

The Use of an Emerging Framework to Explore Students' Cognitive Competency

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Abstract

Background/Objectives: A study on the development of a local framework to develop problem solving activity tasks that assess students' cognitive and metacognitive competency in solving non-routine problems. **Methods/Statistical analysis:** This study investigated the use of an emerging local framework called *MODEL* (*Meanings, Organise, Develop, Execute, Link*), in exploring students' cognitive competency in solving non-routine problems at junior college in six levels. The confidence levels of these students when attempting the problems were also examined. A total of 167 junior college students in Brunei Darussalam were involved in the study. **Findings:** The level of cognitive competency evaluated using the *MODEL* framework, revealed that the maximum level attained by the students was level 4 (*Execute*). Although the students managed to obtain the mathematical solutions and contextualised their solutions, all failed to justify reaching the validation level (*Link*). Students are most confident in solving problems with familiar settings that they have experienced and majority of the students have the abilities to display cognitive process of applying realistic considerations to achieve level 3 (*Develop*) of the *MODEL* framework. Majority of the Brunei junior college students possess the abilities and skills to solve non-routine problems by applying prior knowledge, rules, procedures and experiences in the context of the problem to obtain solutions, but all of them had failed to justify or validate their solutions in the realities of the problem contexts. Students acquiring more abstract knowledge of the mathematics with maturity of thinking skills in a higher year group performed better in non-routine problems that are most significant to them. **Application/Improvements:** A simple integration of solving non-routine (real-life) problems into the curriculum might just be the solution for improving mathematical literacy.

Keywords: Cognitive Competency, Curriculum, Junior College, *MODEL* Framework, Problem-Solving

1. Introduction

Schoenfeld¹ stated that the National Council of Teachers of Mathematics presaged, "problem solving must be the focus of school mathematics" in its *Agenda for Action* held in 1980. Eleven years later, in 1991, to expand NCTM's vision of a high quality mathematics education for every child, the *Professional Standards for Teaching Mathematics* was designed as the support and development of teachers and teaching². Consequently, the NCTM Standards require

teachers to be professionals who are able to produce high quality experiences in mathematics classrooms. It is only in 2014 that Brunei Darussalam launched its own framework for teachers called Brunei Teachers' Standards (BTS) underlining the Teachers' Professional Knowledge and Skill as its main core element and teachers are expected to adopt best practices in impacting students' learning³. It took 23 years for Brunei to embark on embracing the NCTM Standards to produce its own. Thereafter in 2015,

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the theme for NCTM was *Effective Teaching to Ensure Mathematical Success for All*, that statement includes strands to support teachers as learners and students as learners, through solving worthwhile problems and integrating mathematics with other disciplines⁴. Accordingly, what is the next direction for Brunei's education then?

Solving real-world problems is one of the most significant cognitive and metacognitive process in understanding the real mathematics embedded in the real world. This process involves students having to understand the problem, make a plan to develop methods and strategies and apply all heuristics to seek for the valid solutions⁵. Generally, most students would seek known rules and procedures, and their knowledge of abstract mathematics in solving problems will result in exclusive solutions in the context of the taught curriculum. These students are classified as good problem solvers of routine problems. How about non-routine problems? Real-world problems are classified as one of the type of non-routine problems⁶ that require students not only to apply their mathematical knowledge but to critically evaluate strategies, approaches and also validate their solutions to be realistically meaningful in the real-world.

Problem-solving in mathematics education has evolved from earlier work by Polya⁷ on the process of solving problems, to the success or failure of problem-solving discussed by Schoenfeld⁸, and an evolving cognitive and meta cognitive framework⁹ in analysing meta cognitive aspects of mathematical performance. Lester¹⁰ attributed factors like knowledge, control, beliefs and socio-cultural contexts as significant as general heuristics in problem-solving performance. He noted that it is the characteristics of the problem solver that determines the elegant of the solutions produced. Many problem-solving frameworks, such as the Multidimensional Problem Solving (MPS) Framework by Carlson and Bloom¹¹ and the theory of goal-oriented problem solving by Schoenfeld¹² have also evolved as sophisticated development to include theoretical approach in analysing the process of problem solving. Subsequently to more recent researches contributing to the success of problem solving include, aspects of attributes in developing problem-solving taxonomy¹¹ and strategies^{13,14} used by problem solvers in solving mathematical problems. The recent trend and development in problem-solving in mathematics education has made a spectacular progress but requires more work to be done

