# An Improved Brown's Method Applying Fractal Dimension to Forecast the Load in a Computing Cluster for Short Time Series

#### Vladimir Vyacheslalovich Kopytov, Viacheslav Ivanovich Petrenko, Fariza Bilyalovna Tebueva\* and Natalia Vasilievna Streblianskaia

Department of Applied Mathematics and Computer Security, Department of Organization and Technology of Information Security, North Caucasus Federal University, Stavropol, Russia fariza.teb@gmail.com

### Abstract

Background/Objectives: The study considers the class of short time series possessing persistency. The investigation is focused on selecting and adapting mathematical tools to forecast such type of time series. Methods/Statistical Analysis: Adaptive prediction models are capable of adjusting their structures and parameters to changing conditions. Adaptive prediction methods opted for Brown's method for time series of the load in a computing cluster. The time series under investigation possess persistency. Fractal dimension of time series should be defined applying the Hurst exponent averaged through these time series. The obtained fractal dimension should be taken as the smoothing ratio for Brown's method. Findings: Applied forecasting tasks are often comprised of too short samples that do not allow obtaining statistically valid predictions. To forecast short time series is a problem of current importance; and to solve this problem it is required to have an idea of the process described by these time series. One of such series is dynamic measurement of load in a computing cluster, the statistics of which cannot be modeled for a long-term period. In Brown's method, the predicted value is found applying the average weighted smoothing ratio within the range from zero to one. Selection of optimum smoothing ratio is mostly done experimentally, by sorting out all possible values within this range. This procedure can become quite laborintensive. Besides, there were ideas of more precise forecasting if the smoothing ratio is selected within the range from one to two. This suggestion has to be justified. This study offers a prediction method improving Brown's method and confirms that the smoothing ratio should be selected within the range from one to two. Improvements/Applications: The suggested method provides theoretically precise calculated value of the smoothing ratio instead of the experimentally selected one and solves the problem of the measuring lag between forecast and actual values.

Keywords: Brown's Method, Fractal Dimension, Hurst Exponent, Persistency, Short Time Series

# 1. Introduction

In adaptive prediction models<sup>1-4</sup> time-related characteristics of the parameters are used together with the characteristics of the interrelations existing between consecutive terms of time series and thus, the relevant models are rebuilt as the data deteriorate. These models are appropriate for more precise response to the changing terms of the time series affected by casual interference and they apply correcting elements for aligning basic model work and actual data. Prediction adaptive models are founded on two patterns: Moving Average (MA-models) and Autoregressive (AR-models)<sup>5,6</sup>. Within the moving average pattern, evaluation of the current level is represented by the weighted average value of all previous levels; furthermore, in the process of observation, the weights are counted with respect to their remoteness from the last level, i.e. the closer

\*Author for correspondence

those observations are to the end of the observed interval, the higher is the informational value of those observations. Such models are good at reflecting the changes occurring in a trend, but in their pure form, they cannot reflect the fluctuations. In the models built according to MA principle, the processes of responding to prediction errors and discounting the levels of the time series are implemented by means of applying smoothing (adaptation) parameters whose values can alter within the range from zero to one<sup>7,8</sup>. In Autoregressive pattern (AR-model) the evaluation of the current level is represented by the weighted sum of not all but several preceding levels, in addition, the weighted quotients of observation are not ranked. Informational values of observation are determined not by their proximity to the modeled level, but by the strength of correlations between them. AR-models are most appropriate for stationary fluctuating processes, while MA-models are better fit for non-stationary evolutional processes with the intrinsic properties of fractality<sup>9,10</sup> and multi-fractality<sup>11</sup>. The load in a computing cluster is a non-stationary evolutionary process, thus, in this case, MA-models will be considered for the purposes of prediction<sup>12</sup>.

## 2. Concept Headings

Suggested prediction method is applied to persistent<sup>13,14</sup> time series, i.e. to the series, where the Hurst exponent is within the range of [0.7; 1]. The property of persistency means that this particular section of a time series is prone to behave according to the trend as follows: If the values of a series used to increase over the previous period, it is probable that they would continue increasing over the following period as well. Persistent time series possess long-term memory therefore there are long-term correlations between current and future events.

Fractal dimension is a value that describes the space filled with an object and it is a fractional number by contrast to topological dimension (which is always an integral number). Fractal dimension D of a time series correlates with the Hurst exponent H as follows:

$$D = 2 - H \tag{1}$$

Where H is the Hurst exponent, which is a measure of displacement in partially Brownian movements.

