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An Experimental Study of Porous Medium Flow in Converging Boundary

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Abstract

Objectives: Since 1856, Darcy's equation is used to analyse the ground water flow. This equation can be applied only in laminar regime of flow. No suitable equation is available to analyse other regimes of flow. During the pumping the water from well, the water around well flows in converging pattern towards well. The analysis of flow of water around wells is a problem in converging boundary through porous soils or sands. Generally, Dupuit's equation is used to analyse the flow around the well. One of the drawback in Dupuit's equation is it can be used only in laminar flow, since, this equation used Darcy's concept. No equation is available to analyse other than laminar flow in converging pattern towards well. Therefore, this situation motivated to think to design a sector of well model to conduct experiments on porous medium flow in converging boundary for all regimes of flow to estimate the discharge of a well. **Methodology:** In order to achieve the objective, a converging permeameter (a sector of a well model) is planned, designed and experimented. In this study, experiments have been conducted on porous medium in converging flow permeameter for all regimes of flow. From the experimentation of converging boundary, data like discharge, hydraulic gradient and velocity are calculated. Further, a new non-dimensional form of equation is also derived relating hydraulic gradient with Reynolds number. **Findings:** A set of graphs are drawn between hydraulic gradient and velocity, using Darcy's range of velocities and corresponding hydraulic gradient. These graphs are compared with Darcy's graph and the validity of Darcy's equation is verified. The trends of the present graph is coinciding with that of Darcy's graph such that the validation of this experimentation is checked. Another set of graphs are also drawn with hydraulic gradient versus velocity for all regimes of flow. A polynomial second degree equation ($i=a_1 V_b^2 + k'V_b$) has been proposed relating hydraulic gradient with bulk velocity in laminar and turbulent regimes. Hence this project result proves that, using this equation and corresponding constants discharge of a well can be estimated even if the flow is laminar and turbulent. **Applications:** The relation between hydraulic gradient and Reynolds number is obtained as $i=0.0004\ Re^{0.5}$. Substituting hydraulic gradient in this equation from field observations, Reynolds number can be find out. From the Reynolds number seepage velocity can be find out and corresponding discharge may be calculated in a well for any regime of flow which superfluous Dupuit's equation.

Keywords: Darcy's Equation, Hydraulic Gradient, Porous Medium, Reynolds Number, Velocity of Flow

1. Introduction

Water plays an energetic part in all categories of life on the world from simplest living organisms to the most complete human systems[1]. Water is a prime basic human need and a national asset. Rain occurs, very much or very less causing flood or drought. The main source of water is rainfall, out of which, a portion of it is penetrated beneath the ground surface and occupying the void space within the geologic stratum, which forms as ground water. The ground water in its natural state is invisibly moving ever from higher potential to lower potential

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level with in hydraulic principals. When surface water is exhausted, ground water is only the source of water. Groundwater hydrology is the science of the occurrence, distribution, and movement of water below the surface of the earth. The analysis of ground water is important but it is complicated[2]. Ground water flow requires careful investigations using characteristics of the porous medium and other hydraulic conditions. Several theoretical studies, modelling approaches, laboratory and field tests, and mathematical models have been developed to find the relationship between various porous medium flow properties[3].

In 1856, flow of water through porous structure was demonstrated first by French Civil Engineer, **Henry Darcy**. The pioneering Darcy's experiment on the flow of water through filter sands have appeared on the subject of porous media flow, thereby, subjecting it to continuous exploitation in various aspects by different investigators[4–7]. Darcy postulated a mathematical law that governs the flow of ground water, which later becomes most widely referred and used as Darcy's law. A linear relationship is obtained between bulk velocity 'V_b' and hydraulic gradient as well-known Darcy's equation, which is

$$V_b = ki \tag{1}$$

where 'V_b' is bulk velocity, 'k' is the co-efficient of permeability of the soil matrix to conduct a fluid flow through it or slope of the graph and 'i' is the hydraulic gradient.

The velocity 'V_b' and 'i' bear a definite linear relation depicted by a straight line with a positive slope. Therefore, each size of the particle has different 'k' values and with same size and different porosities. All these relations are formed without considering seepage velocity, porosity, and viscosity. In many of ground water flow situations flow is not only laminar ($V_b^{\,1}$) however other regimes of flow is also exist. In such situations, Darcy's linear relation cannot used. Therefore this subject has been continuously explored in various aspects by different investigators.

2. Well Hydraulics

During pumping, the water around the well flows in converging pattern towards well. The analysis of flow of water around wells is an important problem in converging boundary through porous soils or sands. Very few literature is available in converging flow and it also complex as various past investigators adopted various methods of expressing the results of their work[8]. Furthermore, most of approaches are found to lack in describing complete wide range of seepage flow[9]. Therefore, this situation is motivated to think to design a sector of well model to conduct experiments on porous medium flow in converging boundary for all regimes of flow to estimate the discharge of the well[10].