as described by Voskoglou¹⁵ because he argued that the mind of an individual is more complex than the body. Irrefutably, we echoed Voskoglou's¹⁵ views on problem-solving and we further believe that the effectiveness of a human mind in working to solve a problem is defined by three competencies: cognitive, meta cognitive and affective (beliefs, attitudes and values).

In Brunei, majority of the students are not exposed to the learning of mathematics involving non-routine problems in the context of real-world settings. This could be attributed to, the lack of awareness and capacity in this field of applied mathematics in teachers and pre-service teachers, similarly reported in South Africa¹⁶, in Singapore^{17,18} and in Indonesia¹⁹. However, there has been increasing awareness in improving students' numeracy skills in Brunei's curriculum through one of the initiatives called the Numeracy Programme, featured in the Ministry of Education Strategic Plan 2012-2017. The Ministry of Education²⁰ initiated this programme in 2010 to provide professional development courses to teachers in order to help them develop the necessary mathematical skills for the teaching and learning of Mathematics in school. The main focus of the Numeracy Programme is to develop teachers' mathematical skills with relevance to the concepts and operations of the curriculum. We argued that the programme should also include teaching and learning with relevance to context: teaching and learning through understanding of what, how and why? That is, the pedagogical practices that promotes critical thinking, reasoning, innovation, communication and performance. Therefore, tasks in classroom should now progress from curriculum-based learning²¹ to solving non-routine problems connecting mathematics to the real-world, a framework which draws out some understanding and interpretations of the real world and not just evoking students' learning through teaching curriculum-based problems. Hence, this study aims to explore junior college students' mathematical cognitive competency in solving non-routine problems using the emerging local *MODEL* framework, which evaluate students' solutions from their understanding, to the implementation, and validation of mathematical knowledge and competency in *real* situations. Then we could put forward this framework for further developing and exploring the efficacy of heuristics, self-regulation and effects of students' competencies.

2. Cognitive Competency and the MODEL Framework

In general, competency in an individual is having the abilities and skills to complete tasks successfully by means of different approaches and strategies and becoming more proficient in the latter, by means such as training or self-efforts²². Blomhøj²³ defined an individual as competent when he or she possesses the mental capacity to cope with a certain type of challenge in a knowledgeable and reflective way. In mathematical modelling, the seven phases (constructing, simplifying, mathematising, working mathematically, interpreting, validating and exposing) of the modelling cycle encompass the cognitive competency pertaining to the conscious activities students are involved²⁴. Collectively, cognitive competency in problem-solving refers to an individual's capability and skills in displaying process-oriented, problem solving approaches⁵ and exhibit flexibility in using powerful

content-related resources²⁵ and possess high level of self-awareness of own strengths and weaknesses²⁶. Hence, cognitive competency forms the main focus of this study in attempting to understand the developmental status of the students' thinking and reasoning skills correlating to their confidence level in solving the non-routine problems. This study employed a pen and paper test involving four non-routine problems essentially to describe the students' cognitive competency.

A simple local framework was designed and used in this exploratory study called *MODEL* (Figure 1). This framework explores students' cognitive competency in six levels, in their application of abstract mathematical knowledge into non-routine problems. This framework was designed underpinning Carlson and Bloom¹¹ MPS Framework of four phases: orientation, planning, executing and checking; and also Garofalo and Lester⁹ cognitive and meta cognitive framework, such that the different levels of the problem-solving process are simplified and idealised into six comprehensive levels.