The Hurst indicator H can be calculated applying the algorithm of R/S - analysis or the normalized Hurst range. According to this algorithm, the given time series  $Z = \langle z_i \rangle$ ,  $i = \overline{1, n}$  is split into starting sections  $z_{\tau}$ , where  $\tau = \overline{1, n}$ . For those sections, the range is calculated as follows:  $R = R(\tau) = \max_{1 \le t \le \tau} Z_{\tau,t} - \min_{1 \le t \le \tau} Z_{\tau,t}$  and then this range is normalized to standard deviation  $S(\tau)$ . . The Hurst exponent amounts to the value as follows:  $H(\tau) = \frac{(\log (R(\tau)/S(\tau)))}{\log(\tau/2)}$ .

Applying this algorithm, the values of the Hurst exponent  $H(\tau)$  for starting sections  $\tau = \overline{3,n}$  are found. Now, the averaged Hurst exponent for all starting sections  $\tau = \overline{3,n}$  shall be found as follows:

$$\widetilde{H} = \frac{\sum_{\tau=3}^{n} H(\tau)}{n-2}$$
(2)

This study suggests that the value of fractal dimension D of the time series should be assigned to the smoothing ratio  $\alpha$ 

$$\alpha = D = 2 - H \tag{3}$$

Inserting value  $\alpha$  of (3) into (1), the calculation formula of an improved Brown's method applying fractal dimension will be as follows:

$$\hat{x}_{n+1} = D \cdot x_n + (1 - D) \cdot x_{n-1} \tag{4}$$

To solve the issue of the lags existing between the prediction and the relevant actual values, it is suggested that forecasting should be carried out according to the following formula:

$$\hat{x}_{n+1} = D \cdot \hat{x}_n + (1 - D) \cdot x_{n-2}$$
(5)

It is now possible to evaluate the percentage accuracy of prediction, by means of subtracting the relative deviations of the resulting predicted values from the actual values.

$$\delta_i = \frac{|x_i - x_i|}{x_i} \cdot 100\% \tag{6}$$

Cumulative prediction error shall be the average value  $\widetilde{\delta}~$  of all relative deviations (6).

# 3. Results

For the purposes of carrying out forecasting applying the suggested method, consider four short time series of load on computing cluster with similar lengths  $i = \overline{1, 21}$ :

 $X^1 = \langle x_i^1 \rangle$  – Time series of the load in a computing cluster, Gflops;

 $X^2 = \langle x_i^2 \rangle$  – Time series of the load in processor No. 1 of a computing cluster, Gflops;  $X^3 = \langle x_i^3 \rangle$  – Time series of the load in processor No. 2 of a computing cluster, Gflops;

 $X^4 = \langle x_i^4 \rangle$  – Time series of the load in processor

No. 3 of a computing cluster, Gflops.

The results of calculating the Hurst exponent for these time series have been represented in Table 1 (values have been calculated accurate to five decimal places).

Figure 1 shows histograms of the time series under consideration; Table 2 gives the calculated averaged values of the Hurst exponent and fractal dimensions.

**Table 1.**  $X^1$ ,  $X^2$ ,  $X^3$ ,  $X^4$  time series and the relevant values of the Hurst exponent  $H^1(\tau)$ ,  $H^2(\tau)$ ,  $H^3(\tau)$ ,  $H^4(\tau)$  for all starting sections

