left(\frac{\partial f}{\partial\eta}\frac{\partial\varphi}{\partial x}-\frac{\partial f}{\partial x}\frac{\partial\varphi}{\partial\eta}\right) \quad (18)$$

with the dimensionless boundary conditions are

at $$\eta=0: f'(0)=0, f(0)=\frac{2}{1+m}\left(S-\frac{x\partial f}{\partial x}\right), \quad S=-v_0\sqrt{\frac{(1+m)x}{2\nu U}}, \theta(0)=1, \varphi(0)=1 \quad (19)$$

at $$\text{at } \eta\to\infty: f'(\infty)=1, \theta(\infty)=0, \varphi(\infty)=0, \quad (20)$$

where prime sign (′) in the equations (16) – (18) and boundary conditions (19) – (20) represent the derivative with respect to variable η. Reynolds number, parameter porosity and Schmidt number are defined by $\mathrm{Re}_x=Ux/\nu$, $\lambda=\nu/KA$ and $Sc=\nu/D$, respectively. Parameter S is a suction if $S>0$ and S is an injection if $S<0$. Dimensionless parameter of Forchheimer number is denoted by Fn and dimensionless parameter of reference temperature is given by $\theta_r=(T_r-T_\infty)/(T_w-T_\infty)$.

Now, introducing a dimensionless parameter ξ representing the distance along the wedge defined by

$$\xi=kx^{(1-m)/2} \quad (21)$$

where k is a positive constant and substitute into the equations (16) – (18) with the boundary conditions (19) – (20) yields the system of partial differential equations (22) – (24) subject to boundary conditions (25) – (26),

$$f'''-\frac{\theta'f'}{\theta-\theta_r}+\frac{2}{1+m}\left(\frac{\mathrm{Re}_x}{\mathrm{Re}_k^2}Fn+m\right)\left(\frac{\theta-\theta_r}{\theta_r}\right)[f'^2-1]-\frac{\theta-\theta_r}{\theta_r}ff''+\frac{2\lambda}{1+m}\xi^2\frac{\theta-\theta_r}{\theta_r}(f'-1)=\frac{1-m}{1+m}\frac{\theta-\theta_r}{\theta_r}\xi\left(\frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial\xi^2}-\frac{\partial f}{\partial\xi}\frac{\partial^2 f}{\partial\eta^2}\right) \quad (22)$$

$$\theta''-\frac{2n}{1+m}\Pr\left(f'\theta+f\theta'\right)+\frac{2}{1+m}\Pr Ec_1\xi^{\frac{4m_1}{1+m}}f''^2=\frac{(1-m)\Pr}{1+m}\xi\left(\frac{\partial\theta}{\partial\xi}f'-\theta'\frac{\partial f}{\partial\xi}\right) \quad (23)$$

$$\varphi''-\frac{2n}{1+m}Scf'\varphi-\tau Sc\left(\varphi\theta''+\theta'\varphi'\right)+Scf\varphi'=\frac{(1-m)Sc}{1+m}\xi\left(\frac{\partial\varphi}{\partial\xi}f'-\varphi'\frac{\partial f}{\partial\xi}\right) \quad (24)$$

subject to boundary conditions

$$f'(\xi,0)=0, \frac{1}{2}(1+m)f(\xi,0)+\frac{1-m}{2}\xi\frac{\partial f(\xi,0)}{\partial\xi}=-S, \theta(\xi,0)=1, \varphi(\xi,0)=1 \quad (25)$$

$$f'(\xi,\infty)=1, \theta(\xi,\infty)=0, \varphi(\xi,0)=0 \quad (26)$$

The main physical quantities of interest are local skin friction coefficient, the Nusselt number and Sherwood number which are defined by

$$C_f=\frac{2\tau_s}{\rho U^2}=\sqrt{2(m+1)}\frac{f''(\xi,0)}{\mathrm{Re}_x^{1/2}} \quad (27)$$

$$Nu_x=-\sqrt{\frac{m+1}{2}}\mathrm{Re}_x^{1/2}\theta'(\xi,0) \quad (28)$$

$$Sh_x=-\sqrt{\frac{m+1}{2}}\mathrm{Re}_x^{1/2}\varphi'(\xi,0) \quad (29)$$

where is the viscous friction on the wedge surface is determined by

$$\tau_s=\mu\frac{\partial u}{\partial y}\bigg|_{y=0}=\sqrt{\frac{(m+1)\nu\rho^2U^3}{2x}}f''(\xi,0) \quad (30)$$