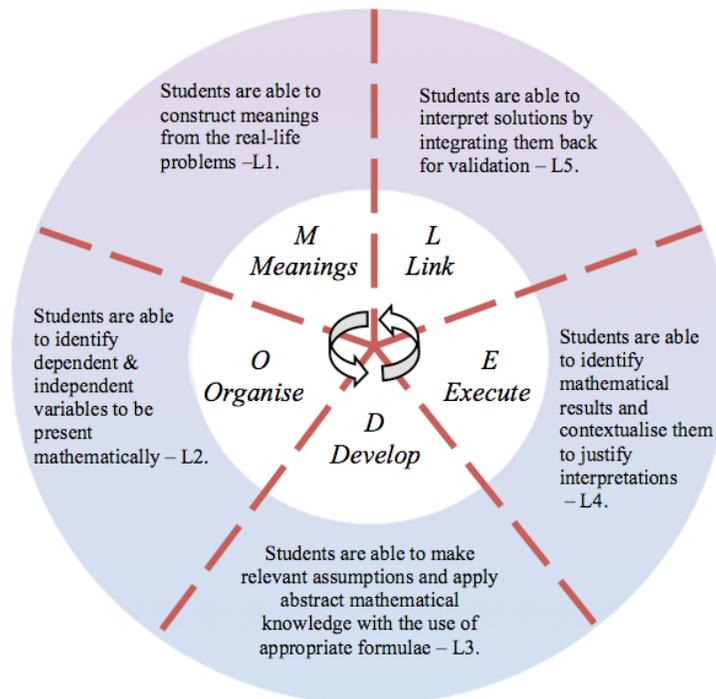


Figure 1. The MODEL framework.

In L0 – students did not attempt the problem. At L1 – *Meanings (M)*, students present some fragments of their abstract knowledge into either diagrammatic representation of the problem using concept map, mind map, flowchart, diagrams of all sorts and also any relevant figures or brief explanation and description of their understandings. At this level, students will demonstrate memory recall and reinforced prior knowledge or learning into the real-life problem posed. This level is considered to be the vital level of the problem-solving skill, because students who successfully reached L1 will be able to develop the skill to reach L3 of the *MODEL* cycle.

In L2 – *Organise (O)*, students must identify the dependent and independent variables in the non-routine problems posed. They will explore and generate ideas, parameters and break down the problem into simpler task by asking questions and linking ideas.

In L3 – *Develop (D)*, students make relevant assumptions based on their ideas and decide which variables are feasible and possible to solve this problem. Students will learn creative decision-making at this level by choosing the appropriate mathematical formulae to use in solving the problem.

In L4 – *Execute (E)*, students will obtain mathematical solution(s) at this level, and will need to contextualise the solution(s) in order to justify for interpretations in the next level. The learning outcome at this level is that students will demonstrate their cognitive competency in reflecting back into the problem.

The fifth level, L5 – *Link (L)*, the meta cognition level, and students must be able to link and validate their solution(s) to the problem and finally reflecting on any error(s) encountered. Students are also expected to synthesise their findings including interpretation of their written work in integrating back into the *real* situations for validation and exposing. However, it must be stressed that aim of designing the *MODEL* framework was to idealise and simplify the complex mathematical problem-solving cycle into a schematic representation of the cycle in six simple levels.

3. Methodology

A total of 167 junior college students (equivalent to 11th and 12th grades in American schooling or Year 12 and Year 13 in the context of Brunei high schools) in eight different classes were involved in a pen and paper test that aim to explore their cognitive competency in problem-solving process. The one-hour test was to examine the students' cognitive competency in their transfer skills of abstract knowledge to non-routine problems using the *MODEL* framework. Four of the classes involved were students studying in their Year 12 (Y12), while the remaining four classes are in Year 13 (Y13). The classes were quite heterogeneous with multi-cultural, multi-disciplinary and age range of 16-20 years old. Eight different teachers, who followed the same departmental scheme for teaching strategies, but with variation in the delivery of their respective lessons, taught all eight of the classes.

