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Unscented Kalman Filter with Application to Bearings-only Passive Target Tracking

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Abstract

Background/Objectives: Target tracking is an age old problem which demands robust statistical estimators which can effectively track the target within the acceptable limits of errors in target motion parameters. The objective of this paper is to develop a novel estimation algorithm based target tracking simulator for underwater target tracking applications. **Methods/Statistical Analysis**: The unscented transformation developed by Julier, et al., is applied to the body of Kalman filter to synthesize Unscented Kalman filter. The modeling of target state and measurement vectors is carried out. Unscented Kalman filter is integrated into the model to result in evolution of simulator. Extensive performance evaluation of UKF with respect to bearings-only target tracking problem in Monte-Carlo simulation is carried out and the results are presented. **Findings:** UKF algorithms effectively track the target with encouraging convergence time which is proved from the results obtained in single run and Monte-Carlo simulation. It is observed that UKF is suitable algorithm for bearings-only target tracking problem. **Application/Improvements:** The results obtained are satisfactory and UKF can be used in futuristic submarines in Indian Navy owing to its advantages as envisaged in this paper.

Keywords: Estimation Theory, Kalman Filter, Simulation, Sonar, Target Tracking

1. Introduction

In maritime environment, target motion analysis using bearings-only measurements in two-dimensional plane is usually carried out. It is assumed that the observer generates a sequence of bearing measurements of the target. The observer typically performs S-maneuver on line of sight as shown in Figure 1 for range observability. In Bearings-Only Tracking (BOT), range measurements are unavailable[1].

The conventional Kalman filter[1–7] works well for linear models. Kalman filter is extended for non-linear processes like bearings-only target trackingand it is called as Extended Kalman filter EKF. Modified Gain Extended Kalman Filter (MGEKF)[6] modifies the gain eliminating the problem of divergence in EKF. The main ideology of MGEKF is that nonlinearities are "modifiable". It exhibits certain similarities that of PLE but it differs in its functionalities. In PLE, the gain is dependent on previous and arrived measurements. On contrary to PLE, the gain function in MGEKF depends only on previous measurements. The problem of bias is eliminated in MGEKF by having no co-relation between gain function and measurement noise[6,7]. A simplified version of the modified gain function is available[8]. This algorithm using bearing and altitude measurements is

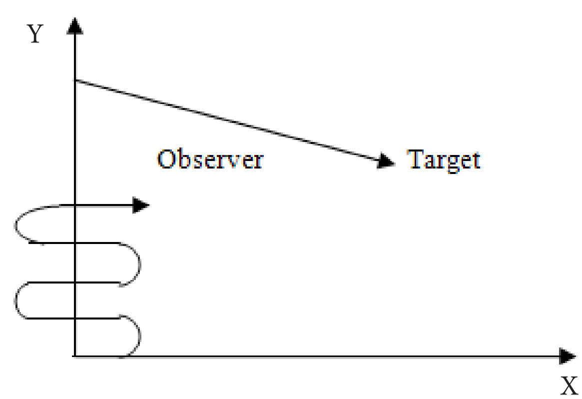

Figure 1. Observer in S-maneuver.

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extended to underwater applications using bearings-only measurements and it is called Modified Gain Bearings-only Extended Kalman Filter (MGBEKF)[9]. Unscented transformation is a novel transformation based on the intuition that it is easier to approximate a probability distribution than it is to approximate anarbitrary nonlinear function or transformation[10,11].

The unscented transformation is applied to skeleton of Kalman filter and Jeffrey Uhlmann coined this filter as Unscented Kalman Filter (UKF). UKF works well only if noise measurements have Gaussian probability density functions. In nutshell, UKF can be utilized for target tracking which a non-linear problem is provided only if the measurements are Gaussian[12]. Particle Filter (PF) is the advanced filter[12,13], overcomes the disadvantages of classical UKF and PF effectively deals with non-Gaussianity in measurement noise. In Bayesian framework, PF uses numerous samples, termed as particles (weighted samples of the state) to effectively predict the posterior probability distribution. The Probability Density Function (PDF) of true state can be accurately predicted as the particles approach infinite number which increases computational effort but produces the best state estimates for non-Gaussian noise and non-linear applications like bearings-only tracking. PF is cascaded along with classical filters such as EKF, UKF and then resampled (if necessity arises)[12] to overcome the sample impoverishment. Many researchers from TMA community are effectively and actively contributing to this emerging field.