4. Local Non-similarity Solution

To derive equations for second level truncation, it is convenient to define the following new variables,

$$\chi=\partial f/\partial\xi,\quad \chi'=\partial f'/\partial\xi,\quad \chi'''=\partial f'''/\partial\xi,\quad h=\partial\chi/\partial\xi,$$
$$h'=\partial\chi'/\partial\xi,\quad h''=\partial\chi''/\partial\xi,\quad \Theta=\partial\theta/\partial\xi,\quad \Theta'=\partial\theta'/\partial\xi,$$
$$\Theta''=\partial\theta''/\partial\xi,\quad \Theta'''=\partial\theta'''/\partial\xi,\quad l=\partial\Theta/\partial\xi,\quad l'=\partial\Theta'/\partial\xi,$$
$$l''=\partial\Theta''/\partial\xi,\quad \Phi=\partial\varphi/\partial\xi,\quad \Phi'=\partial\varphi'/\partial\xi,\quad \Phi''=\partial\varphi''/\partial\xi,$$
$$\Phi'''=\partial\varphi'''/\partial\xi,\quad q=\partial\Phi/\partial\xi,\quad q'=\partial\Phi'/\partial\xi,\quad q''=\partial\Phi''/\partial\xi. \tag{31}$$

At the second level truncation, the simultaneous governing equations (22) – (24) are retained without approximation. Subsidiary equations for χ, Θ, Φ and their boundary conditions are obtained by taking the derivatives of equations (22) - (24) with respect to ξ, and terms involving ξh, $\xi h'$, ξl and ξq are ignored. Now, the governing equations of the second level of truncation consists of the coupled of former equations and subsidiary equations.

$$f'''-\frac{\theta' f'}{\theta-\theta r}+\frac{2}{1+m}\left(\frac{\mathrm{Re}_x}{\mathrm{Re}_k^2}Fn+m\right)\left(\frac{\theta-\theta r}{\theta r}\right)\left(f'^2-1\right)-\frac{\theta-\theta r}{\theta r}ff''$$
$$+\frac{2}{1+m}\lambda\xi^2\frac{\theta-\theta r}{\theta r}(f'-1)=\frac{1-m}{1+m}\xi\frac{\theta-\theta r}{\theta r}\left(f'\chi'-\chi f''\right) \tag{32}$$

$$\theta''-\Pr\left(\frac{2n}{1+m}f'\theta-f\theta'-\frac{2}{1+m}Ec_1\xi^{\frac{4m_1}{1+m}}f''^2\right)=\frac{1-m}{1+m}\Pr\xi(f'\Theta-\chi\theta') \tag{33}$$

$$\varphi''-\frac{2n}{1+m}Scf'\varphi-\tau Sc(\varphi\theta''+\theta'\varphi')+Scf\varphi'$$
$$=\frac{1-m}{1+m}Sc\xi(f'\Phi-\chi\varphi') \tag{34}$$

$$\chi'''+\frac{\Theta'}{\theta-\theta_r}-\frac{\theta'\Theta}{(\theta-\theta_r)^2}+\frac{\theta'\chi'}{\theta-\theta_r}+\frac{2}{1+m}\left(\frac{\mathrm{Re}_x}{\mathrm{Re}_k^2}Fn+m\right)\times$$
$$\frac{1}{\theta_r}\left[\Theta\left(f'^2-1\right)+2(\theta-\theta_r)f'\chi'\right]-\frac{\Theta ff''}{\theta_r}+\frac{\theta-\theta_r}{\theta_r}(\chi f''+f\chi'')$$
$$+\frac{2}{1+m}\lambda\left[2\xi\left(\frac{\theta-\theta_r}{\theta_r}\right)(f'^2-1)+\frac{\xi^2}{\theta_r}[\Theta(f'-1)+(\theta-\theta_r)\chi']\right]$$
$$=\frac{1-m}{1+m}(\chi'f'-\chi f''+\xi(\chi'^2-\chi\chi'')) \tag{35}$$