In addition, all of the students are studying General Certificate of Education (GCE) Advanced (A) level mathematics with a pre-requisite of at least grade C in GCE Ordinary (O) level mathematics. Since the study was conducted in the final term of an academic year, all the students would have already covered all the topics in their designated syllabus, which is the pre-requisite of the test. It must also be noted that the sample of students in this exploratory study had not had prior experience of solving non-routine problems in everyday classroom routine or any training course, or experiencing of mathematical modelling prior to taking the test. The design of the test was carefully scrutinised to include a wide range of different non-routine problems in order to assess students' cognitive competency in a systematic way as proposed by the *MODEL* framework. In addition, the test items selected were such that students are well acquainted with the mathematical concepts, rules and procedures required in solving these problems. This included major topics covered in the A Level syllabus for Y12 level:

- Coordinate geometry - knowledge and understanding of linear function and plotting of graph;
- Quadratics and functions - knowledge and understanding of surface area of cuboids, maximum and minimum values;
- Algebra and Series - knowledge and understanding of simple algebra and patterns;

- Calculus – knowledge of gradient of a function to examine the maximum or minimum value(s) for maximum or minimum volume;
- Representation of data – knowledge and understanding of collecting, sorting and comparing data.

The data source of this exploratory study consisted of four non-routine problems conducted as a test completed by 167 junior college students in Brunei. The test focused on the students’ level of cognitive competency and knowledge transfer skills assessed in six levels of the *MODEL* framework, where the test items were based on theoretical mathematical facts that all the students were familiar with. Each class had exactly 60 minutes to attempt four questions (Q1, Q2, Q3 and Q4) in the test. The questions were selected based on the relevance to the A Level syllabus content and restricted to a maximum of an hour for completion of all four questions. The first and second

questions, Q1 and Q2, were adapted from Ang²⁷, the first author created Q3, while Q4 was adapted from Blum and Borromeo Ferri²⁸. The intended use of well-researched test questions (3 out of the 4 test items) in this study was to support the idea that existing items are internally and externally reliable and data collected can be used to compare with the existing norms²⁹. In addition, both authors also pointed out that using a recognised standardised test could eliminate observer subjectivity.

This study aimed to explore the cognitive competency of junior college students using the *MODEL* framework in six levels. As a result, the test items were further validated using the six concrete principles of Model Eliciting Activities (MEAs) as was described by Lesh and Doerr³⁰, and Inversen and Larson³¹, so as to provide suitable insights into students’ abilities – what they know, understand and able to achieve, as shown below in Table 1.

Table 1. The mathematical modelling problems in the test validated using the six concrete principles of MEAs

	Principles of MEAs	Q1 – Linear function	Q2 – Biggest box problem	Q3 – Tiling of a floor area	Q4 – The giant shoe
1	The Reality Principle – Does the situation appear to be meaningful to the students, and does it build on to their former experiences?	This is a simple problem requiring knowledge on linear function and graphing. All the students have learnt the concepts of coordinate geometry at O level mathematics.	This problem requires students to have sound knowledge in algebra, quadratics, graphing and calculus. All the students at A level have already learnt these concepts.	This is a simple arithmetic problem already learnt in primary school education. All the students at A level can easily understand the concepts of floor area and square tiles.	This is a simple arithmetic problem. All the students have already learnt the concepts of heights and dimensions of shoes in early years of their secondary schooling.
2	The Model Construction Principle – Does the situation create a need to develop significant mathematical constructs?	Students need to understand the concepts of linear function and also able to interpret graph in order to solve this problem. They also need to make relevant assumptions about the initial water level and the constant rate of water flow.	Students need to choose which methods are easier and more appropriate to solve this problem, and make relevant assumptions about the maximum volume of the box.	Students need to decide which arithmetic operations are required to solve this problem, and make relevant assumptions about the tiles used (e.g. design and how tiling is done).	Students need to decide which arithmetic operations are required to solve this problem, and make relevant assumptions about the relationship between height and shoes size.