The aim of this paper is to select a better algorithm from the existing algorithms for BOT. In this paper, it is assumed that the bearing measurements are contaminated with zero mean Gaussian noise of constant variance and this assumption is valid as the latest sonars are equipped with highly sophisticated signal processing algorithms. So, PF is not necessary and hence EKF or UKF can be used for this application. It is well known that UKF outperforms than that of EKF and so UKF can be used here safely. Immediately after the publication of UKF, Ito and Xiong[13,14] raised that the UKF can be considered as a special case of so-called Gaussian filters, where the non-linear filtering problem is solved using Gaussian assumed density approximations. SimoSarkka[15] mentioned that this generalized framework enables the usage of various powerful Gaussian quadrature and cubature integration methods[16] for the realization new non-linear filters like Gauss-Hermite Kalman filter and Cubature Kalman filter. Jouni, et al.[17] has also demonstrated that UKF can be seen as a generalization of CKF. So, far UKF is generally used for non-linear applications like BOT. The authors are motivated by the paper written by Pie. H. Leong et al[18]. Their effort is highly appreciated. However, the algorithms are evaluated using target position errors only. Target course error and target speed error are also required for passive target tracking applications. As this research work is useful for surveillance in sea waters, some more information like errors in target course and speed are also required.

In weapon guidance algorithms, the future position of target location is determined by present position as well as course and speed of the target. Target position error does not provide any information regarding the course and speed errors. Various tactical scenarios also need to be considered for performance evaluation of the bearings-only target tracking algorithms.

The bearings-only tracking algorithm can be used for submarine and ship observers. The target can be submarine, ship or torpedo. All the scenarios can be described in terms of Angle on Target Bow (ATB). ATB is defined as the difference in line of sight and target course. The algorithms are to be evaluated when observer is approaching the target at low (0^o-30^o), ii) medium (31^o-40^o) and iii) high (41^0-90^0) ATBs. In general, if ATB is more than 90^o (opening range scenarios), the observer is not interested in the estimation of target path at high ranges.

In [18] the observer carried out one maneuver and the length of each leg is 15 minutes approximately. In general one observer maneuver is not sufficient. The first observer maneuver makes the process observable and the second maneuver is required to get better solution and third maneuver is required to obtain the required solution. Additionally fourth observer maneuver may be required in case of highly noisy bearing measurements or if the range increases with time (range is opening). The sampling interval[18] chosen is 1 minute which leads to availability of fewer bearing measurements. In real time environment, integration time at the sonar is 1 second approximately for bearing measurement. In this paper, it is chosen as 1 second. The scenarios chosen in [18–23] are far away from practical viewpoint. In highly nonlinear scenario, the turning rate of observer is infinite which is practically not feasible. In real time environment, the maximum possible submarine observer turning rate is 0.5^o/s. Another important issue is initialization of the target state vector in [18] where, the observer course is assumed to be target course with 180 degrees addition, in reality, target course and target speed are not available initially. In this paper, it is tried out to evaluate the UKF algorithms with different scenarios covering all types of ATB's and possible observer and target speeds.

Section 2 describes mathematical modeling of the bearing measurements, target state & measurement equations and UKF algorithm. In section 3, initialization of target state vector and its covariance matrix of UKF is elaborated. Performance evaluation of these algorithms is discussed in section 4. Summary and conclusion are presented in section 5.

2. Mathematical Modeling

Let $X_s(k)$ be

$$X_s(k)=\begin{bmatrix} \dot{x}(k) & \dot{y}(k) & R_x(k) & R_y(k) \end{bmatrix}^T \tag{1}$$

B_m is modeled as

$$B_m(k+1)=\tan^{-1}\left(\frac{R_x(k+1)}{R_y(k+1)}\right)+\zeta(k) \tag{2}$$

The measurement matrix is given by

$$H(k+1)=\begin{bmatrix} 0 & 0 & \hat{R}_Y(k+1|k)/R^2(k+1|k) & \hat{R}_x(k+1|k)/R^2(k+1|k) \end{bmatrix} \tag{3}$$