3. Radial Flow Around Well

Generally, Dupuit's equation is used to analyse the flow around the well[11]. Based on Darcy's Law, Dupuit (1863), derived an equation to estimate the rate of flow of water to a well for steady radial flow and given in equation 2. Dupuit's well discharge (Q) equation is

$$Q = \pi k \frac{h_2^2 - h_1^2}{ln(r_2 - r_1)} \tag{2}$$

and rearranging to solve for the hydraulic conductivity

$$k = \frac{Q}{\pi(h_2^2 - h_{11}^2)} ln \frac{r_2}{r_1} \tag{3}$$

where, 'k' is co-efficient of permeability in laminar region, h_1 and h_2 are the piezometric heads, r_1 and r_2 are the radius of influence.

In Dupuit's equation, the co-efficient of permeability (k) is used to find out the discharge of the well. This 'k' can be estimated from experimental program of a well. One of the drawback in Dupuit's equation is that equation can be used only in laminar flow, since, this equation used Darcy's concept. This equation does not reflect the actual drawdown curve because large vertical flow components contradict Dupuit's assumptions. Thiem (1906) modified Dupuit's equation to calculate hydraulic properties of an aquifer by means of a pump test.

4. Properties of Porous Medium

Porous medium is a solid matrix, which contains inter connected pore space, allows the fluid to flow through it (Figure 1). Usually aggregate, sand, soil are considered as porous medium. Present study, two sizes of aggregates (15 mm and 3.5 mm) are used as porous medium as initial investigation in converging flow. This type of porous medium used in water purification plant and filling material in and around bore holes. A porous medium is characterized by a variety of properties, like surface area, volume diameter, porosity, pore diameter, bulk area, flow area, surface area of fluid contact, hydraulic mean radius. Aggregates are sieved through a set of I.S.I. sieves and that passing through a larger sieve and retained by on the next lower size sieve. The volume mean diameter (d_p) of the porous medium, is the diameter of a sphere having a volume equal to that of an irregular shaped particle, is used as the volume diameter of the particle. The porosity n of a given Porous medium is the ratio of the volume of voids to the total volume of permeameter.

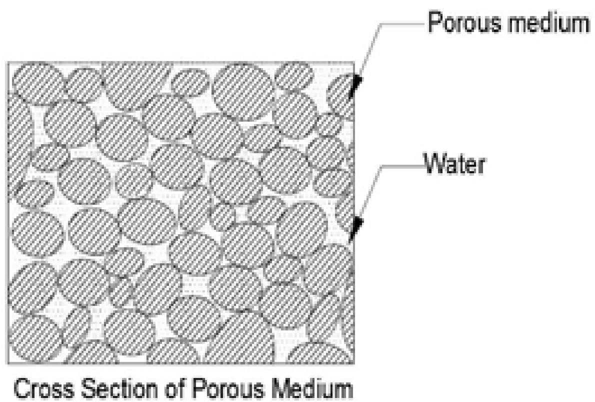

Figure 1. Cross section of porous medium.

5. Experimental Setup (Converging Permeameter a Well Model)

To overcome the aim and objectives of this study, a sector of (9°) well model (Figure 2) is designed and fabricated to conduct experiments on porous medium flow in converging boundary for all regimes of flow to estimate the discharge of a well. The length of converging permeameter is 15 m and larger width is 3.33 m and smaller width is 1 m with a test section of 13 m and the depth of permeameter is 1.3 m. This permeameter is fabricated using I-section, angle and M.S sheet. The experimental setup rests on fourteen R.C.C pillars. Piezometric tapings are provided at the bottom of the permeameter at one metre interval to measure the pressure heads. An experimental room is provided over the sump to accommodate pumping units and manometer board. The details of experimental setup are shown in Figures 3, 4, 5, and 6. Arial photo graphic view of converging permeameter and experimental setup room are shown in Figure 7.

