

Skeletonisation Approaches for Determining Paths in an Image: A Review

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Abstract

Skeletonisation is an effective way to provide an intact representation of an object's shape. This approach has great applications in the fields of motion planning, object representation, tracking, information retrieval, topology representation, sensor distribution and location, facility location, forestry application, building constructions etc. Skeletons naturally provide a pixel to boundary mapping through which the path between points can be found in a terrain map. Many approaches for determining the skeleton of a terrain exist. In this paper, the different methods for Skeletonisation are discussed. Initially the various categorizations of Skeletonisation is discussed which is further followed by application of those algorithms for path determining. Quantitative measures for validating the skeleton are also discussed. Finally, the applications of Skeletonisation in various domains are elaborated.

Keywords: Distance Map, Distance Transform, Skeletonisation, Thinning, Voronoi Diagram

1. Introduction

Numerous research is on the rise in the field of image processing. One such area is Skeletonisation. In the recent years, more focus is been put on this field due to its versatility in applications. The concept of skeleton begun initially from the Blum's Prairie Fire Analogy¹. According to him, when a field containing grass is set fire, the fire propagates and meets exactly at the medial axis which is the skeleton of the area. It is inferred that skeleton is the set of quench points where the fire fronts meet. This classic definition led to many other definitions. Skeletonisation can be defined as the process of reducing the foreground regions to set of quench points called skeleton whereby most of the foreground pixels are thrown away but their connection to original boundary is preserved. Medial axis is the set points so that at least one is near to boundary. A formal definition for skeleton can

be stated as loci of all centers of maximal hyper sphere inscribed. Skeleton can also be viewed as a result of Medial Axis Transform (MAT). A point is set to belong to a skeleton only if it has two closest boundaries. Skeletonisation has a greater range of applications and benefits². Because of its ability to make out a compact representation of an object, it has greater affinity towards the fields of shape matching, shape recognition, object tracking, handwriting recognition. Skeletonisation will be able to represent a topological map in its most abstract and precise form. Thus, it has greater applications in the fields of path finding, path tracking, motion planning, sensor distribution, facility location. Skeletonisation has greater impact in medical field because of its shape description capability. The exact structure of blood vessels can be got after which the flow can be studied further. The skeleton structure is greatly bound by the presence of noise in the image. Common noises are the surface noise³ and contour noise³.

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Henceforth the raw skeleton got from the process must be processed to remove noise. Morphological operations like dilation and erosion are applied to remove noises. In the field of path planning, the process of Skeletonisation has the following stages⁴. Initially the binary mask is applied on the source image to extract skeleton. Binary mask is dependent on the structural element used in the operation. The Figure 1 shows the procedural structure of Skeletonisation for determining path.

The skeleton can be classified into two types as Surface skeleton and Curve skeleton. Surface skeleton⁵ are two dimensional skeletons got from 3D images. This show the structure of skeleton with respect to surface. Basically it is a union of surfaces and curves. They are generic and the image can be reconstructed from them again. They are widely used in volumetric animation, surface smoothing, and registration. Curve skeleton⁵, on the other hand are one dimensional skeleton for 2D images. They project the skeleton in terms of curves in its simple form. The curve corresponds to the centre of the object boundary. They can also be got as a result of dimension reduction from surface skeleton. They look like a stick figure where the details would be abstract. They are used in animations, morphing, and geometric processing. There also exists another method for classifying the skeleton according to the reference used. Automatic skeletons are those skeletons that do not have any input parameters. Semi automatic skeletons are those that require set of human validated parameters. There are two most important properties^{6,7} for skeletons:

- Skeletons must preserve the topological structure.
- Skeletons must preserve the geometrical nature of the image.

Additional properties include hierarchical structure, robustness, uniformity, inclusion, clarity^{8,9}. There are many different approaches for Skeletonisation.

The rest of the paper discusses various approaches for Skeletonisation, validation of the skeleton and the applications of Skeletonisation.