Sequential number of Time Series	$X^1$	$H^1(\tau)$	$X^2$	$H^2(\tau)$	X <sup>3</sup>	$H^{3}(\tau)$	$X^4$	$H^4( au)$
1	16,590	-	23	-	10.2	-	27.4	-
2	16,484	-	22	-	12.1	_	20.4	-
3	19,524	0.85,359	31	0.84,202	9.8	0.82,165	21.1	0.84,455
4	21,476	0.91,718	26	0.77,761	9.8	0.77,095	21.6	0.77,544
5	19,371	0.88,795	28	0.82,522	11.3	0.66,498	13.7	0.53,943
6	21,007	0.87,374	24	0.86,553	13.0	0.64,683	27.8	0.78,130
7	19,938	0.85,452	18	0.84,354	12.2	0.70,833	21.2	0.71,180
8	19,872	0.83,675	20	0.85,590	13.1	0.81,880	27.6	0.79,263
9	21,688	0.80,974	25	0.80,842	10.7	0.82,160	31	0.82,352
10	20,832	0.81,221	19	0.82,368	12.5	0.76,241	35.2	0.82,763
11	22,256	0.80,229	28	0.74,851	10.4	0.79,282	31.8	0.86,672
12	23,534	0.77,455	35	0.75,304	9.3	0.82,203	29.7	0.88,134
13	27,454	0.78,720	28	0.76,699	12.9	0.69,722	32.6	0.89,469
14	31,419	0.81,451	31	0.78,153	10.6	0.71,321	25.3	0.87,263
15	31,194	0.86,280	22	0.73,502	12.7	0.63,591	24.6	0.86,291
16	30,478	0.90,076	27	0.73,781	11.3	0.63,401	18.3	0.86,628
17	29,568	0.92,515	22	0.72,057	9.6	0.65,670	18.5	0.86,994
18	29,685	0.94,128	22	0.72,368	10.1	0.67,507	25	0.86,109
19	31,777	0.95,513	30	0.68,252	12.0	0.64,577	25.2	0.85,222
20	14,825	0.94,457	17	0.67,717	10.6	0.65,504	24.5	0.84,691
21	21,497	0.94,063	24	0.67,637	10.0	0.68,809	25.3	0.83,903



**Figure 1.** Graphic representation of time series: a)  $X^1$ , b)  $X^2$ , c)  $X^3$ , d)  $X^4$ .

Table 2. Averaged values of the Hurst exponent and corresponding fractal dimensions

Calculated values	$X^1$	$X^2$	$X^3$	$X^4$
Averaged Hurst exponent, $\widetilde{H}$	0.86,813	0.77,080	0.71,744	0.82,158
Fractal dimension, D	1.13,187	1.22,920	1.28,256	1.17,842

**Table 3.** Results of  $X^1$ ,  $X^2$ ,  $X^3$ ,  $X^4$  time series forecasting applying an improved Brown's method with fractal dimension

Sequential number of Time Series	Time seri computin Gflops.	es $X^1$ of th ig cluster,	ne load in a	Time series $X$ No.1 of a comp Gflops.	<sup>2</sup> of the load in <sub>j</sub> puting cluster,	processor	Time se processo cluster, Gflops.	ries $X^3$ of $th$ or No.2 of a c	ie load in omputing	Time sei processo cluster, Gflops.	ries $X^4$ of the or No.3 of a c	e load in omputing
	Actual value	Forecast value	Relative deviation	Actual value	Forecast value	Relative deviation	Actual value	Forecast value	Relative deviation	Actual value	Forecast value	Relative deviation
1	16,590	1	1	23		1	10.2	I	1	27.4	I	1
2	16,484	16,470	0.1	22	22	1.0	12.1	12.6	4.4	20.4	19.2	6.1
3	19,524	19,925	2.1	31	33	6.7	9.8	9.2	6.6	21.1	21.2	0.6
4	21,476	21,733	1.2	26	25	4.4	9.8	9.8	0.0	21.6	21.7	0.4
5	19,371	19,093	1.4	28	28	1.6	11.3	11.7	3.8	13.7	12.3	10.3
6	21,007	21,223	1.0	24	23	3.8	13.0	13.5	3.7	27.8	30.3	9.0
7	19,938	19,797	0.7	18	17	7.6	12.2	12.0	1.9	21.2	20.0	5.6
8	19,872	19,863	0.0	20	20	2.3	13.1	13.4	1.9	27.6	28.7	4.1
6	21,688	21,927	1.1	25	26	4.6	10.7	10.0	6.3	31	31.6	2.0
10	20,832	20,719	0.5	19	18	7.2	12.5	13.0	4.1	35.2	35.9	2.1
11	22,256	22,444	0.8	28	30	7.4	10.4	9.8	5.7	31.8	31.2	1.9
12	23,534	23,703	0.7	35	37	4.6	9.3	9.0	3.3	29.7	29.3	1.3
13	27,454	27,971	1.9	28	26	5.7	12.9	13.9	7.9	32.6	33.1	1.6
14	31,419	31,942	1.7	31	32	2.2	10.6	100	6.1	25.3	24.0	5.1
15	31,194	31,164	0.1	22	20	9.4	12.7	13.3	4.7	24.6	24.5	0.5
16	30,478	30,384	0.3	27	28	4.2	11.3	10.9	3.5	18.3	17.2	6.1
17	29,568	29,448	0.4	22	21	5.2	9.6	9.1	5.0	18.5	18.5	0.2
18	29,685	29,700	0.1	22	22	0.0	10.1	10.2	1.4	25.0	26.2	4.6
19	31,777	32,053	0.9	30	32	6.1	12.0	12.5	4.5	25.2	25.2	0.1
20	14,825	12,590	15.1	17	14	17.5	10.6	10.2	3.7	24.5	24.4	0.5
21	21,497	22,377	4.1	24	26	6.7	10.0	9.8	1.7	25.3	25.4	0.6
Prediction		19,432			22			9.6			25.6	
Deviation			1.8 %			5.4%			<b>4.0</b> %			3.1 %

