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Flow and Heat Transfer on a Moving Flat Plate in a Parallel Stream with Constant Surface Heat Flux: A Stability Analysis

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Abstract

Numerical solutions for the moving flat plat in a parallel stream with heat flux is to be constant have been studied. By using non-similar transformation, the governing equations be able to reduced to an ordinary differential equation. Then, results of the equations can be solved by shooting method with maple implementation. Numerical results reveal that the velocity ratio parameter λ<0, the non-unique solutions do exist. Then, the analysis of stability is carried out into two non-unique solutions to determine which is more stable between both of the solutions by bvp4c solver in Matlab. From the result of stability analysis, the eigenvalues for the first solution is positive at the same time as second solution is negative.

Keywords: Dual Solution, Heat Flux, Moving Flat Plate, Parallel Stream, Stability Analysis

1. Introduction

There exists two ways of motion of heat transfer from a surface, either it is moving or stationary fluid. Whereas in engineering, the heat transfer's motion can be applied in many areas[1]. To the best of our knowledge, in[3–5] was the first who did a research about a moving surface in the boundary layer flow. Then, the problem within moving surface has been studied in different situations by many researchers such as[6–9,11]. However, as reported by[10], the papers by[12–15] shows the dual solution in their numerical results.

In general, constant for both wall temperature and surface heat flux are common applications with in heat transfer problem by researchers. For constant wall heat flux, the temperature is increasing with distance along the wall while for constant the wall temperature, the wall temperature is constant. In this study, the boundary condition is constant heat flux is considered.

Recently, a study regarding stability analysis had sparked an interest in research. This analysis is important in this fluid dynamics to identify which solution is stable if there are non–unique solutions exist in computation. However, the papers regarding this problem are limited in view since it is very new in our research. Some papers can be viewed towards this interest such as papers by[12,15–21]. In this present study, the aim is to examine the stability of the existence dual solution reported by[10].

	Nomenclature		Greek symbols
g	acceleration due to gravity	α	thermal diffusivity
f	non-dimensional stream function	ψ	stream function
T	fluid temperature	η	similarity variable
T_∞	ambient uniform temperature	λ	velocity ratio parameter
q_w	local heat flux	μ	dynamic viscosity
T_w	surface temperature	ρ	fluid density
Pr	Prandtl number	ν	kinematic viscosity
C_f	skin friction coefficient		
Re	Reynolds number		
Nu	Nusselt number		

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2. Mathematical Formulation

Let consider a viscous, an incompressible fluid two – dimensional boundary layer flow on a rigid or constantly moving flat surface. The plate moves is assumed in similar or reverse direction to the free stream and the surface is a constant heat flux. The velocities for both also assumed be constant. By the assumptions, the governing equations for this study are[10]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

and the conditions at the boundary are

$$u=U_w,\ v=0,\ \frac{\partial T}{\partial y}=-\frac{q_w}{k}\ \text{at}\ y=0$$

$$u\to U_\infty,\ T\to T_\infty\ \text{as}\ y\to\infty \tag{4}$$

where u is the velocity along the x- direction whereas v is the velocity along the y-directions. U_∞ is free stream velocity and U_w is plate velocity with both are constants. Furthermore, α is refer to thermal diffusivity, k is denotes the thermal conductivity and q_w is surface heat flux. In order to find a similarity solution of Equation (1) - (4), we apply the following similarity transformations

$$f(\eta)=\frac{\psi}{(\nu xU)^{1/2}},\ \theta(\eta)=\frac{k(T-T_\infty)}{q_w}\left(\frac{U}{\nu x}\right)^{1/2},\ \text{and}\ \eta=\left(\frac{U}{\nu x}\right)^{1/2}y, \tag{5}$$

Where $U=U_w+U_\infty$ is the composite velocity. Further, ψ is known as stream function and usually declare as $u=\partial\psi/\partial y$ and $v=-\partial\psi/\partial x$. Employing the similarity variables Equation (5), the Equation (1) is automatically convinced while the Equations (2) and (3) change to the ordinary differential equations as below:

$$f'''+\frac{1}{2}ff''=0 \tag{6}$$

$$\frac{1}{\Pr}\theta''+\frac{1}{2}f\theta'-\frac{1}{2}f'\theta=0 \tag{7}$$

Then, the boundary conditions in Equation (4) reduce to the forms:

$$f(0)=0,\ f'(0)=\lambda,\ \theta'(0)=-1$$

$$f'(\eta)\to 1-\lambda,\ \theta(\eta)\to 0\ \text{as}\ \eta\to\infty \tag{8}$$

with Pr is defined as $\Pr=\nu/\alpha$, primes stand for differentiation respect to η and velocity ratio parameter λ define as

$$\lambda=\frac{U_w}{U} \tag{9}$$

The skin friction coefficient C_f along with the local Nusselt number Nu_x acknowledged as

$$C_f=\frac{\tau_w}{\rho U^2/2},\ Nu_x=\frac{xq_w}{k(T_w-T_\infty)}, \tag{10}$$

with the wall shear stress τ_w and the local heat flux q_w are known as

$$\tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}\ q_w=-k\left(\frac{\partial T}{\partial y}\right)_{y=0}, \tag{11}$$