2. Different Approaches for Skeletonisation

The approaches for Skeletonisation can be classified into three types.

- Approaches based on Geometric Continuities.
- Approaches based on object reduction.
- Approaches based on Curve propagation.

Also, there is another method for classification where the methods are classified as Parallel⁷ and Sequential methods⁷. In parallel methods, both the process of distance computation and pixel reduction takes place simultaneously. In sequential methods, first the distance map of each pixel is formed. Then, the pixels are reduced accordingly. There is also a narrow categorization of approaches for Skeletonisation where the methods are categorized according to the constraints they face⁸. The figure below shows one such categorization.

The sections below elaborate the wider categorization of different approaches for Skeletonisation.

2.1 Approaches based on Geometric Continuities

Skeletonisation is achieved through geometric continuities by means of Voronoi diagrams¹⁰ and Graph based approach^{8,9}. According to Voronoi diagram, the image plane is portioned into zones, and each zone has one seed point or generating point. The other points are put into zones based on their distance to the seed point. Voronoi diagram follows incremental construction. Generally Voronoi diagram is the intersection of Voronoi diagram

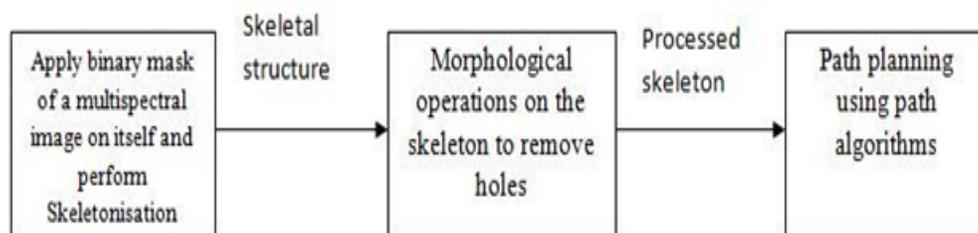


Figure 1. Procedural structure of Skeletonisation for determining path.

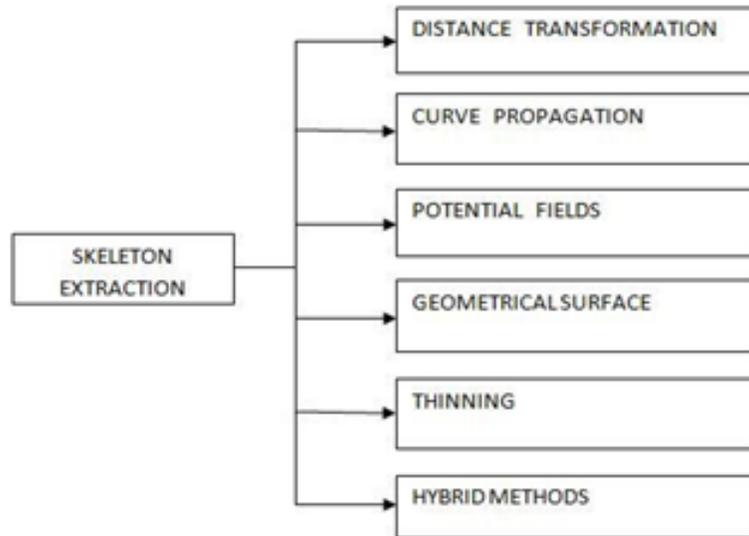


Figure 2. Narrow categorization of Skeletonisation algorithms.

of the boundary and the image¹¹. In Voronoi diagrams, changes in the boundaries are not exactly represented in the inner skeletons. Thus it has less sensitivity towards change response. But Voronoi diagrams retain the topological and the geometrical nature of the image^{12,13}. One of the major constrain in generating Voronoi diagram is calculating the distance between the geospatial features¹⁴. Voronoi diagram is greatly applied in fields like facility location, land planning, proximity analysis, building Geographic Information Systems. Another approach is Graph based approach for Skeletonisation. This approach follows collapsing and merging procedures where the zones are initially randomly portioned after which they are merged iteratively.