The prediction models for the time series under study are given below:

$$\hat{x}_{n+1}^{1} = 1,131866 \cdot x_{n} - 0,131866 \cdot x_{n-1}, \qquad (7)$$

$$\hat{x}_{n+1}^2 = 1,22920 \cdot x_n - 0,22920 \cdot x_{n-1};$$
(8)

$$\hat{x}_{n+1}^3 = 1,282556 \cdot x_n - 0,282556 \cdot x_{n-1} \tag{9}$$

$$\hat{x}_{n+1}^4 = 1,178418 \cdot x_n - 0,178418 \cdot x_{n-1}.$$
 (10)

To obtain prediction for the reference point n = 22, formula (5) shall be applied:

$$\hat{x}_{22}^{1} = 1,131866 \cdot x_{21} - 0,131866 \cdot x_{19}, \qquad (11)$$

 $\hat{x}_{22}^2 = 1,22920 \cdot x_{21} - 0,22920 \cdot x_{19}; \qquad (12)$ 

$$\hat{x}_{22}^3 = 1,282556 \cdot x_{21} - 0,282556 \cdot x_{19};$$
 (13)

 $\hat{x}_{22}^4 = 1,178418 \cdot x_{21} - 0,178418 \cdot x_{19} . \tag{14}$ 

Prediction error can be evaluated by subtracting the predicted values for reference points  $n = \overline{3, 21}$  and by

obtaining average value  $\tilde{\delta}$  of relative deviations  $\delta_3, ..., \delta_1$  according to formula (6).

Table 3 shows initial values, forecast values and the deviations of the forecast values from the actual values as percentages for each of the time series under investigation.

### **4.Discussion**

To compare the results of applying Brown's method<sup>15,16</sup> with the results of applying the improved Brown's method, the forecasting shall be carried out for the four time series under study using Brown's method. In the process of prediction applying Brown's method, the following smoothing quotients have been obtained experimentally:

$$\alpha = 0,61$$
 for time series  $X^1$ ;

$$\alpha = 0,45$$
 for time series  $X^2$ ;

 $\alpha = 0,36$  for time series  $X^3$ ;

$$\alpha = 0,65$$
 for time series  $X^4$ .





applying Brown's method
$, X^4$
$X^3$
$X^2$ ,
$X^{1}$ ,
able 4. Results of forecasting time series

