

Multi-Sensor Submarine Surveillance System using MGBEKF

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Abstract

Background/Objectives: The modern submarines use multiple sensors for tracking multiple targets in sea environment. In general, multiple sensor data can be handled in two ways: measurement fusion and state vector fusion. **Methods/Statistical Analysis:** Measurement fusion is not practical for implementation due to various reasons and hence state vector fusion is proposed. Target tracking using Modified Gain Bearings only Extended Kalman Filter in generic two-dimensional platform is carried out in each channel. **Findings:** In this approach, a state vector and its corresponding covariance matrix are extracted from the sensor measurements by an estimator equipped on each sensor. The output of each channel is transported via a data link in to order to be reach the fusion center. **Applications/Improvements:** A composite target state vector is obtained by performing track-to-track correlation and fusion at the fusion center. Monte-Carlo simulation is carried out and the results are presented for a typical scenario.

Keywords: Estimation, Fusion, Kalman Filter, Simulation, Tracking

1. Introduction

Submarines use different types of sensors like (hull mounted array, towed array etc.) to track the target. The modern submarines remain in passive mode of listening for most of the time. In this work it is assumed that observer submarine is using hull mounted array and towed array in passive mode to track target ships/submarines. Therefore, each sensor has its own set of targets in the track. The complexity lies in how to know whether two different tracks originating from two different systems represent one target.

If yes, another problem is track fusion (also called data fusion)¹⁻³.

There are two different kinds of data fusion, namely measurement fusion and state vector fusion. In

measurement fusion, optimal target state estimate is obtained by combining the measurement from the sensors. However, it is not practically viable for many reasons due to the enormous sensor data transfer from various stations to the data fusion center^{4,5}. For field implementation, state vector fusion is viable for data fusion. In state vector fusion, the state vector along with its covariance matrix is extracted from sensor measurements by the estimator employed by the sensor. Each channel output is transported to a fusion center via a data link where obtain a composite target state vector is obtained using track-to-track correlation and state vector fusion. The fusion process is described in Figure 1².

Let us consider a situation of tracking a target moving at constant velocity in an ocean environment. (For analysis, constant velocity target model is assumed. The

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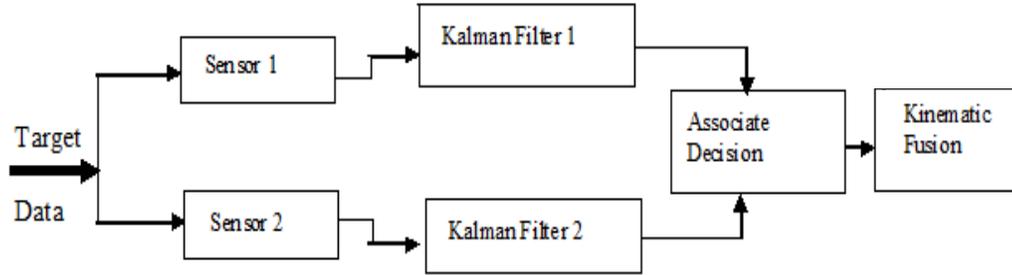


Figure 1. Block Diagram of state vector fusion.

same algorithm can be extended easily for maneuvering targets) and it is being tracked by a Hull Mounted Array (HMA) and a Towed Array (TA) operating from an observer ship. Two Kalman filters, as shown in Figure 1 process the HMA and TA data in two different channels and two estimated target state vectors are available. First it is decided whether these estimates are related to single target and if it is so, then all these estimates are combined to get a better or more smoothed estimate. Data fusion is abundantly available in literature² and author tries to extend the theory for sea environment.

2. Mathematical Modeling

It is proposed to do data fusion of hull-mounted array and towed array state vector outputs. These sensors generate bearings of the target. Modified Gain Bearings only Extended Kalman Filter (MGBEKF) is used to estimate the target motion parameters of the target using the above-mentioned measurements. Whenever the input data is available, the filter estimate is updated. The mathematical modeling of input measurements, Kalman filter and the outputs is as follows^{5,6}. The alternative derivation of the modified gain function^{3,4} of Song and Speyer's extended Kalman filter is slightly modified as follows. Let the target state vector be $X_s(k)$ where

$$X_s(k) = [\dot{x}(k) \quad \dot{y}(k) \quad R_x(k) \quad R_y(k)]^T \quad (1)$$

Where $\dot{x}(k)$ and $\dot{y}(k)$ are target velocity components and $R_x(k)$ and $R_y(k)$ are range components respectively. The target state dynamic equation is given by

$$X_s(k+1) = \phi X_s(k) + b(k+1) + \Gamma \omega(k) \quad (2)$$

where ϕ and b are transition matrix and deterministic vector respectively.