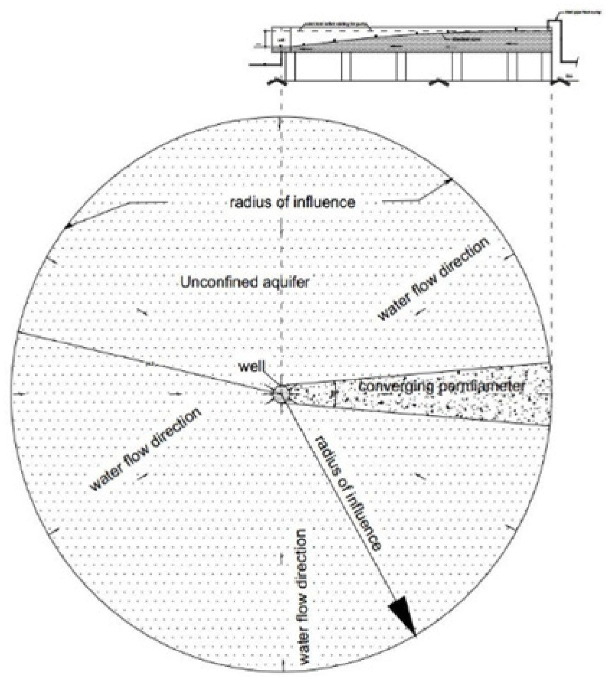

Figure 2. Layout of a well model.

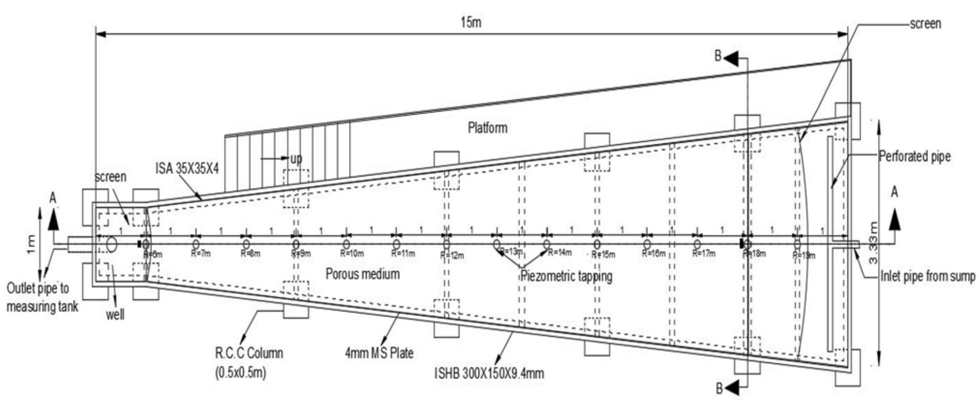

Figure 3. Plan of Converging Permeameter.

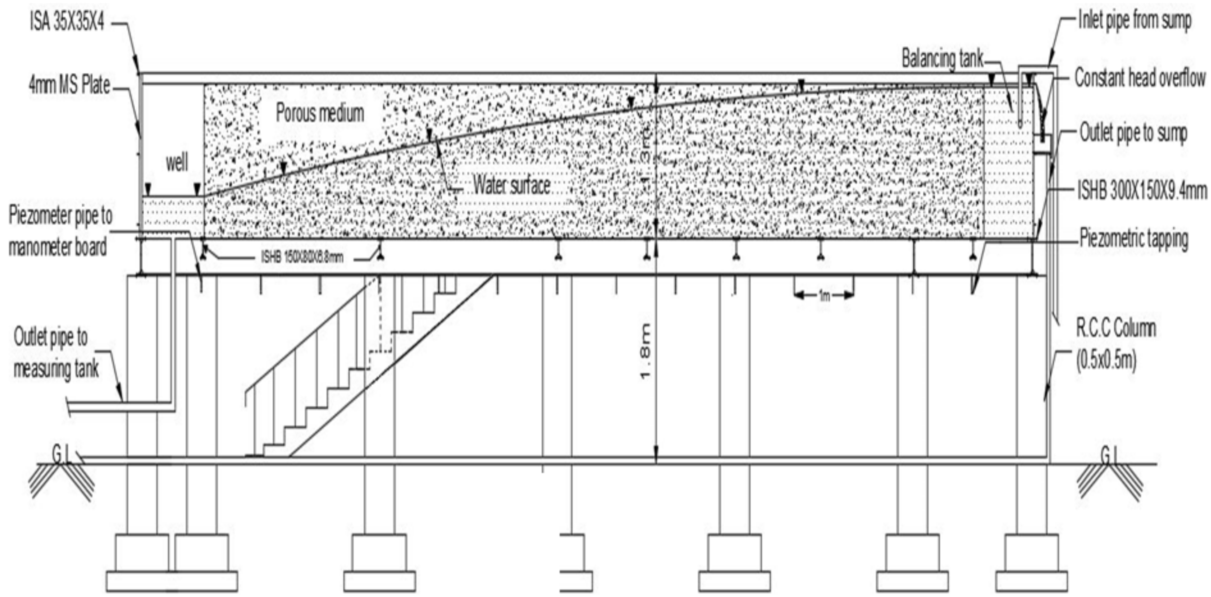

Figure 4. Cross section of converging permeameter.