Then, apply the Equation (5) into Equation (11), which becomes

$$\frac{1}{2}C_f\,\mathrm{Re}_x^{1/2}=f''(0)$$

$$Nu_x/\mathrm{Re}_x^{1/2}=\frac{1}{\theta(0)}, \tag{12}$$

Where, $\mathrm{Re}_x=Ux/\nu$ is known as Reynolds number

3. Solution of Stability

The first step to analyze a stability analysis is to consider the problem is an unsteady problem. Then, Equations (2) and (3) are replaced by

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2} \tag{13}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{14}$$

Where, t refers to time. As identified by the variable in Equation (5), the new dimensionless variable for the unsteady problem can be introduced as

$$\psi=\sqrt{\nu xU}\,f(\eta,\tau),\ \theta(\eta,\tau)=\frac{k(T-T_\infty)}{q_w}\left(\frac{U}{\nu x}\right)^{1/2},$$

$$\eta = \sqrt{\frac{U}{\nu x}}y \text{, and } \tau = \frac{Ut}{x} \quad (15)$$

so that Equations (2) and (3) will become as

$$\frac{\partial^3 f}{\partial \eta^3} + \frac{1}{2} f \frac{\partial^2 f}{\partial \eta^2} - \frac{\partial^2 f}{\partial \eta \partial \tau} = 0 \quad (16)$$

$$\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial \eta^2} - \frac{1}{2}\frac{\partial f}{\partial \eta}\theta + \frac{1}{2} f \frac{\partial \theta}{\partial \eta} - \frac{\partial \theta}{\partial \tau} = 0 \quad (17)$$

follows with the boundary conditions

$$f(\eta,\tau) = 0,\ \frac{\partial f}{\partial \eta}(\eta,\tau) = \lambda,\ \frac{\partial}{\partial \eta}\theta(\eta,\tau) = -1 \text{ at } \eta = 0$$

$$\frac{\partial f}{\partial \eta}(\eta,\tau) \to 1-\lambda,\ \theta(\eta,\tau) \to 0 \text{ as } \eta \to \infty \quad (18)$$

With a view to analysis the stability for the steady flow solution $f(\eta) = f_0(\eta)$ as well as $\theta(\eta) = \theta_0(\eta)$ comply with the boundary – value problem Equation (1) - (4), we put in writing[16–19]

$$f(\eta,\tau) = f_0(\eta) + e^{-\gamma\tau} F(\eta,\tau)$$

$$\theta(\eta,\tau) = \theta_0(\eta) + e^{-\gamma\tau} G(\eta,\tau) \quad (19)$$

With $F(\eta,\tau)$ is small relative to $f_0(\eta)$ while $G(\eta,\tau)$ is small relative to $g_0(\eta)$ and γ is an unknown eigenvalue $\theta_0(\eta)$. Introducing Equation (19) into Equation (16) and Equation (17), the result is

$$\frac{\partial^3 F}{\partial \eta^3} + \frac{1}{2}\left[f_0 \frac{\partial^2 F}{\partial \eta^2} + f_0'' F \right] + \gamma \frac{\partial F}{\partial \eta} - \frac{\partial^2 F}{\partial \eta \partial \tau} = 0 \quad (20)$$

$$\frac{1}{\Pr}\frac{\partial^2 G}{\partial \eta^2} - \frac{1}{2}\left(f_0' G + \frac{\partial F}{\partial \eta}\theta_0 - f_0 \frac{\partial G}{\partial \eta} - F\theta_0' \right) + \gamma G - \frac{\partial G}{\partial \tau} = 0 \quad (21)$$

and the new boundary conditions

$$F(\eta,\tau) = 0,\ \frac{\partial F}{\partial \eta}(\eta,\tau) = 0,\ G(\eta,\tau) = 0 \text{ as } \eta = 0$$

$$\frac{\partial F}{\partial \eta}(\eta,\tau) \to 0,\ G(\eta,\tau) \to 0 \text{ as } \eta \to \infty \quad (22)$$