$$\Theta''-\frac{2n}{1+m}\Pr(\chi'\theta+f'\Theta)+\Pr(\chi\theta'+f\Theta')$$
$$+\frac{2}{1+m}\Pr Ec_1\xi^{\frac{4m_1}{1+m}}\left[\frac{4m}{(!+m)\xi}f''^2+2f''\chi''\right]$$
$$=\frac{1-m}{1+m}\Pr(f'\Theta-\theta'\chi+\xi(\chi'\Theta-\Theta'\chi) \tag{36}$$

$$\Phi''-\tau Sc[\Phi\theta''+\varphi\Theta''+\Theta'\varphi'+\theta'\Phi']+Sc(\chi\varphi'+f\Phi')$$
$$-\frac{2nSc}{1+m}(\chi'\varphi+f'\Phi)=\frac{1-m}{1+m}Sc(f'\Phi-\varphi'\chi+\xi(\chi'\Phi-\Phi'\chi)) \tag{37}$$

subject to the following boundary conditions

$$f'(\xi,0)=0,\frac{1+m}{2}f(\xi,0)+\frac{1-m}{2}\xi\frac{\partial f(\xi,0)}{\partial\xi}=-S,$$
$$\theta(\xi,0)=1,\varphi(\xi,0)=1 \tag{38}$$

$$f'(\xi,\infty)=1,\theta(\xi,\infty)=0,\varphi(\xi,0)=0 \tag{39}$$

In the local non-similarity approach for second level truncation, the approximation is introduced in a subsidiary equation, that is, in equations (35) – (37). Hence, it yields more accurate result than those of the local similarity solution. However, the exact solution for this problem are complicated, so numerical scheme is preferred. In this article, the numerical scheme from Runge-Kutta-Gill with modified Newton-Raphson shooting method can be applied to find the values of skin friction $f''(0)$, rate of heat transfer $\theta'(0)$ and rate of mass transfer $\phi'(0)$. Now, the governing equations (32) – (37) with all initials values are called a system of initial value problems. Finally, the Fourth order Runge-Kutta in conjuction to Fehlberg method can be applied to solve this problem. The solution profiles are shown graphically to show the interesting aspect of the solution.

5. Results and Discussion

The problem of convective heat and mass transfer in non-similar Hiemenz flow of Newtonian, viscous and incompressible fluid over a porous wedge which are described by governing equations (32) – (37) subject to boundary conditions (38) – (29) are solved numerically after finding the values of skin friction, rate of heat and mass transfer by applying the Classical Fourth-order Runge-Kutta Fehlberg Scheme with respect to some prescribed parameters such as Prandtl number, Forchheimer number, Stanton number for various values of thermophoretic and viscosity parameters. To validate our this method, we already compared results of Stanton number $St_m \mathrm{Re}_x^{1/2}$ to those of Mills et al.[16], Tsai[17] and Chamkha[18] for various values of suction $f(0)$ and thermophoretic parameter τ with $\Pr=0.7$, $Sc=0.0001, Ha^2=0, \Delta=0, Ec=0$, and $\theta'(0)=-0.4138$ which is presented in Table 1. The current results are found in excellence agreement with the previously published works. Hence, we are very confident to analyse and to draw conclusion based on this methods presented in this article.

Table 1. Comparison the values of Stanton number $St_m \mathrm{Re}_x^{1/2}$ to those of previous published works

		$St_m \mathrm{Re}_x^{1/2}$			
τ	$f(0)$	Mills et all15	Tsai16	Chamkha17	Present works
0.01	1.0	0.7091	0.7100	0.7098	0.7100
0.01	5.0	0.3559	0.3565	0.3590	0.3565
0.01	0.0	0.0029	0.0029	0.0030	0.0029
0.1	1.0	0.7265	0.7346	0.7241	0.7346
0.1	0.5	0.3767	0.3810	0.3810	0.3810
0.1	0.0	0.0277	0.0275	0.0280	0.0275
1.0	1.0	0.8619	0.9134	0.8932	0.9133
1.0	0.5	0.5346	0.5598	0.5450	0.5598
1.0	0.0	0.2095	0.2063	0.2120	0.2062