The target state dynamic equation is given by

$$X_s(k+1)=\varphi X_s(k)+b(k+1)+\Gamma w(k) \tag{4}$$

$$\varphi=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ t & 0 & 1 & 0 \\ 0 & t & 0 & 1 \end{bmatrix} \tag{5}$$

$$b(k+1)=\begin{bmatrix} 0 & 0 & -(x_0(k+1)-x_0(k)) & -(y_0(k+1)-y_0(k)) \end{bmatrix} \tag{6}$$

$$\Gamma=\begin{bmatrix} t & 0 \\ 0 & t \\ t^2/2 & 0 \\ 0 & t^2/2 \end{bmatrix} \tag{7}$$

$\omega(k)$, is assumed to be zero mean white Gaussian with

$$E\left[\Gamma(k)w(k)w^T(k)\Gamma^T(k)\right]=Q\delta_{ij} \tag{8}$$

where

$$\delta_{ij}=\sigma_w^{\ 2} \text{ if i=j}$$
$$=0 \text{ otherwise} \tag{9}$$

$$Q=\begin{bmatrix} t_s^{\ 2} & 0 & t_s^{\ 3}/2 & 0 \\ 0 & t_s^{\ 2} & 0 & t_s^{\ 3}/2 \\ t_s^{\ 3}/2 & 0 & t_s^{\ 4}/8 & 0 \\ 0 & t_s^{\ 3}/2 & 0 & t_s^{\ 4}/8 \end{bmatrix} \tag{10}$$

3. Initialization of Target State Vector and its Covariance Matrix

As bearing angle measurements are only available and there is no scope to guess the target speed and its course, the velocity components of the target submarine are assumed to be around (20 m/sec for torpedo target, 10m/s for ship target and 5 m/sec for submarine target) 8m/s (average of speeds of all possible targets). So, the initial estimate of the target state vector is given by

$$X_S(0|0)=\begin{bmatrix} 8 & 8 & \text{SRD}\sin B_m(0) & \text{SRD}\cos B_m(0) \end{bmatrix} \tag{39}$$

Where $B_m(0)$ is initial measured bearing. Initial estimate is assumed to be with uniform density function and so the elements of initial co-variance matrix is given by

$$P(0|0)=\begin{bmatrix} \frac{4\dot{x}(0|0)}{12} & 0 & 0 & 0 \\ 0 & \frac{4\dot{y}(0|0)}{12} & 0 & 0 \\ 0 & 0 & \frac{4R_x^{\ 2}(0|0)}{12} & 0 \\ 0 & 0 & 0 & \frac{4R_y^{\ 2}(0|0)}{12} \end{bmatrix} \tag{40}$$

Same initialization of the target state vector and its covariance matrix are chosen in both algorithms.

4. Performance Evaluation of the Algorithm

As the aim of this paper is to evaluate the performance of UKF, it is assumed that favorable situation prevail at the time of experiment and hence the measurements are available continuously for every 1 second. The algorithm is implemented using MATLAB in a PC environment. The noise in the bearing measurement is assumed to be following white Gaussian with Standard Deviation (SD) of 0.5°. So, each bearing measurement available at every 1sec is added with white Gaussian of SD, 0.5°.

The observer travels for 2 minutes in the first leg at 90° course and then turns towards 270^0 course. In the second leg, it travels for 4 minutes and then turns towards 90° course. The third and fourth legs are just like second leg except in the third leg, observer course is 270^0 and in fourth leg it is 90°course[19–23]. The length of each run is 42 minutes (2520 samples) covering 5 legs with 4 maneuvers. The algorithms are evaluated using scenarios given in Table 1, Table 2 and Table 3. The targets are assumed to be submarine, ship and torpedo in Table 1, Table 2 and Table 3 scenarios respectively. The target range and speeds are chosen which are close to realistic values.

The algorithm is said to be converged in the range, course and speed, once the solution (target location is tracked) is accepted. The convergence time in seconds of UKF is shown in the Table 4 to 6 with single runmode. The errors in estimate of range(R-error), course(C-error) and speed(S-error) in case oflow, medium and high ATBs are shown in Figure 2 to 4 respectively. For example in Table 1, the scenario describes a low ATB submarine target moving at 5 m/s at the course of 175°. Initial range between the target and submarine is 5500m and the target moves at an initial bearing of 0°. The observer moves at a speed of 5m/s in S-maneuver as shown in Figure 1.