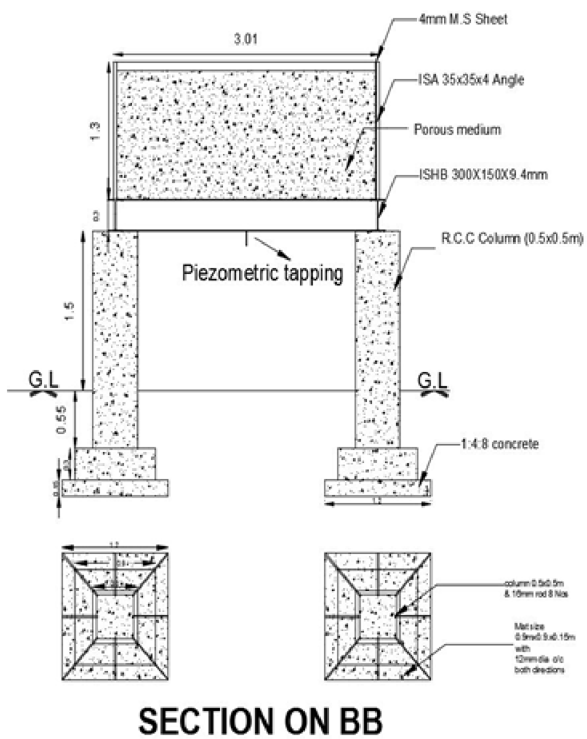

Figure 5. Foundation and RCC details of experimental setup.

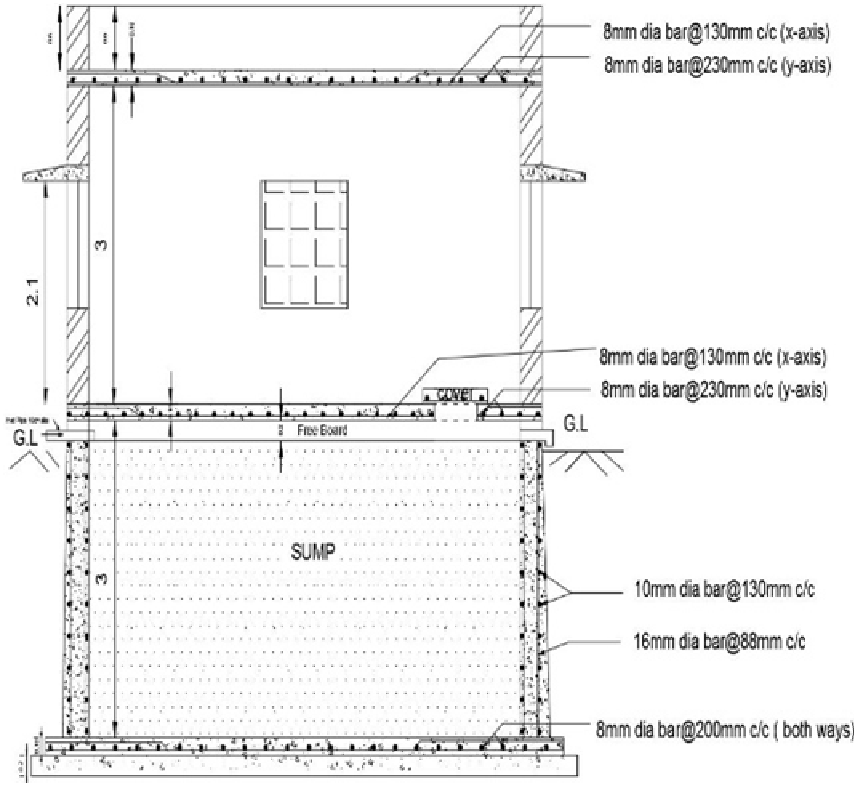

Figure 6. Cross section and RCC detail of sump and laboratory room.

Figure 7. Arial view of converging permeameter and experimental set-up room.