2.2 Approaches based on Object Reduction

Object reduction techniques are fairly called as thinning methods¹⁵. As the name implies, thinning methods aims at reducing the pixels directly. The pixels are rather deleted to get the final skeletal structure. The pixels are generally classified into three types. Line pixel³, Border pixel³, Euler Invariant pixel³. Border pixels are those that have at least one neighbor without Intensity value. Line pixels are those that are not located at the end of the line i.e., more than one pixel will have intensity value. Euler Invariant pixels are those pixels whose deletion will not cause any change in the Euler characteristic. Euler characteristic (χ) will describe the topological space's in a shape. The idea of thinning is that erode the above three pixels continuously

until n further change occurs. The result of thinning will be single wide pixel skeleton¹⁵. The Figure 3 below shows the sample results of thinning of a sea shore area.

Thinning is advantageous because,

- It preserves the topology of the original image.
- It does not change the shape through origin.
- It produces a single pixel width skeletal structure.
- It produces the skeleton along the medial axis.

2.3 Approaches based on Curve Propagation (Distance Vector)

The formation of Distance map for the process of Skeletonisation is the most basic procedure. According to Curve propagation, the distance of a pixel from its nearest boundary is calculated and encoded in a map form called distance map. The Distance Transform (DT) forms the basic metrics for calculating the separation between points. From the distance map, the local maximal points are connected to form the skeleton. Curve propagation approaches have methods like, Active Region Contour¹⁶, Shock Identification, Vector Field Identification¹⁷, Potential Field model¹⁸, Edge Strength model¹⁵, Linear Diffusion¹⁹. Distance Transformation uses three metrics for evaluation. They are Euclidean distance metric¹⁹, Chessboard distance metric¹⁹, City block distance metric¹⁹. The Table 1 below summarizes the three metrics.

The notion of wave propagation led the way to Distance transform. In wave front propagation, the

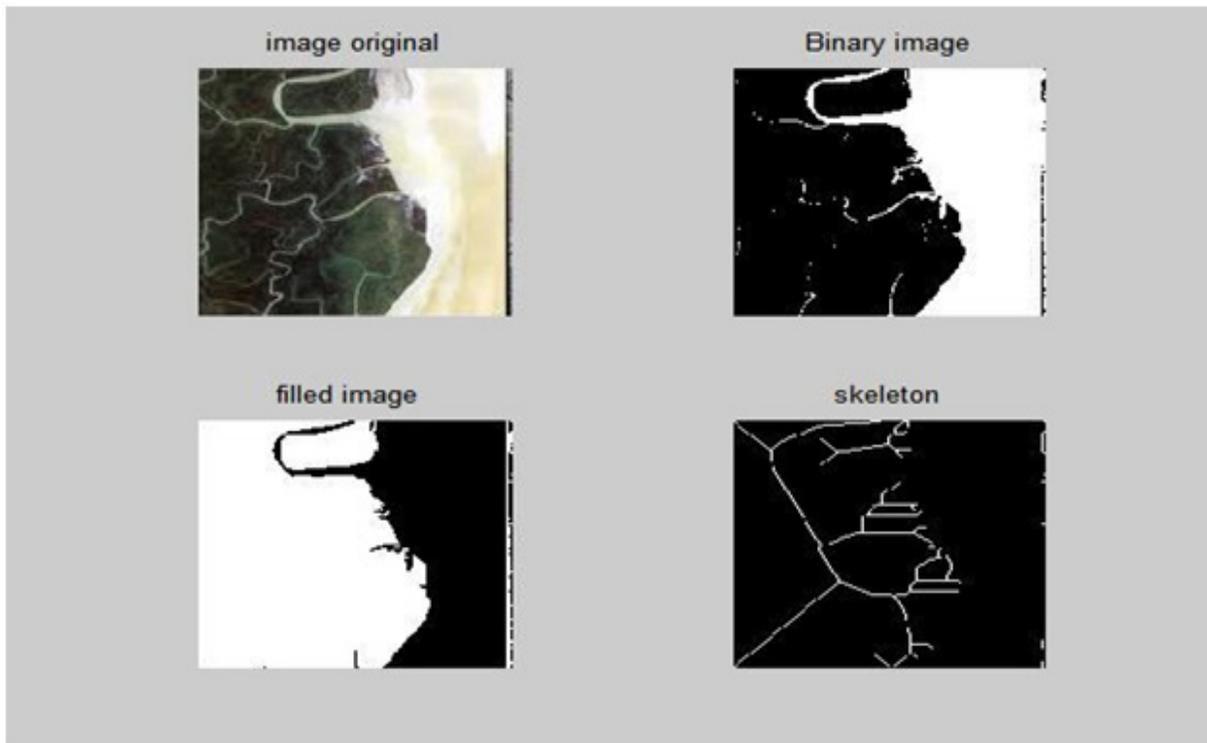
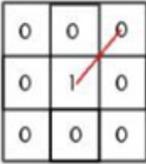
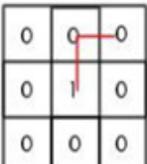
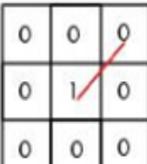
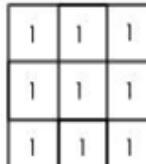