Table 1. Scenarios chosen for low ATB

Scenario	Initial range (m)	Initial bearing (deg)	Target speed (m/s)	Observer speed (m/s)	Target course (deg)
1.(Submarine to submarine)	5500	0	6	5	175
2.(Submarine to ship)	10500	0	12	5	175
3.(Submarine to torpedo)	25000	0	22	5	175

Table 2. Scenarios chosen for medium ATB

Scenario	Initial range (m)	Initial bearing (deg)	Target speed (m/s)	Observer speed (m/s)	Target course (deg)
1.(Submarine to submarine)	5500	0	6	5	140
2.(Submarine to ship)	10500	0	12	5	140
3.(Submarine to torpedo)	25000	0	22	5	140

Table 3. Scenarios chosen for high ATB

Scenario	Initial range (m)	Initial bearing (deg)	Target speed (m/s)	Observer speed (m/s)	Target course (deg)
1.(Submarine to submarine)	5500	0	6	5	115
2.(Submarine to ship)	10500	0	12	5	115
3.(Submarine to torpedo)	25000	0	22	5	115

Table 4. Convergence time in seconds for low ATB scenarios with single run

Scenario	UKF			
	R	C	S	Total solution
1.(Submarine to submarine	362	361	242	362
2.(submarine to Ship)	422	337	482	482
3.(submarine to torpedo)	543	602	757	757

Table 5. Convergence time in seconds for medium ATB scenarios with single run

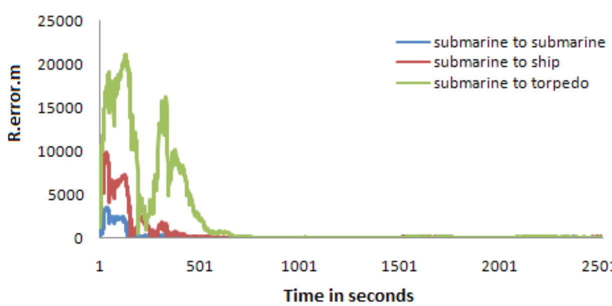
Scenario	UKF			
	R	C	S	Total solution
1. (Submarine to submarine	337	452	382	452
2. (submarine to Ship)	396	483	506	506
3. (submarine to torpedo)	529	529	756	756

Table 6. Convergence time in seconds for high ATB scenarios with single run

Scenario	UKF			
	R	C	S	Total solution
1. (Submarine to submarine	303	563	348	563
2. (submarine to Ship)	427	584	565	584
3. (submarine to torpedo)	543	645	665	665

With UKF algorithm, the estimated range, course and speed of the target are converged at 362,361 and 242 seconds and hence the total solution is said to be converged at 362 seconds.

According to this criterion, the convergence of solution is obtained in Monte-Carlo simulation for 100 runs are shown in Table 7 to 9. The graphs obtained in Monte-Carlo simulation are shown in Figure 5 to 7. It can be observed that for submarine to torpedo scenarios at medium and high ATBs the solution is not converged due to low bearing rate. However for low ATB scenarios, in which bearing rate is more. UKF is efficient for BOT as UKF provides tuning parameters like α and β to fine tune the solution. In UKF, β and α are chosen as 2 and 0.09 respectively. These parameters can be still fine tuned on trial and error basis. Hence, for passive bearings-only target tracking, in which the solution is observable after the observer's first maneuver, it is better to choose the UKF algorithm.



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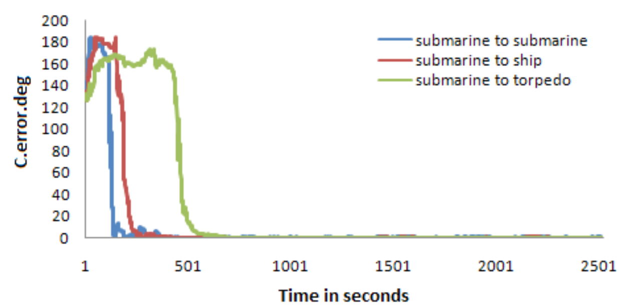

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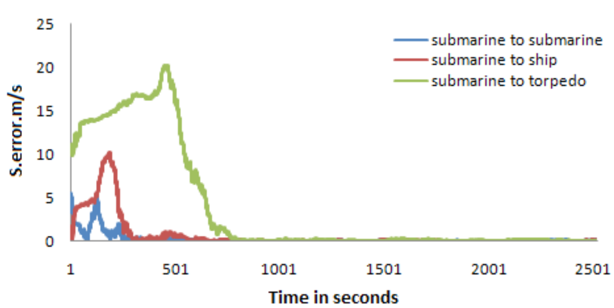