continue

	Principles of MEAs	Q1 – Linear function	Q2 – Biggest box problem	Q3 – Tiling of a floor area	Q4 – The giant shoe
3	The Self-evaluation Principle – Does the situation demand that the students assess their own elicited models continuously?	Given the conditions, students should be able to construct the equation of straight line based on their assumptions of the variables used and reflect back to the graph given.	Given the conditions, students need to ensure that the volume of the box is calculated to the desire dimensions in making the biggest box.	Given the conditions, students need to decide how the arrangements of the tiles will be best suited to the floor area based on different patterns and designs.	Given the conditions, students need to use several heights and shoes sizes as the basis of comparison.
4	The Construct Documentation Principle – Does the situation demand the students to reveal what they think while they work on solving the problem?	This is essentially a simple model aims at eliciting students understanding of linear relationship. For instance, increasing the flow rate will result in steeper gradient.	This is essentially a design problem where students need to recognise that to maximise volume of the box, the dimensions of x must be small. A 'peak' on the graph will indicate this.	This is essentially a design problem where students are required to put together the ideas of tiling a floor area based on the aspect of recognising patterns.	Students are required to have some knowledge of gigantism in humans and the different phases of their growth. Only then, they can valid assumptions.
5	The Construct neralization Principle – Can the elicited model be generalized to other similar situations?	This problem is simple to understand and comparable to simple problems carried out at home such as filing a bucket of water, time management and financing of household budgets.	This problem is simple to understand in context but difficult to solve by concepts. Students can easily relate this to other real-life settings such as fencing a boundary, building of swimming pool and so on.	This problem is simple to understand and comparable to designing a house plan including room spaces for furniture fittings and even for town planning and development.	This problem is simple to understand but requires complex process for the validating (Link) level. Similar situations range from building of big suspension bridge to simple concept of proportionality.
6	6. The Simplicity Principle – Is the problem situation simple?	This is a simple problem to calculate and solutions are easily attainable.	This problem requires students to have sound knowledge of calculus and algebra. Solutions are attainable but discrepancies can occur at the link level (S5).	This is a simple problem to calculate and solutions are easily attainable.	This is a simple but 'rare' real-life situation. All of the students will have no exposure to gigantism especially in Brunei. Nevertheless, the basis of comparison can be made to most grown adults.

One characteristic of the non-routine problems used in the test is the simplicity of the arithmetic involved in which solution is easily attainable. What made the problems eligible as good models is that it required students to recognise mathematical patterns and understanding the different aspects of real-life settings – a simple contextual problem and the relevance of the problems for the students. Similarly, Zöttl, Ufer and Reiss³² reported that

several different problems in a test had to serve as a scale of items considering different mathematical contexts for more reliable information on students' cognitive competency. For example, Q3 in this test is a simple question about tiling of floor but a good starting point to solve this question is to understand the very simple saying of “do not paint yourself to the corner”, or never tile a floor from the corner. If we do a quick search on tiling of floors from

the Internet, reliable websites will advise that tiling should start from the exact centre of the floor, measurable by ignoring any irregularities or offsets of the room. Just as the calculation of tiles required in this problem required students to understand that if the distance between the last tile and the wall is less than a piece of square tile, then cutting of tiles will be necessary, and this will incur additional tiles if uneven cuts and breakage occurred. This constituted the final level of the *MODEL* framework, the L5 - *Link* where students had difficulty in validating the solution. Brown and Schafer³³ also used similar question in eliciting the teaching approach of mathematical modelling to teachers in their study. They found that the teachers did enjoy creating a design for tiling of the floor, but only half of the teachers on the course were able to calculate the number of tiles needed for the hall floor. They had doubtful results at the last level of the modelling cycle (the verification level). This is where the teachers managed to state the product of rows and columns was a useful and efficient summary in this question, but they did not interpret this product in terms of area³³. Similarly, in this study, most students managed to reach a reasonable estimate of the number of tiles required, but all failed to contextualise their solutions in S5 (*Link*) of the *MODEL* framework.