$$\phi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ t & 0 & 1 & 0 \\ 0 & t & 0 & 1 \end{bmatrix}$$

where t is sample time

$$b(k+1) = [0 \ 0 -\{x_0(k+1) - x_0(k)\} -\{y_0(k+1) - y_0(k)\}]$$

and

$$\Gamma = \begin{bmatrix} t & 0 \\ 0 & t \\ \frac{t^2}{2} & 0 \\ 0 & \frac{t^2}{2} \end{bmatrix} \quad (3)$$

where x_0 and y_0 are ownship position components. The plant noise $\omega(k)$ is assumed to be zero mean white Gaussian.

The bearing measurement, B_m is modeled as

$$B_m(k+1) = \tan^{-1} \left(\frac{R_x(k+1)}{R_y(k+1)} \right) + \zeta(k) \quad (4)$$

where $\zeta(k)$ is measurement error.

The measurement matrix is obtained as

$$H(k+1) = \begin{bmatrix} 0 & 0 & \check{R}_y(k+1|k) / \check{R}^2(k+1|k) \\ -\check{R}_x(k+1|k) / \check{R}^2(k+1|k) \end{bmatrix} \quad (5)$$

The covariance prediction is

$$P(k+1|k) = \phi(k+1|k)P(k|k)\phi^T(k+1|k) + \Gamma Q(k+1)\Gamma^T \quad (6)$$

The Kalman gain is

$$G(k+1) = P(k+1|k) H^T(k+1) [\sigma^2 + H(k+1)P(k+1|k)H^T(k+1)]^{-1} \tag{7}$$

The state and its covariance corrections are given by

$$X(k+1|k+1) = X(k+1|k) + G(k+1) [B_m(k+1) - h(k+1, X(k+1|k))] \tag{8}$$

Where $h(k+1, X(k+1|k))$ is the bearing using predicted estimate at time index $k+1$

$$P(k+1|k+1) = [I - G(k+1)g(B_m(k+1), X(k+1|k))] P(k+1|k) [I - G(k+1)g(B_m(k+1), X(k+1|k))]^T + \sigma^2 G(k+1)G^T(k+1) \tag{9}$$

where $g(\cdot)$ is modified gain function as defined in [4]. The value of g is

$$g = \begin{bmatrix} 0 & 0 & \cos B_m / (\check{R}_x \sin B_m + \check{R}_y \cos B_m) \\ -\sin B_m / (\check{R}_x \sin B_m + \check{R}_y \cos B_m) \end{bmatrix} \tag{10}$$

2.2 Fusion of State Vectors

Two separate identical MGBEKF's (Modified Gain Bearings Only Extended Kalman Filter) are used to process the HMA and TA data and at this stage it is assumed that the two estimates of target are available. Now it is to be decided, whether the HMA and TA are tracking different targets or same target. This process is called data fusion and is described as follows. Let $\hat{X}^i(k)$ be the state of a target estimated by a HMA and let $P^i(k)$ be covariance of $\hat{X}^i(k)$. Let $\hat{X}^j(k)$ be the state of a target estimated by TA and let $P^j(k)$ be the covariance of $\hat{X}^j(k)$. Both are current estimates. It is useful to test the hypothesis that these estimates are for the same target or not. The difference in these estimates is given by

$$\check{\Delta}^{ij}(k) = \check{X}^i(k) - \check{X}^j(k) \tag{11}$$

Similarly the difference in the true states is given by

$$\Delta^{ij}(k) = X^i(k) - X^j(k) \tag{12}$$

Let

$$\begin{aligned} \tilde{\Delta}^{ij}(k) &= \Delta^{ij}(k) - \check{\Delta}^{ij}(k) \\ &= (X^i(k) - \check{X}^i(k)) - [X^j(k) - \check{X}^j(k)] \\ &= [\check{X}^i(k) - \check{X}^j(k)] \end{aligned} \tag{13}$$

The following hypothesis is used.