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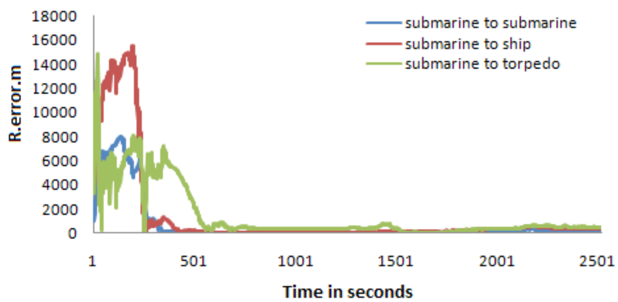
Figure 2. (a) R Error, (b) C Error, (c) S Error Figure.
Figure 2, errors in estimates for low ATB (Table 4).



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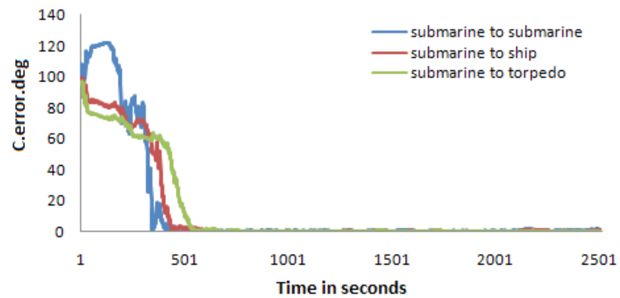

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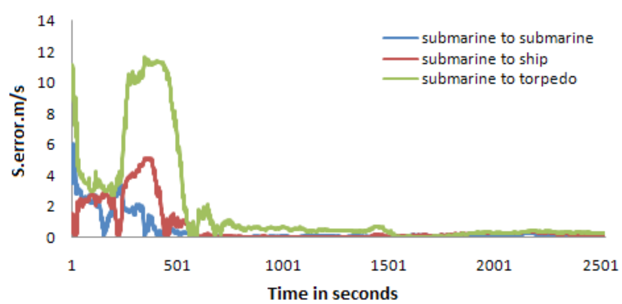

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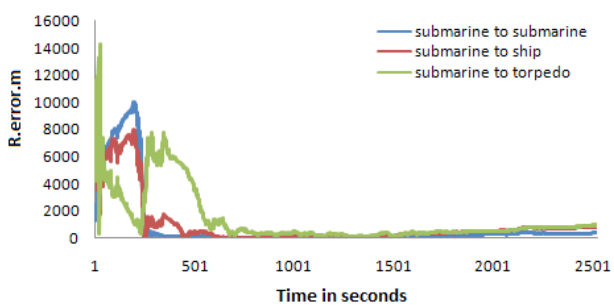
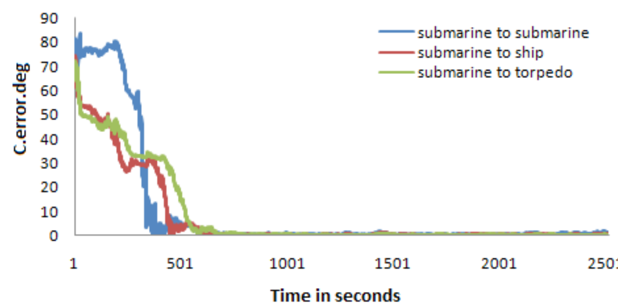
Figure 3. (a) R Error, (b) C Error, (c) S Error.
Figure 3, errors in estimates for medium ATB (Table 5).



4(a)



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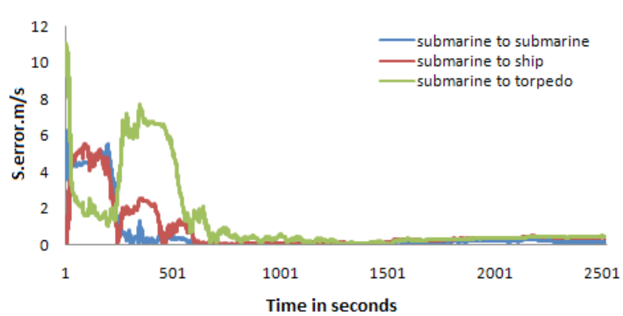


4(c)

Figure 4. (a) R Error, (b) R Error, (c) S Error.
Figure 4, errors in estimates for high ATB (Table 6).