4. Results and Discussions

This exploratory study is part of a larger research project on the development of questionnaire, lesson plans, problem-solving activities and a local framework in integrating solving of non-routine problems into Brunei's curriculum. From the total of 167 respondents, 88 students (52.7%) were studying in Y12 and 79 students (47.3%) were studying in Y13. Each item in the test paper is rated from minimum score of 0 for L0 to a maximum score of 5 for achieving L5. Thus, the total maximum score of the test paper is 20. Data in Table 2 shows the descriptive statistics of the test for both year groups. Since the students were from different year groups, their levels of abstract mathematics were not comparative, such that students in Y13 had more exposure of in-depth calculus than students in Y12. In light of this, the test items were designed to suit the Year 12 syllabus content so as both year groups will have equal opportunity to present their results. Nevertheless, Y13 students still achieved a higher mean of 9.62 compared to Y12 students only achieving a mean score of 8.10. A number of factors influencing these results might be influenced by Y13 students already exposed to substantial concepts of higher mathematics compared to Y12 students; also maturity of the Y13 students' thinking skills; and that the non-routine problems in this test was designed based on the recommendation of problems used should be done with the previous year's mathematics made by Burley and Trowbridge³⁴. Hence, Y13 students would have more exposure to the concepts required in solving these problems.

Table 2. The Descriptive statistics of Y12 and Y13 students

Year Group	n (%)	Mean (SD)	Variance	Minimum Score	Maximum Score
Y12	88 (52.7)	8.10 (2.595)	6.737	2	13
Y13	79 (47.3)	9.62 (2.377)	5.649	2	14

In exploring students' cognitive competency using the *MODEL* framework, when students do not attempt the question, then they will be assigned L0 for that particular problem. When they were able to create *meanings* and understand the problem by either diagrammatic representation or brief explanation, they will be assigned L1. The next level, L2 require students to be able to explore and generate ideas, parameters and break down the problem into simpler problem by linking ideas. At L3, students make relevant assumptions based on their ideas

and choose to formulate the appropriate mathematical formulae to use in solving the problem. Subsequently, at L4, students will obtain mathematical solution(s) and are required to contextualise the solution(s) in order to justify for interpretations in the next level. The sixth level, L5 is attainable when students are able to link and validate their solution(s) to the problem and finally reflecting on any error(s) encountered. The indexed competency of the Y12 and Y13 students assessed using the *MODEL* framework are shown in Table 3 and Figure 2.

Table 3. The cognitive competency levels evaluated using the MODEL framework

Cognitive competency levels	Y12 n (%)	Y13 n (%)
L0	48 (13.6)	26 (8.2)
L1	56 (15.9)	29 (9.2)
L2	91 (25.9)	73 (23.1)
L3	153 (43.5)	167 (52.8)
L4	4 (1.1)	21 (6.6)
L5	0 (0)	0 (0)