If $H_0 : \Delta^{ij}(k) = 0$, then there is only one target

And if $H_1 : \Delta^{ij}(k) \neq 0$ then there are two different targets

Assuming that $\tilde{X}^i(k)$ is independent of $\tilde{X}^j(k)$, the covariance of $\tilde{\Delta}^{ij}(k)$ is

$$\begin{aligned} T^{ij}(k) &= E \left\{ [\tilde{X}^i(k) - \tilde{X}^j(k)] [\tilde{X}^i(k) - \tilde{X}^j(k)]^T \right\} \\ &= E [\tilde{X}^i(k)\tilde{X}^i(k)^T + \tilde{X}^j(k)\tilde{X}^j(k)^T] \\ &= P^i(k) + P^j(k). \end{aligned} \tag{14}$$

Assuming that the estimation errors are Gaussian, the test of H_0 vs H_1 is as follows:

H_0 is accepted if $d = \hat{\Delta}^{ij}(k)^T [T^{ij}(k)]^{-1} \hat{\Delta}^{ij}(k) \leq \delta$, else H_1 is accepted. The threshold is such that $P\{d > \delta | H_0\} = \alpha$, where α is say 0.05. The choice of this threshold is based on the fact that the above Gaussian assumption 'd' has a chi-square distribution with n_x degrees of freedom. If H_0 is accepted, then the two estimates $\hat{X}^i(k)$ and $\hat{X}^j(k)$ can be combined as follows.

$$\hat{X}^{ij} = \hat{X}^i + P^i(P^i + P^j)^{-1}(\hat{X}^j - \hat{X}^i) \tag{15}$$

The covariance associated with the fused estimate is given by

$$M^{ij} = P^i(P^i + P^j)^{-1}P^j \tag{16}$$

3. Simulation and Results

To find out the initial target state estimate, the target velocity is assumed to be 10m/sec as only bearing only measurements are available. Let range of the day be 15000 metres,

$$X(0|0) = [10 \ 10 \ 15000 * \sin B_m \ 15000 * \cos B_m]^T \tag{17}$$

It is assumed that the initial estimate, $X(0|0)$ is uniformly distributed. Then the elements of initial covariance diagonal matrix can be written as

$$\begin{aligned} P_{00}(0|0) &= \frac{4 * \dot{x}^2(0|0)}{12} \\ P_{11}(0|0) &= \frac{4 * \dot{y}^2(0|0)}{12} \\ P_{22}(0|0) &= \frac{4 * R_x^2(0|0)}{12} \end{aligned} \tag{18}$$

$$P_{33}(0|0) = \frac{4 * R_y^2(0|0)}{12} \tag{19}$$

The observer is performing ‘S’ maneuver . The speed of the observer is 7 knots with a turning rate of 1 deg./sec. The observer course is 90 degrees for two minutes and changed to 270 degrees. The course of the observer changes from 270 to 90 degrees at 9th minute, 90 to 270 degrees at 16th minute and 270 to 90 degrees at 23rd minute respectively. Let the range and bearing initially be 20000 metres and zero degrees with respect to the observer respectively. Also let the target velocity be 25 knots.

The targets close to the observer are the area of interest and therefore, the angle between target course and LOS is always less than 55 degrees . Different scenarios are obtained with varying target course by 1 degree everytime. The bearing measurements are obtained every second from HMA and TA . These measurements are corrupted by noise level having a r.m.s level of 1 degree. The Monte-Carlo simulation is performed for various scenarios.

The results inferred that after first maneuver , the target parameters are observable. The limit on range error estimate is 8%, bearing error estimate is 0.2 degrees and velocity error estimate is 3m/sec. After the second observer maneuver, 80% of the solution is realized . After third, 95 % of the solution is realized atmost. Figure 2 shows the simulation results with single sensor and target course of 140 degrees. The computational time required is 30 minutes. Monte Carlo simulation is performed with 100 runs and the results are shown in figure 2 respectively. Here, R-error denotes error in range estimates, C-error denotes error in course estimate, B-error denotes error in bearing estimate and S-error denotes error in speed