Table 7. Convergence time in seconds for low ATB scenarios with 100 Monte-Carlo runs

Scenario	UKF			
	R	C	S	Total solution
1. (Submarine to submarine	410	379	282	410
2. (submarine to Ship)	466	379	484	484
3. (submarine to torpedo)	581	598	732	732

Table 8. Convergence time in seconds for medium ATB scenarios with 100 Monte-Carlo runs

Scenario	UKF			
	R	C	S	Total solution
1.(Submarine to submarine	343	406	356	406
2.(submarine to Ship)	396	451	448	451
3.(submarine to torpedo)	564	579	731	731

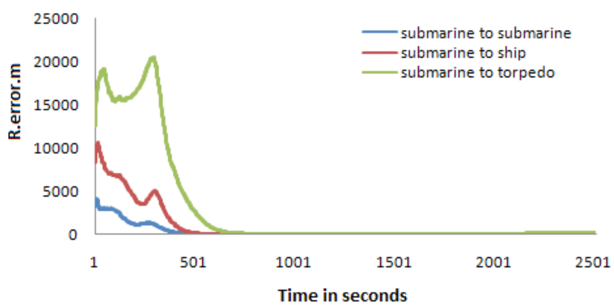
Table 9. Convergence time in seconds for high ATB scenarios with 100 Monte-Carlo runs

Scenario	UKF			
	R	C	S	Total solution
1.(Submarine to submarine	292	472	237	472
2.(submarine to Ship)	402	491	432	491
3.(submarine to torpedo)	587	628	743	743

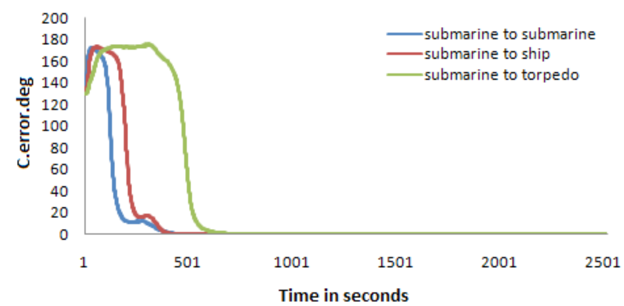
NC: Not Converged, R: Range, C: Course, S:Speed

5. Summary and Conclusion

In underwater, nine scenarios are considered covering low, medium and high ATB. The UKF algorithm is found out be efficient for bearings-only passive target tracking due to fine tuning of its parameters. Simulation results agree with the inference and hence, UKF is very effective for BOT.



5(a)



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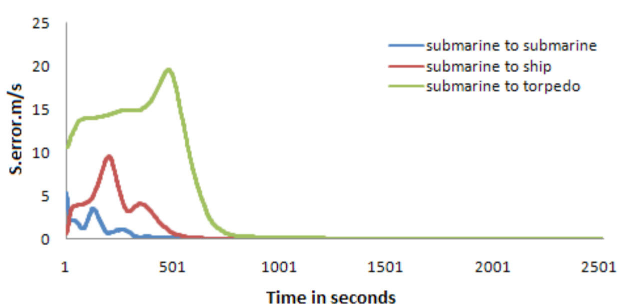
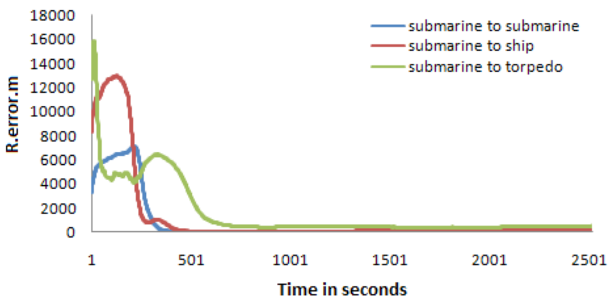
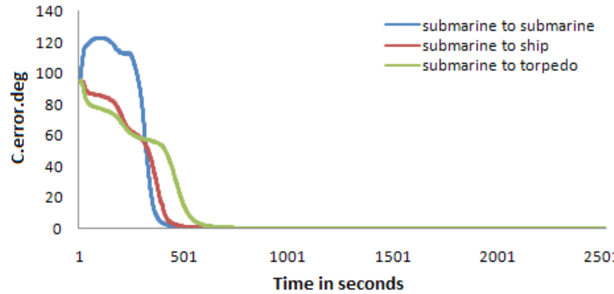


5(c)