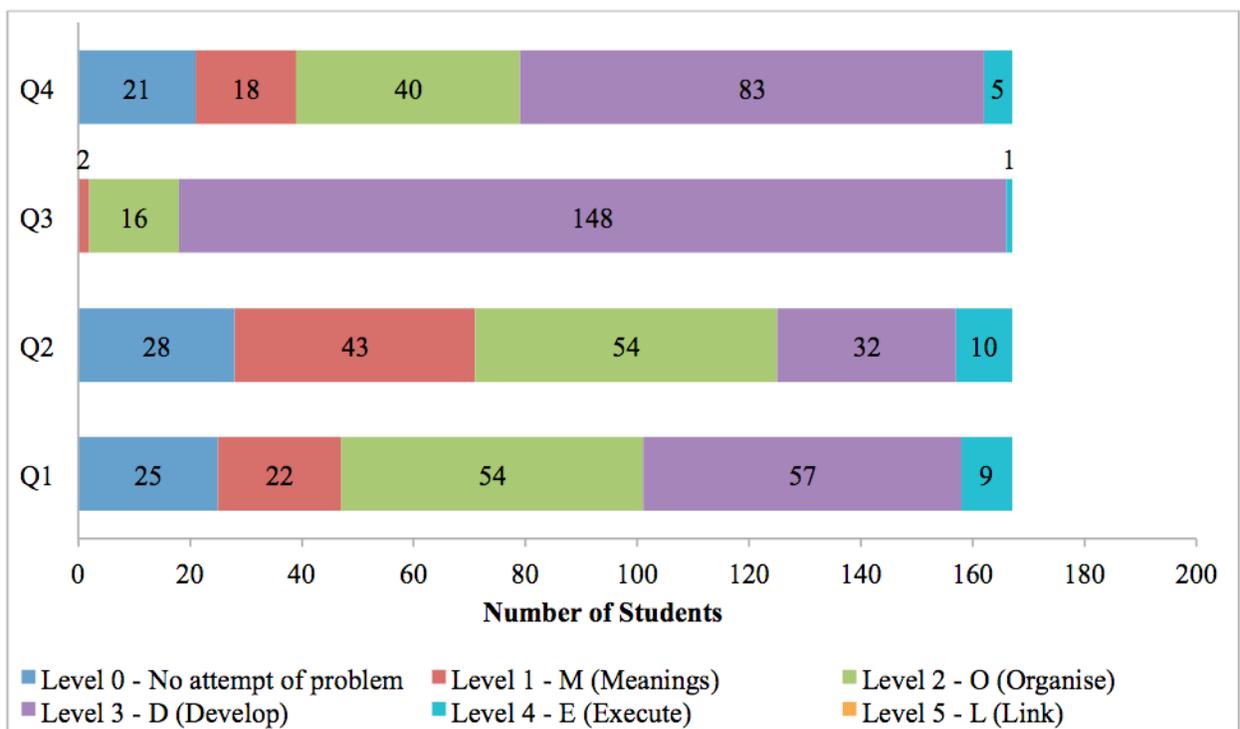


Figure 2. The indexed cognitive competency of students evaluated using the MODEL framework.

With regard to the level of cognitive competency evaluated using the *MODEL* framework, the maximum level attained by the students was Level 4 - *Execute* (E), where students managed to obtain mathematical solutions, and contextualised their solutions, but all failed to justify reaching for the validation level (L5 - *Link*). This is

the fundamental level in the *MODEL* framework, which corresponded to the critical level of the MPS Framework (checking) proposed by Carlson and Bloom¹¹. Only 4 Y12 students (1.1%), shown in Table 3, had achieved the learning outcome of demonstrating their competency in reflecting back into the problem (L4). This may be

attributed to the fact that students are expected not only to have abstract knowledge of mathematics but a sound contextual knowledge of a real-life setting in solving these non-routine problems. However, a staggering number of 167 students (52.8%) had achieved L3 (*Develop*) level of the *MODEL* framework for Y13, where they are able to solve the problem by using appropriate mathematical operations or formulae. These findings indicated that majority of the students in this study had the competency of transferring abstract mathematical knowledge into real-life situations through creating meanings of the problem, organising assumptions and ideas, developing appropriate mathematical formulae to solve it, and executing the plan to obtain the solutions. Similar observations were also reported in Brown and Schafer³³ study of modelling competency in teachers. They reported that it was necessary to develop effective skills in relating mathematics and context, mathematical idealisation and idealising contextual patterns, and relations to form mathematical structures. It is believed that only through these skills that an individual can interpret mathematical concepts in relation to a context.

Subsequently, the inter-rater reliability (IRR) was also assessed using a two-way mixed, consistency, average measures intra-class correlation (ICC) to measure the degree of consistency between the raters in assessing the students' mathematical modelling competency using the *MODEL* framework in five levels. The ICC was calculated using SPSS v.20 and generated a value of .89, indicating excellent range³⁵ and a high degree of agreement between raters in evaluating students' competencies using the six levels of the *MODEL* framework. The *MODEL* framework defined in six levels of this study was therefore deemed to be suitable for use in evaluating the students' cognitive competency in the test.

The relationship between students' confidence level in the attempted method/workings and solutions, and epistemic actions for problem-solving achieved by the students in this study was investigated using Pearson product-moment correlation coefficient (see Table 4).