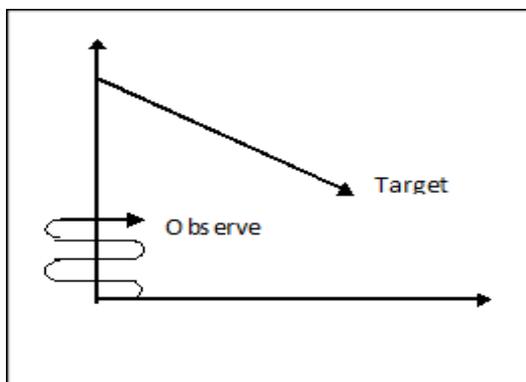


Figure 2. TMA with single observation platform in S - maneuver.

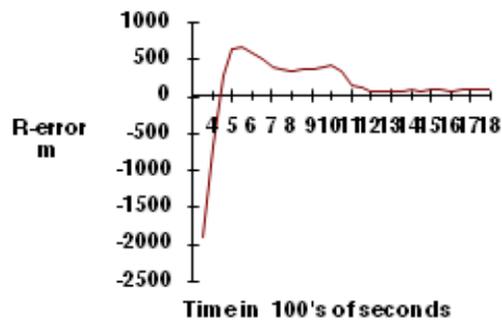


Figure 2(a). R-Error.

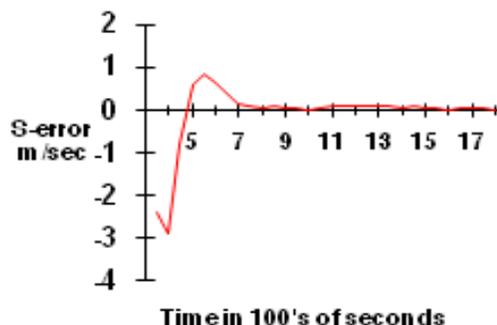


Figure 2(b). S-Error.

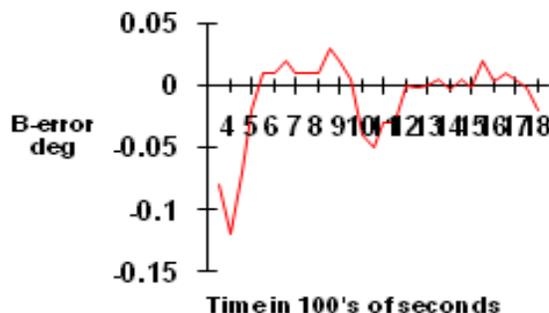


Figure 2(c). B-Error.

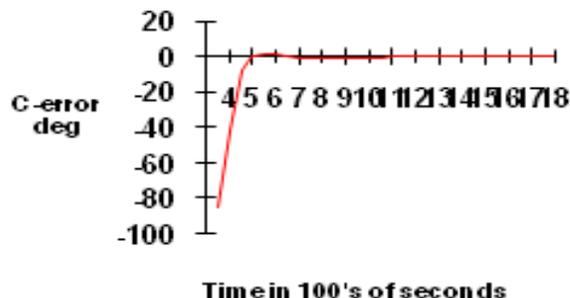


Figure 2(d). C-Error.

estimate. Let us consider, two sensors are considered instead of one in the previous case. This implies that two Kalman filters are working in parallel. There may be different noise magnitudes with which the bearing measurements are corrupted while the Gaussian distribution and variance remains same. The bearing measurements are corrupted with different magnitudes of noise, but maintaining the same Gaussian distribution and variance.

The parameter 'd' follows chi-square distribution and its theoretical value with four degrees of freedom is 0.96, for $\alpha = 0.05$. During Monte-Carlo simulation, d does not exceed the threshold value 0.96. Hence it is decided that the two sensors are tracking the same target. So, the target state and its covariance are combined as per Equation (15) and (16). Now let us assume that HMA is tracking a target described in the previous scenario and TA is tracking another target with the same characteristics of previous target but at initial range of 19500 meters. In this case d is 1.34, more than the threshold 0.96. Hence it is declared that the two sensors are tracking two targets.

4. Conclusion

The MGBEKF is used for undersea bearings only tracking with data fusion, in particular state vector fusion, to

find out whether there are two targets or one target. If it is declared that only one target is present, then a better estimate of that target is presented.

5. References

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