Figure 3. Sample Skeleton of seashore inward.

Table 1. Summarization of the three metrics of Distance Transform

Metric	Description	Illustration	Formula
Euclidean distance	Calculates the distance between two points in a straight line	 Image  Distance Transform	$D_{Euclid} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
City Block	Calculates the 4-neighborhood distance.	 Image  Distance Transform	$D_{city} = x_2 - x_1 + y_2 - y_1 $
Chess Board	Calculates the 8 neighborhood distance	 Image  Distance Transform	$D_{chess} = \max(x_2 - x_1 , y_2 - y_1)$

wave propagates through the medium. The distance of the point at a particular position is taken. But this does not guarantee skeleton. As the wave propagates, cluster gets formed where the centers of cluster would be the local maxima. Thinning the cluster would result in skeleton structure. The presence of surface noises will cause parasite branches. Henceforth a detail search must be performed to identify the local maxima. This method of Skeletonisation ensures complete connectivity of the skeleton. The figure below shows some sample results of Skeletonisation through Distance Transform using all the three metrics. The differences in the structuring element¹⁹ of the metrics results in slightly varying skeletal structures. Figure 4 shows the application of various distance measures on an image.

The Table 2 below summarizes all the approaches and compares them.

From the above table, it is inferred that, geometrical continuities approach yield good results. But they are not practically suitable for large topological spaces. Thus when curve propagation approaches and object reduction approaches are combined together, accurate skeleton structure can be got.

3. Quantitative Metrics for Validating Skeletons

Every procedure performed in a process is not worthy unless validated. The results of Skeletonisation must be validated. There are three quantitative metrics for validating the skeleton. They are Thinness²⁰, Connectedness²⁰, and Localization Quality²⁰.

3.1 Thinness

Thinness is one of the most important qualities of skeleton. It is a constraint that the skeleton must be single pixel

Table 2. Comparison of the three metrics of Distance Transform ¹⁹

Method	Topological Structure	Geometrics of the region
Curve Propagation Approaches	Yes	No
Geometrical continuities approaches	Yes	Yes
Object reduction approaches	No	Yes