Table 4. The Pearson correlations between the confidence level of students in the attempted methods and solutions

	Total score for 4 problems in the test	Confidence Level in workings
Total score for 4 problems in the test	-	.427**
Confidence Level in workings	.427**	-
Confidence Level in solutions	.423**	.883**

** Correlation is significant at the 0.01 level (2-tailed).

The overall students' performance showed that there was a medium, positive correlation between the total score achieved by the students and their confidence level in attempted workings, $r = .427$, $n = 167$, $p < .0005$. Similar findings are recorded between the total score achieved by the students and their confidence level in solutions obtained with $r = .423$, $n = 167$, $p < .0005$. These results indicated that the higher levels of confidence might associate with high levels of transfer skill of abstract knowledge into complex real-life situations. While the correlation between the confidence levels of students in their attempted workings and solutions have a strong, positive correlation, $r = .883$, $n = 167$, $p < .0005$. This indicated that students who are confident in working mathematically are also confident in the solutions obtained. When analysis was done individually for each problem in the test, it showed that students are most confident in attempting problems with familiar real-life situations (Q4 – giant shoes size dimension versus height) with $r = .337$, $n = 167$, $p < .0005$ as students are able to display realistic considerations of comparing his/her height to own shoe size as the basis for comparison.

It is unsurprising to record that students lack most confidence in attempting Q3 (tiling of floor) with $r = .157$, $n = 167$, $p < .0005$ because they might be unfamiliar with the contexts of tiling of floor in real settings, even though they have excellent concepts of the arithmetic in solving this problem. Contrasting to Bayazit⁵ observations that students excluded real-world knowledge and experiences from their solutions and their performance worsen when the problem request more cognitive demands and flexibil-

ity in thinking in his study. This study has further shown that Brunei students have the abilities and knowledge to include realistic considerations from their prior knowledge and experiences in solving non-routine problems.

5. Conclusions

This study does not provide a complete picture of students' competencies in problem-solving due to lack of analysis in the meta cognitive and affective aspects of problem-solving process. On the other hand, the test instrument was adequate to fulfil the aim of the study to explore students' cognitive competency using the *MODEL* framework in six levels by creating a proxy of the real situations for the students through different mechanisms of the concepts and contexts of the curriculum. As the results and analysis have shown that majority of the Brunei junior college students possess the abilities and skills to solve non-routine problems by applying prior knowledge, rules, procedures and experiences in the context of the problem to obtain solutions, but all of them had failed to justify or validate their solutions in the realities of the problem contexts. It can be concluded that these students are excellent routine problem solvers and this gives an opportunity to further develop them into intermediate problem solvers of non-routine problems. The results also revealed that students acquiring more abstract knowledge of the mathematics with maturity of thinking skills in a higher year group performed better in non-routine problems that are most significant to them.

As discussed by Haines³⁶, the current mathematics teaching emphasises the applications of mathematics and students' abilities to address real-world problems. Imagine a classroom where students can actively explore, constructively engaged with learning materials and openly discuss strategies and ideas with peers and teacher. These students need a simple and innovative approach in the teaching and learning of mathematics through a simple *link* of abstract mathematics and the real-world settings. They need this association or connection to enhance and to appreciate the learning of mathematics, and integrating problem solving of non-routine problems into the curriculum can foster students to competently use and apply their mathematical knowledge. Furthermore, we believed that when a student is engaged in problem-solving activities, their mathematical reasoning processes should

improve. Likewise, de Oliveira and Barbosa³⁷ examined the works by Doerr³⁸ and Leiß³⁹ and reflected that by promoting mathematical modelling, in the context of real-world settings in the classroom have incited transformations in the pedagogical relationship between the teacher and his or her students. Perhaps, for the benefit of Brunei's perspectives, a simple integration of solving non-routine (real-life) problems into the curriculum might just be the solution for improving mathematical literacy.

6. List of Abbreviations

MODEL Framework – Meanings (M), Organise (O), Develop (D), Execute (E), and Link (L) L0 – Level 0, L1 – Level 1 and so on.

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