Sequential number	Time seri	ties $X^1$ , smo	othing	Time sei	ries $X^2$ , sm $\alpha = 0.6$	oothing 1	Time serie: $\alpha = 0.2$	s X <sup>3</sup> , smoothi	ng coefficient	Time serie $\alpha = 0.6$	$X^4$ , smooth	ing coefficient
	coemcien	μ α – ν, -	2	coemcie	$\alpha = \alpha^{2}$	1	α - υ,			α – ν, ν		
of Time	Actual	Forecast	Relative	Actual	Forecast	Relative	Actual	Forecast	Relative	Actual	Forecast	Relative
Series	value	value	deviation	value	value	deviation	value	value	deviation	value	value	deviation
1	16,590	1	1	23	I	I	10.2	I	I	27,4	I	1
2	16,484	16,525	0.3	22	22.5	27.3	12.1	10.9	10.0	20.4	22.9	12.0
3	19,524	18,338	6.1	31	26.1	0.5	9.8	11.3	15.0	21.1	20.9	1.2
4	21,476	20,715	3.5	26	28.7	2.5	9.8	9.8	0.0	21.6	21.4	0.8
5	19,371	20,192	4.2	28	26.9	12.2	11.3	10.3	8.5	13.7	16.5	20.2
6	21,007	20,369	3.0	24	26.2	45.3	13.0	11.9	8.4	27.8	22.9	17.8
7	19,938	20,355	2.1	18	21.2	6.2	12.2	12.7	4.2	21.2	23.5	10.9
8	19,872	19,898	0.1	20	18.9	24.3	13.1	12.5	4.4	27.6	25.4	8.1
6	21,688	20,980	3.3	25	22.3	17.4	10.7	12.2	14.4	31	29.8	3.8
10	20,832	21,166	1.6	19	22.2	20.6	12.5	11.3	9.2	35.2	33.7	4.2
11	22,256	21,701	2.5	28	23.1	33.9	10.4	11.7	12.9	31.8	33.0	3.7
12	23,534	23,036	2.1	35	31.2	11.5	9.3	10.0	7.6	29.7	30.4	2.5
13	27,454	25,925	5.6	28	31.8	2.5	12.9	10.6	17.9	32.6	31.6	3.1
14	31,419	29,873	4.9	31	29.4	33.5	10.6	12.1	13.9	25.3	27.9	10.1
15	31,194	31,282	0.3	22	26.9	0.5	12.7	11.4	10.6	24.6	24.8	1.0
16	30,478	30,757	0.9	27	24.3	10.5	11.3	12.2	7.9	18.3	20.5	12.0
17	29,568	29,923	1.2	22	24.7	12.3	9.6	10.7	11.3	18.5	18.4	0.4
18	29,685	29,639	0.2	22	22.0	26.7	10.1	9.8	3.2	25.0	22.7	9.1
19	31,777	30,961	2.6	30	25.7	51.1	12.0	10.8	10.1	25.2	25.1	0.3
20	14,825	21,436	44.6	17	24.0	0.1	10.6	11.5	8.5	24.5	24.7	1.0
21	21,497	18,895	12.1	24	20.2	16.0	10.0	10.4	3.8	25.3	25.0	1.1
Forecast		17,308			18.4			10.5			25.1	
Deviation			5.3 %			13.6 %			9.0 %			6.2 %

Use  $\tilde{\omega}^1$ ,  $\tilde{\omega}^2$ ,  $\tilde{\omega}^3$ ,  $\tilde{\omega}^4$  to express the relevant prediction errors of time series  $X^k$ ,  $k = \overline{1,4}$  according to the basic Brown's method. Table 4 shows actual values, predicted values and relative deviations. Figure 2 represents histograms of actual and predicted values for time series  $X^1$ , obtained by applying the improved Brown's method.

Table 5 gives the values n = 22 and the relevant average prediction errors for the predictions carried out applying two methods.

Table 5 confirms that improved Brown's method gives better prediction accuracy as compared to conventional Brown's method in each time series. This fact proves that the suggested method makes it possible to take into account the preceding values more comprehensively to obtain the predicted value of a time series. The efficiency of the suggested improved Brown's method can be evaluated by merit value  $\tilde{\delta}$  for the time series of load in a computing cluster under investigation:

$$\Delta \widetilde{\delta} = \widetilde{\omega}^k - \widetilde{\delta}^k. \tag{15}$$

Efficiency evaluations of the prediction method suggested within the framework of this study as compared to conventional Brown's method are given below:

$$\Delta \widetilde{\delta}^{1} = \widetilde{\omega}^{1} - \widetilde{\delta}^{1} = 5.3\% - 1.8\% = 3.5\%;$$
  

$$\Delta \widetilde{\delta}^{2} = \widetilde{\omega}^{2} - \widetilde{\delta}^{2} = \mathbf{J} \cdot .6\% - 5.4\% = 8.2\%;$$
  

$$\Delta \widetilde{\delta}^{3} = \widetilde{\omega}^{3} - \widetilde{\delta}^{3} = 9.0\% - 4.0\% = 5.0\%;$$
  

$$\Delta \widetilde{\delta}^{4} = \widetilde{\omega}^{4} - \widetilde{\delta}^{4} = 6.2\% - 3.1\% = 3.1\%;$$

Thus, it proved possible to improve the prediction error of time series  $X^1$  by 3.5%; that of time series  $X^2$ by 8.2%; of time series  $X^3$  by 5.0%; and of time series  $X^4$  by 3.1%.