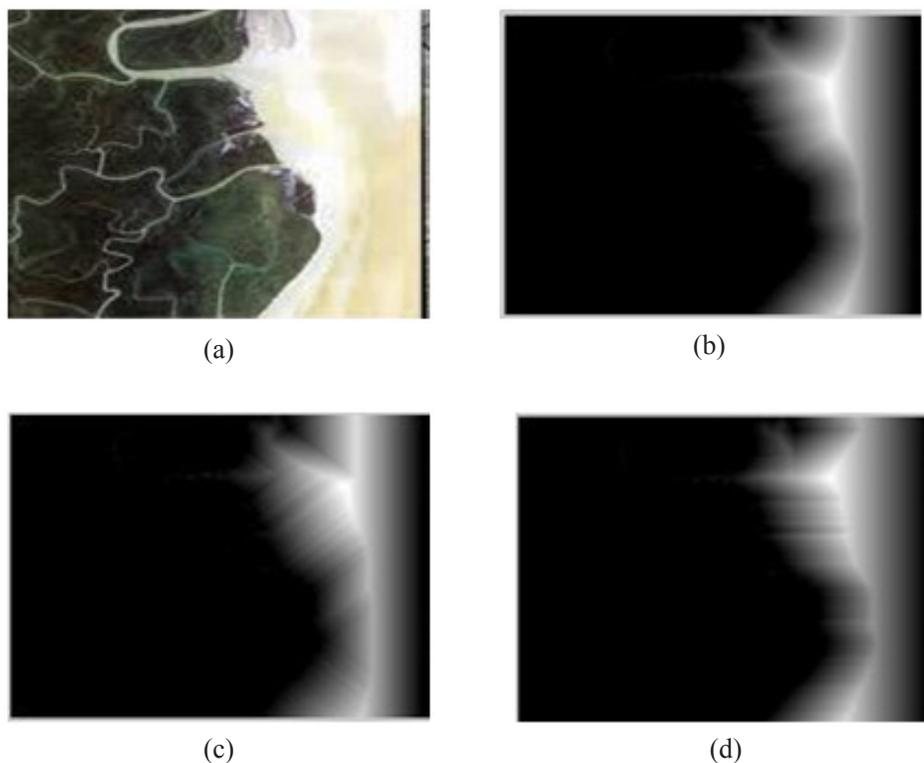


Figure 4. (a) Original image. (b) Skeleton evaluated using Euclidean distance. (c) Skeleton evaluated using Chessboard distance. (d) Skeleton evaluated using City block distance.

wide. A simple formula for calculating the mean thickness of the skeleton is given below:

$$\mu_{dnp} = \frac{1}{T} \sum_{i=0}^T 2dnp - 1 \quad (1)$$

Where μ_{dnp} = mean thickness,
 T = total number of local maxima distance values
 Dnp=distance to the nearest skeletal pixel

3.2 Connectedness

A skeleton is so called only when it is connected in all aspects to all its pixels. A quantity called BettiNumber²⁰ quantifies the information about the number of connected spaces in a skeleton. B_0 specifies that there are no holes i.e., all spaces are connected. B_1 specifies the number of holes in 1D skeleton.

$$B_1 = p - q + M \quad (2)$$

Where B_1 = betti number of 1D skeleton
 p = number of edges
 q = number of nodes
 M = number of connected components

3.3 Localization Quality

A skeleton is the central medial axis of an image space. Thus centerness is the most important criteria for a skeleton to be so. The points must be localized to its position. To crosscheck the localization, the skeleton can be compared against the hand-made ground truth skeleton²⁰.

4. Conclusion

A large number of research is been carried out in the field of Skeletonisation. Skeletonisation is one of the emerging areas of interest in the Image Processing domain. Because of its abstractness, Skeletonisation is been used in many areas of science like geography, biology, astronomy²¹⁻²⁴ etc. There are many different approaches under Skeletonisation which needs to be clarified further and can be experimented in many other fields²⁵. The major challenge in devising algorithms is to preserve the characteristics of the image space both topologically and geometrically. Validation metrics for skeleton is another area which needs to be focused. There are many challenges like skeleton stretching, graph plotting with skeleton,

skeleton merging in the operations with skeleton. In the future, Skeletonisation would definitely be a promising area of research with its challenges getting resolved.

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