The values of the skin friction $f''(0)$, rate of heat and mass transfer, $\theta'(0)$ and $\varphi'(0)$, respectively, are evaluated for some prescribed parameter of interest. These values are very important not only from a major physical interest but also from mathematical aspects in which the governing equations are changed from a system of boundary value problems into a system of initial value problems. Hence, the governing equation can be solved numerically by applying the Fourth-Order Runge Kutta method. It is also noted that the values of Prandtl number $\Pr = 0.72$ represents air at temperature $20^0 C$ and $Sc = 0.62$ corresponds to a diffusion of water vapour in air. The reference temperature θr is negative or positive depends on state of fluid. In case of liquids, the reference temperature is negative, while the reference temperature is positive in case of gas, provided the wall temperature is greater than an ambient temperature. The other parameter values are given for local Reynold number $\mathrm{Re}_x = 5$, porosity parameter $\lambda = 0.1$, suction S = 1, modified Reynold number $\mathrm{Re}_k = 1$, Eckert number Ec = 0.01, power law of temperature n = 0.3 and Hartree pressure gradient $m = 0.0909$.

The influence of thermophoresis and viscosity on dimensionless velocity and mass transfer profiles are displayed in Table 2 and Table 3. It is noticed that the increasing value of thermophoretic and viscosity parameters affect to decrease skin friction, but the rate of heat transfer remain constant. In addition, the mass transfer rate increases because of the increasing thermophoretic parameter and it remains constant with an increase of the viscosity parameter, provided the values of other parameters are kept constant.

Table 2. The values of skin friction, heat and mass transfer rate $f''(\xi,0)$, $-\theta'(\xi,0)$ and $-\varphi'(\xi,0)$ for various values of thermophoretic parameter τ with $\theta_r = -5$, $Fn = 0.1$

τ	$f''(\xi,0)$	$-\theta'(\xi,0)$	$-\varphi'(\xi,0)$
1	4.923522	1.699558	2.679368
5	4.556477	1.687626	6.556575
7	4.465310	1.685227	8.581475
10	4.380982	1.683370	11.659247

Table 3. The values of skin friction, rate of heat and mass transfer $f''(\xi,0)$, $-\theta'(\xi,0)$ and $-\varphi'(\xi,0)$ for various values of reference temperature parameter θr with $Fn = 0.1$ and $\tau = 1$.

θr	$f''(\xi,0)$	$-\theta'(\xi,0)$	$-\varphi'(\xi,0)$
-1.0	5.044627	1.685399	2.506783
-5.0	3.208586	1.654288	2.456865
-10.0	2.969034	1.648642	2.448369

Furthermore, the influence of thermophoretic parameter on the dimensionless fluid velocity, temperature and concentration of fluid profiles are depicted in Figure 2. It is noticed that an increasing thermophoretic value affects to decrease the velocity and concentration profile of fluid but fluid temperature remains constant. This is evaluated at liquid reference temperature $\theta r = -5$. In special case, when the value of Schmidt number is small for which the convection effects is small compared to the Brownian diffusion effect. However, when the values of Schmidt number is large (Sc > 100), the confection effects is maximal compared to diffusion effect, therefore, the concentration boundary layer is expected to be altered by thermophoretic parameter. This is similar to the result of Goren[18] on the effect of thermophoresis on boundary layer flow over a flat plate.