6. Experimentation and Observations

Porous medium is filled up to the top of the permeameter, and the corresponding porosity is measured by standard method. Water is pumped from sump to inlet of the permeameter and it is allowed to flow through porous medium. At the begging of experimentation processes, flow is allowed about two hours to attain steady state flow. The water is collected at the discharge measuring tank where the flow rate is estimated. From the measured discharge, bulk velocities are calculated at every piezometric section of the permeameter. At the same time pressure heads are also measured in the manometer board. Multiplying the observed pressure head with width of the permeameter at that section, the cross sectional area of flow is calculated. Bulk velocities are calculated, at every piezometric section using measured discharge and cross sectional area of that section. Temperatures are measured at the outlet of the permeameter, from which kinematic viscosity is calculated. Experiments are repeated in wide range of discharges; to cover all regimes of flow. From the experimental observations, hydraulic gradient, bulk velocity, radius of piezometric location, width of permeameter at that piezometric section, bottom level, area of cross section, kinematic viscosity, porosity, void ratio, seepage velocity, volume diameter, discharge are calculated. A non-dimensional number like Reynolds number is derived shown in equation 4.

$$Re = \frac{V_s A_b \sqrt{e}(1-n)}{\pi d_p \nu} \qquad (4)$$

where, 'Re' is equal to Non-dimensional parameter (Reynolds number), 'V_s' is equal to seepage velocity, 'A_b' is equal to bulk area, 'e' is equal to void ratio, porosity is 'n', diameter of particle 'd_p', kinematic viscosity is 'υ'.

7. Hydraulic Gradient

The term hydraulic gradient is one of the important parameters in porous medium flow. In parallel flow the hydraulic gradient is constant throughout the length of the permeameter. However in converging flow at every section the cross sectional area changes correspondingly the velocity and hydraulic gradient also changes. Different authors followed different methods to find the hydraulic gradient. In the present investigation, an attempt is made in a new direction to estimate hydraulic gradient, for all regimes of flow in converging boundary. In this study a graphs are drawn between pressure head in y axis and length of travel (radius) in x axis. The piezometric heads are joined and a parabolic curve is drawn and shown in Figure 8, and a quadric type of equation is fit as

$$y = ax^2 + bx + c \qquad (5)$$

where, y = piezometric head, x = distance and a, b, c = constants.

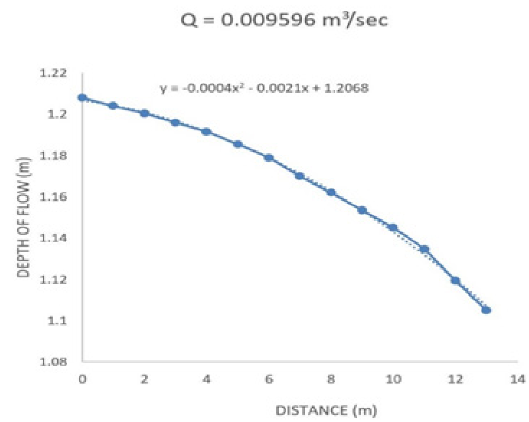

Figure 8. Variation between piezometric head with radius of the permeameter.

Differentiating the equation 5

$$\frac{dy}{dx} = 2ax + b \qquad (6)$$

This equation gives the slope of the curve at any distance x. The slope is nothing but the hydraulic gradient at that point. The hydraulic gradient is calculated for every discharge at every piezometric head and corresponding distance of travel.

8. Validation of Darcy's Equation

From observed data, laminar range of bulk velocities V_b^1 (from 0.0001 to 0.0022 m/sec) are selected from present experimental data and graphs are drawn between bulk velocities in y-axis and hydraulic gradients in x-axis and shown in Figure 9 for 3.5 mm diameter aggregate and in Figure 10 for 15 mm diameter aggregate. The trends of these graphs are similar to that of Darcy's graph, which conforms Darcy's law, and proves that present certain range (laminar) of data follows the Darcy's law. Linear relationship between V_b^1 and 'i' is found to be valid. The slope of the graph is the co-efficient of permeability (k). As the size of media increases there is an obvious increase in the fluid conveying capacity (hydraulic conductivity) of media. The relation 'V_b^{1}' and 'i' are given in equations 7 and 8.

$V_b = 0.242\, i$ (3.5 mm size aggregate) (7)

$V_b = 0.951 i$ (15 mm size aggregate) (8)

The above two individual graphs for 3.5 mm and 15 mm are drawn on a single graph and shown in Figure 11. It is observed from this graph that each diameter of particle forms separate curve. The larger diameter particle is having larger value of co-efficient of permeability and smaller is having lesser value of permeability.

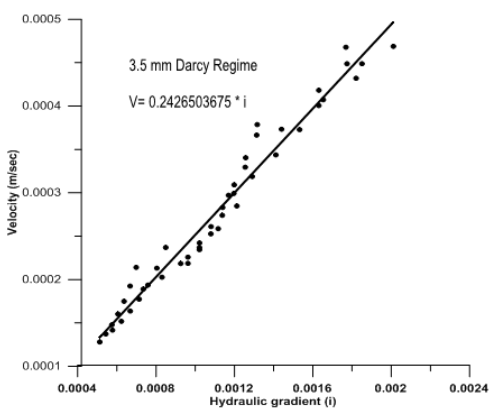

Figure 9. Variation between bulk velocity and hydraulic gradient for 3.5 mm aggregate (Darcy's Range).