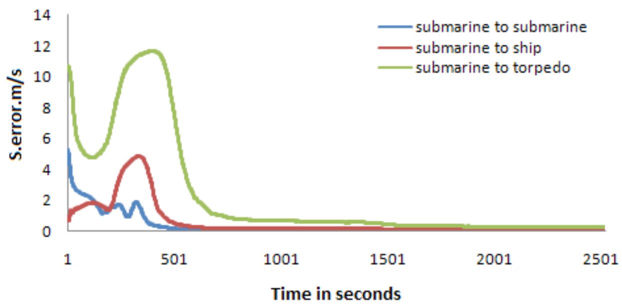
Figure 5. (a) R Error, 5(b) C Error, (c) S Error.
Figure 5, RMS errors in estimates for low ATB (Table 7).



6(a)

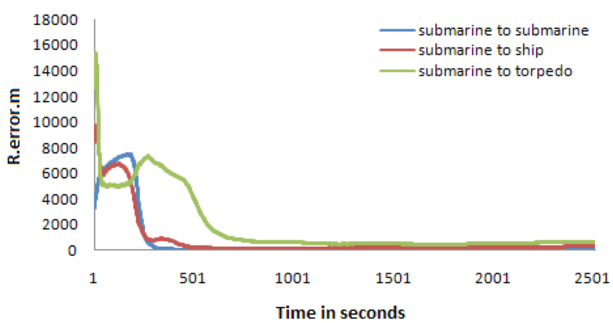


6(b)

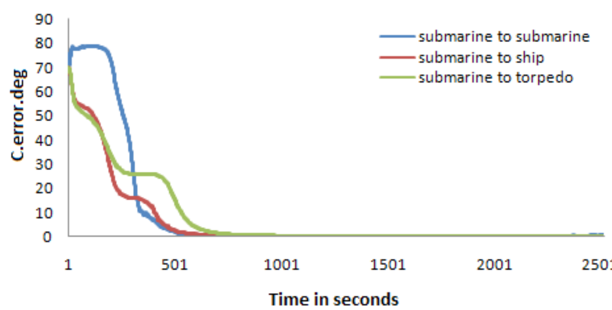


6(c)

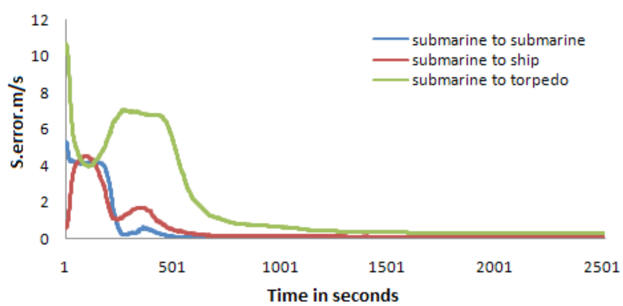
Figure 6. (a) R Error, (b) C Error, (c) S Error.
Figure 6, RMS errors in estimates for medium ATB (Table 8).



7(a)



7(b)



7(c)

Figure 7. (a) R Error, (b) C Error, 7(c) R Error.
Figure 7, RMS errors in estimates for high ATB (Table 9).

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Appendix

Notations

$\dot{x}$	x Component of target velocity, m/s
$\dot{y}$	y Component of target velocity, m/s
R_x	x Component of relative range, m
R_y	y Component of relative range, m
X_{op}	x Component of observer position, m
Y_{op} y	Component of observer position, m
b	Deterministic vector
ϕ	Transition matrix
ω	Plant noise
λ	Scaling parameter
K	Secondary scaling parameter
χ	Sigma point state vector
ζ	Filter weight
Γ	Plant noise gain matrix
B_m	Bearing measurement, deg
C	Target Course, deg
P	Estimated target state vector Covariance matrix

R	Target Range, m
S	Target speed, m/s
W	Inverse of the input measurement covariance matrix
X	Target state vector with sigma points
Y	Predicted measurement vector
P_{xy}	Cross covariance matrix
$W_0^{(m)}$	Weight of initial target state estimate
$W_0^{(c)}$	Weight of initial state covariance matrix
$W^{(m)}$	Weight of target state sigma point vector
$W^{(c)}$	Weight of covariance matrix of target state sigma point vector
P_{yy}	Innovation covariance matrix
y	Predicted bearing measurement
t	Sample time
Q	Covariance matrix of Gaussian white noise