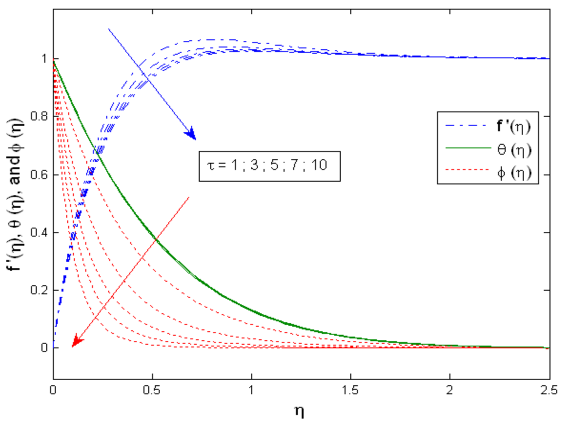

Figure 2. Thermophoretic effect on the dimensionless velocity, temperature and concentration profiles $\Pr = 0.72$, $\mathrm{Re}_x = 5, \lambda = 0.1, Sc = 0.62, S = 1$, $\mathrm{Re}_k = 1, F_n = 0.1, Ec = 0.01$, $n = 0.3, \theta r = -5, m = 0.0909$.

The effects of viscosity parameter θr on dimensionless velocity, temperature and concentration profiles are shown in Figure 3a and 3b. In case of liquid, the increasing values of viscosity parameter will decrease the profiles of dimensionless velocity, temperature and concentration inside the boundary layer which is depicted in Figure 3a. While in the case of gas, when the viscosity parameter increases, then the velocity of fluid increases and it means that the thickness of thermal boundary layer decreases. So, fluid motion is accelerated and the temperature of the fluid along the wall is reduced by the increasing value of viscosity parameter, but the concentration of the fluid remains constant with an increase of the viscosity parameter.

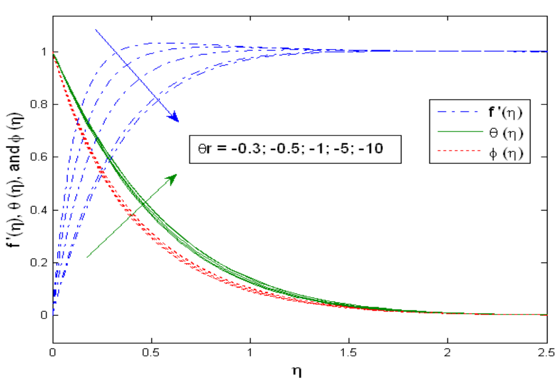

Figure 3(a). In the case of $qr < 0$.

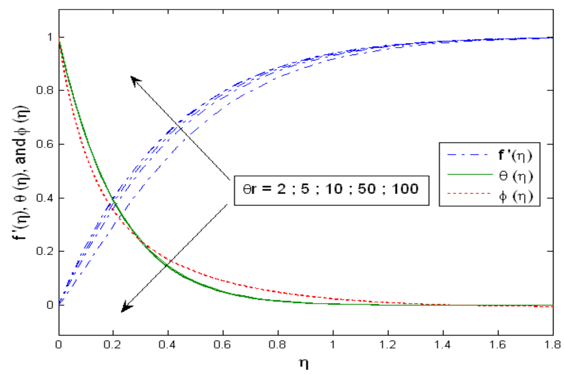

Figure 3(b) In the case of $.\theta r > 0$

Figure 3. Viscosity variable effects on dimensionless velocity, temperature and concentration profile for $\Pr = 0.72$, $\mathrm{Re}_x = 5, Gr = 1, \lambda = 0.1, Sc = 0.62$, $\mathrm{Re}_k = 1, F_n = 0.1, Ec = 0.01$, $n = 0.3, m = 0.0909$.

6. Conclusion

Discussion on this paper focuses on numerical study of the impacts of thermophoresis particle deposition and variable viscosity on forced convective boundary layers Hiemenz flow over a porous wedge. Based on the final form of the mathematical models, there are many parameters involved, but only some effects of parameters are discussed here. For certain values of parameters will determine a certain solution which related to the specific physical meaning. For this purpose, this model will be examined by the impact of thermophoresis and viscosity parameters. Two conclusion are drawn from this study.

- The increase in the value of the termophoretic parameter will accelerate the decline in the concentration of the liquid inside concentration boundary layer. It means that the thickness of concentration boundary layer decreases with an increase of the thermophoretic.
- Fluid motion is accelerated and the temperature of the fluid along the wall is reduced by the increasing value of viscosity parameter, but the concentration of the fluid remains constant with an increase of the viscosity parameter. It is also observed that the velocity boundary layer thickness decreases with the increase of viscosity parameter.

7. References

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