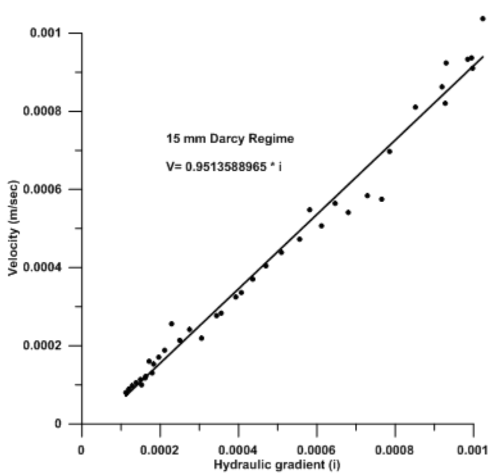

Figure 10. Variation between bulk velocity and hydraulic gradient for 15 mm aggregate (Darcy's Range).

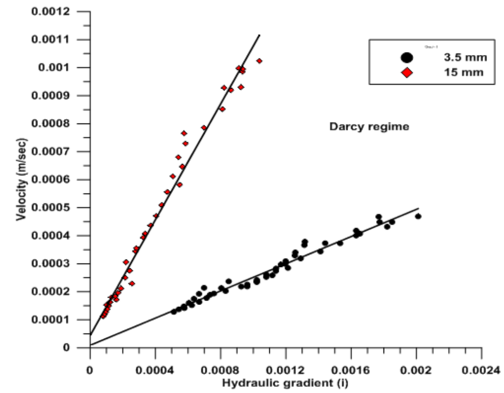

Figure 11. Variation of bulk velocity and hydraulic gradient for 15 mm and 3.5 mm aggregate for Darcy's Regime.

9. Relation between Velocity and Hydraulic Gradient for All Regims of Flow

Using the complete data for all regimes observed from experimentation, graphs are drawn hydraulic gradient 'i' in y-axis and velocity 'V_b^1' in x-axis in log -log graph, and shown in Figures 12, 13, and 14. These graphs are not a straight line like Darcy's graph, however these graphs are parabolic. It is found that each size of particle aligns along a single curve with a relatively flatter slope at lower velocities, with a gradual increase in steepness with the increase in velocity. Some portion of the graph is linear and some other portions are non-linear, which includes Darcy regime, and Forchheimer regime[12–14]. Mathematical analysis are made and, quadric equations are fit between bulk velocity and hydraulic gradient. Each curve fits a[s] polynomial second degree equation as

$$i = a_1 V_b^2 + k' V_b \quad (9)$$

where, i = hydraulic gradient, V_b = Bulk velocity of flow, a_1 = constant, and k'= co-efficient of permeability in all regimes

This equation absorbs Darcy term ($k'\ V_b^1$), Forchheimer term ($a_1\ V_b^2$),. Substituting these co-efficients which is obtained from experimentation, in the above equation, for a known hydraulic gradient 'i', bulk velocity (V_b^1) can be determined. Multiplying the bulk velocity with the corresponding cross sectional area of flow (A_b) the discharge through a well can be estimated irrespective of its regimes of flow through converging boundary. The drawback in this equation is each diameter of particle is having individual Darcy co-efficient and

Forchheimer co-efficient. Another drawback is the effect of porosity, viscosity, diameter of the particles are not included and each diameter of particle align a separate curve. To overcome the above problem a non-dimensional mathematical relation between hydraulic gradient and Reynolds number (*Re*) is derived (equation 4).

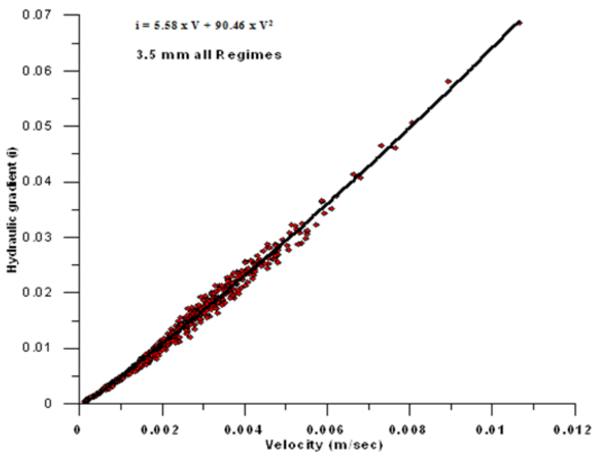

Figure 12. Variation of bulk velocity and hydraulic gradient for 3.5 mm aggregate for all regimes.