### 5. Conclusion

The results of the calculations show that the method suggested within the framework of this study ensures less prediction error at high ultimate precision of sorting out the smoothing ratio as compared to Brown's method. This study illustrates persistent short time series forecasting exemplified by the process of loading in a computing cluster. It should be noted that the improved Brown's method can be applied to short time series of arbitrary character: Natural, social and economical, technical<sup>17,18</sup>. The principle prerequisite is the property of persistency of time series that predetermines successful forecasting. The tool for calculating the smoothing ratio  $\alpha$  applying Brown's method and based on direct calculation of the averaged Hurst exponent in persistent time series, suggested in the framework of this study, is less complicated from computational perspective and makes it possible to achieve better accuracy of predicting the modified method, as compared to basic Brown's model. This can be explained by the fact that the accuracy of classical prediction model is predetermined by the ultimate accuracy of sorting out the smoothing quotient.

### 6. Acknowledgements

The study has been developed in the course of implementing scientific project "Developing cross-platform technology for creating mobile applications with preset contours of integration to improve functional and resource efficiency of corporate information systems" within the framework of Federal Target Program for Research and Development for 2014-2020 (unique identifier of applied research investigation RFMEFI57614X0066) with financial support from the Ministry of Education and Science of the Russian Federation.

### 7. References

- Kantorovich GG. Time series analysis. Economic Journal of HSE; 2003. p. 79–103.
- Hyndman R. Fitting models to short time series. Blog post. 2016. Available from: http://robjhyndman.com/hyndsight/ short-time-series
- Woodward WA, Gray HL, Elliott AC. Applied time series analysis. Statistics: A series of textbooks and monographs. CRC Press; 2011.
- Lyubchyk L, Grinberg G, Rivtis A. Adaptive forecasting of wave non-periodic time series. 58th International Atlantic Economic Conference; Chicago, Illinois. 2004. p. 1–11.
- Yu PL. Adaptive methods for short-term forecasting of time series: Textbook. Moscow: Finansy i Statistika; 2003.
- 6. Siegel E. Practical business statistics. Academic Press; 2011.
- Svetunkov SG, Butukhunov AV, Svetunkov IS. Investigating super-limited cases of Brown's method applied to small samples: Preprint. Saint Petersburg: Publishing House SPbGUEF; 2005.
- 8. Kopytov VV, Tebueva FB. Forecasting anthropogenic emergencies applying short time series. Scientific and Educational Problems of Civil Protection. 2009; 2:33–6.

- Malinetskiy GG, Potapov AB. Non-linearity. New problems, new opportunities. New in Synergetic Science. Enigmas of non-equilibrium structures. Moscow: Nauka, (Series "Cybernetics: unlimited opportunities and potential limitations"); 1996.
- 10. Shelukhin OI, Tenyakshev AM, Osin AB. Fractal processes in telecommunications. Moscow: Radiotekhnika; 2003.
- 11. Shelukhin OI. Multi-fractals. Info-communicational applications. Moscow: Hotline-Telecom; 2011.
- 12. Peters Edgar E. Chaos and order in the capital markets: A new view of cycles, prices and market volatility. (Wiley Finance). Moscow: Mir; 2000.
- 13. Perepelitsa VA, Tebueva FB. Pre-prediction fractal analysis of time series of the industrial production growth index of country and of region. Bulletin of Stavropol State University. 2005; 44:21–9.

- 14. Tebueva FB. Two approaches to realizing fractal analysis of time series. Scientific and Technical Bulletin of Saint Petersburg State Polytechnic University. 2007; 4(2):105–12.
- 15. Mandel AS. Analog method in forecasting short time series: Expert and statistical approach. Automatics and Teleautomatics; 2004. p. 143–52.
- Pylkin AN, Demidova LA, Skvortsov SV, Skvortsova TS. Hybrid models for forecasting short time series. Moscow: Horline-Telecom; 2012.
- Gritsenko AV, Demurchev NG, Kopytov VV, Shulgin AO. Decomposition analysis and machine learning in a workprediction approach to the task scheduling problem for high-loaded distributed systems. Modern Applied Science. 2015; 5(9):38–49.
- Aalen OO, Frigessi A. What can statistics contribute to a causal understanding? Board of the Foundation of the Scandinavian Journal of Statistics; 2007. p. 155–68.