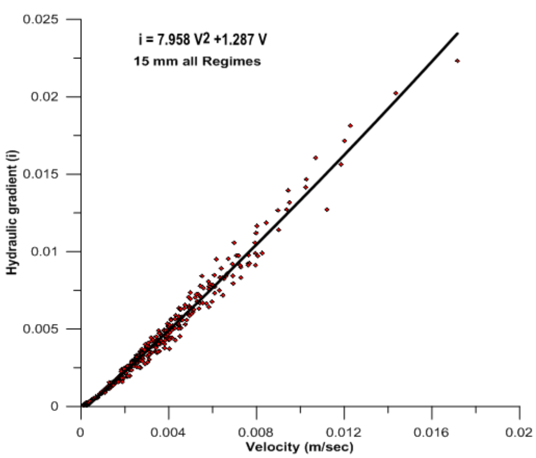

Figure 13. Variation of bulk velocity and hydraulic gradient for 15 mm aggregate for all regimes.

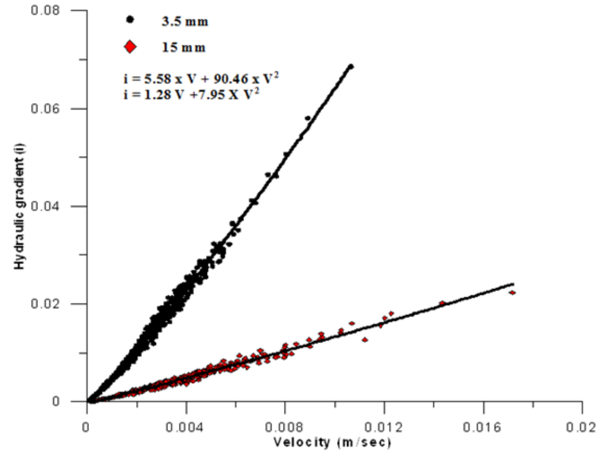

Figure 14. Variation of bulk velocity and hydraulic gradient for 15 mm and 3.5 mm aggregate for all regimes of flow.

10. New Findings/Achievements

The present observed experimental data is substituted in '*Re*' equation (equation 4), and graphs are drawn between hydraulic gradient in y-axis and derived non-dimensional number '*Re*' in x-axis, for both 3.5 mm and 15 mm and shown in Figure 15. A stunning results obtained. Two sizes of aggregate of two curves aligned as unique single curve irrespective of diameter, porosity, viscosity etc. An exponential non-dimensional equation fitted to relate hydraulic gradient and Reynolds numbers for aggregate. The equation obtained is

$$i = 0.0004\ R_e^{0.5} \qquad (10)$$

where, i = hydraulic gradient, *Re* = Reynolds number

Substituting measured field data of hydraulic gradient, in the '*Re*' equation and the value of '*Re*' can be calculated. From the calculated '*Re*' value and substituting the other field data of bulk area, porosity, void ratio, volume diameter of particle, viscosity, and the unknown seepage velocity (V_s) can be estimated. Multiplying this seepage velocity with flow area of the field the discharge from any porous medium can be estimated irrespective of its regimes of flow for any fluids and any particle size. This type of theoretically derived unique non-dimensional relation is not available now to solve problems in porous medium flow.

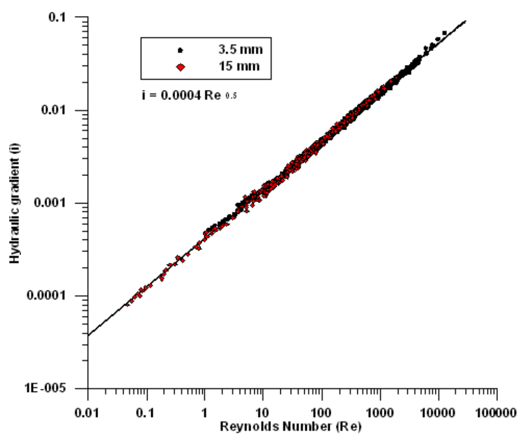

Figure 15. Variation of hydraulic gradient with Reynolds number for 3.5 mm and 15 mm aggregate.

11. Results and Discussions

From this experimental investigations, the following results and conclusions are made. A Converging Permiameter (Well Model) is designed and fabricated. Two sizes of aggregate are used as a porous medium and Water as a fluid medium. Porous medium properties like, volume diameter of particles, Porosity and Void ratio, and hydraulic mean radius are calculated. In this study hydraulic gradients are calculated by a new method. From the observed pressure head and distance of travel, graphs are drawn between pressure head in y axis and length of travel (radius) in x axis. The piezometric heads are joined and a parabolic curve is drawn and a quadric type of equation is obtained using the equation 5. The slope of the curve at radius is obtained using the equation 6 and the slope of the curve gives the hydraulic gradient at that point. The hydraulic gradient is calculated for every discharge at every piezometric head and corresponding distance of travel.

In the present investigation, an attempt is made to study the relationship of hydraulic gradient, '*i*' with velocity of flow 'V_b^1' for a converging boundary for all regimes of flow including Darcy regime. Experiments are conducted in wide range of discharges; as low hydraulic gradient from 0.00001 to high hydraulic gradient 0.1 which covers all regimes of flow. From the observed experimental data, laminar range of bulk velocities V_b^1 from 0.0001 to 0.0022 m/sec are selected and graphs are drawn between bulk velocities and hydraulic gradients. The trends of these graphs are also similar to that of Darcy's graph and it is a straight line, which conforms the Darcy's law concept, and proves that present certain range of data follows the Darcy's law by which it ascertains the validity of this experimentation and analysis of experimental data. From velocity with hydraulic gradient (Darcy's range) equation is fitted as $v= ki$ for converging flow. The equation for 3.5 mm and 15.0 mm size of aggregate is represented by the equations 7 and 8 respectively. Thus, the Co-efficient of Permeability (k) in converging flow is found to be 0.242 and 0.951 m/sec for 3.5 mm and 15 mm aggregate size respectively.

Another set of graphs are drawn between velocity and hydraulic gradient for the complete wide range of experiments conducted for all regimes. It is found that each size of particle aligns along a separate curve with a relatively flatter slope at lower velocities, with a gradual increase steepness curve with the increase in velocity. Some portions of the graph is linear (straight line) and some other portions are non-linear, which includes Darcy's regime, and turbulent regime. A mathematical analysis is made and two term equations are fitted between the bulk velocity and hydraulic gradient as

$$i = 5.58\ V_b + 90.46\ V_b^2 \qquad \text{(3.5 mm size aggregate)} \tag{11}$$

$$i = 1.28\ V_b + 7.95\ V_b^2 \qquad \text{(15 mm size aggregate)} \tag{12}$$

The equation for 3.5 mm and 15.0 mm size of aggregate for all regimes of flow is represented by the equations 11 and 12 respectively. Thus, the Co-efficient of Permeability (k') in converging flow is found to be 5.58 and ç.28 m/sec for 3.5 mm and 15 mm aggregate size respectively.

Each curve fits a polynomial equation covering V_b^2 and V_b^1 terms, which represents the turbulent, and laminar (Darcy's) regimes respectively. This equation absorbs Darcy's term (V_b^1) and turbulent term (V_b^2). It is seen that hydraulic gradient '*i*' increases as the velocity of flow increases. It may also be noted from all the figures, that there is a systematic and regular orientation of verses 'V_b' of different sizes. Therefore, smaller is the size of the medium; larger is the extent of contact of the fluid with the solid matrix, invariably increasing the resistance to flow. It is observed that from present velocity versus hydraulic gradient graphs, each diameter of porous medium forms an individual curve irrespective of its porosity and viscosity since it is related with dimensional parameters.

12. Conclusions

In this study, experiments have been conducted on porous medium in converging flow permeameter for all regimes of flow. From the experimentation of converging boundary, data like discharge, hydraulic gradient and velocity are calculated. A set of graphs are drawn between hydraulic gradient and velocity, using Darcy's range of velocities and corresponding hydraulic gradient. These graphs are compared with Darcy's graph. The trends of the present graph is coinciding with that of Darcy's graph such that the validation of this data is checked in laminar regime. Another set of graphs are drawn with hydraulic gradient versus velocity for all regimes of flow. A polynomial second degree equation has been proposed relating hydraulic gradient with bulk velocity in laminar and turbulent regimes. The co efficient of permeability (k') and turbulent coefficient (a_1) has been estimated for all regimes of flow. In the fitted equation, V_b^2 term represents turbulent regime and V_b^1 term represents laminar regime (Darcy's regime). Hence, this project result proves that, in porous medium flow not only Darcy's equation is applied, however to represent all regimes of flow (turbulent term V_b^2) polynomial type of equation is to be used. Therefore, applicability and validity of Darcy's equation is checked. Another a new non-dimensional form of equation is also derived relating hydraulic gradient with Reynolds number and the experimental data applied in this equation to get unique relation between them. The relation between hydraulic gradient and Reynolds number is $i = 0.0004\ Re^{0.5}$. Substituting hydraulic gradient in this equation from field observations Reynolds number can be find out. From the Reynolds number velocity can be find out and corresponding discharge may be calculated in a well for any regime of flow.

13. Acknowledgement

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