Then, the solutions $f(\eta) = f_0(\eta)$ and $\theta(\eta) = \theta_0(\eta)$ of the steady solutions in Equation (6) and Equation (7) are obtained by replacing the value of τ as 0. Therefore, $F = F_0(\eta)$ and $G = G_0(\eta)$ in Equation (16) and Equation (17) identify initial increment of the solution in Equation (19). For that case, we should deal with the linear eigenvalue problem

$$F_0''' + \frac{1}{2}(f_0 F_0'' + f_0'' F_0) + \gamma F_0' = 0 \quad (23)$$

$$\frac{1}{\Pr} G_0'' - \frac{1}{2}(f_0' G_0 + F_0' \theta_0 - f_0 G_0' - F_0 \theta_0') + \gamma G_0 = 0 \quad (24)$$

and the boundary conditions are

$$F_0(0) = 0,\ F_0'(0) = 0,\ G_0(0) = 0$$

$$F_0'(\eta) \to 0,\ G_0(\eta) \to 0 \text{ as } \eta \to \infty \quad (25)$$

It is necessary to point out that for the specific values of Pr and γ, the stability of the steady flow solutions for both $f_0(\eta)$ as well as $\theta_0(\eta)$ can be tested via the smallest eigenvalue γ. It is necessary to point out that, if an initial increment of interruption during computation, then the flow is declared as unstable once the smallest of eigenvalue shown negative. On the other hand, if an initial decomposition, then the flow is declared as stable when smallest γ eigenvalue is a positive value. As it has been suggested by[22], as a result of relaxing a boundary condition on $F_0(\eta)$ and $G_0(\eta)$, we can determine the possible range of eigenvalues. Then, in this present problem, the condition $F_0' \to 0$ as $\eta \to \infty$ is selected to be relaxed and we solved the Equations (23) and (24) with the updated boundary condition as $F_0''(0) = 1$

4. Result and Discussion

In order to achieve the infinity boundary conditions asymptotically, the thickness of boundary layer known as η_∞ was used between 5 and 15 values. By guessing the difference between initial values for $f''(0)$ and $-\theta'(0)$, the dual solutions are obtained where both profiles satisfy the boundary condition in Equation (8). The Pr and tl number is kept constant at Pr = 1 with various value of velocity ratio parameter λ. The numerical results have shown in good agreement with previously work by[2,3,10] as shown in Table 1. In this study, the solution is unique when $0 \le \lambda \le 1$, dual solution when $-0.5482 \le \lambda \le 0$ and no solution when $\lambda \le -0.5482$.

Further, we perform a stability analysis since the numerical computation admits dual solutions. The stability of the flow can be tested by looking at the polarity of the smallest eigenvalue itself. As has been noted, the flow will be stable if only if the smallest eigenvalue have shown positive result. On the other hand, when the smallest eigenvalue γ is shown negative, then the flow is surely unstable. Table 2 displays the smallest eigenvalue γ for

Table 1. The values of $f''(0)$ for certain values of λ

λ	Blasius[2]	Sakiadis[3]	Ishak[10]		Present results	
			$f_1''(0)$	$f_2''(0)$	$f_1''(0)$	$f_2''(0)$
–0.5			0.3990	0.1710	0.3978	0.1710
–0.4			0.4357	0.0834	0.4356	0.0834
–0.3			0.4339	0.0367	0.4339	0.0367
–0.2			0.4124	0.0114	0.4124	0.0114
–0.1			0.3774	0.0010	0.3774	0.0001
0	0.332		0.3321		0.3221	
0.5			0		0	
1		–0.44375	–0.4438		–0.4438	

Table 2. The smallest eigenvalues γ at certain values of λ

λ	Upper Branch	Lower Branch
–0.40	0.19915	–0.09825
–0.30	0.26622	–0.09554
–0.28	0.27795	–0.09298
–0.26	0.28920	–0.08995
–0.24	0.30006	–0.08637
–0.22	0.31053	–0.08224
–0.20	0.32062	–0.07679

the variation values of λ with Pr = 1. We can see clearly that the value of γ for the upper branch is real and positive and the value of γ for the lower branch is real and negative.

The velocity and temperature profile are illustrated in Figures 1 and 2 with velocity ratio parameter λ is –0.3. From the figures, we can see the chosen value of λ has shown non-unique solutions and it is supported by results and discussions in paper[10]. The thickness of boundary layer is thicker on behalf of lower branch and thinner towards upper branch for both velocity and temperature profile. It is as expected due to physically significant and stable for upper branch and it is unstable and physically insignificant for the lower branch. Finally, the most important, both profiles show the solutions satisfy the boundary conditions in Equation (11) as $f'(\eta) \to 1$ and $\theta(\eta) \to 0$ asymptotically.

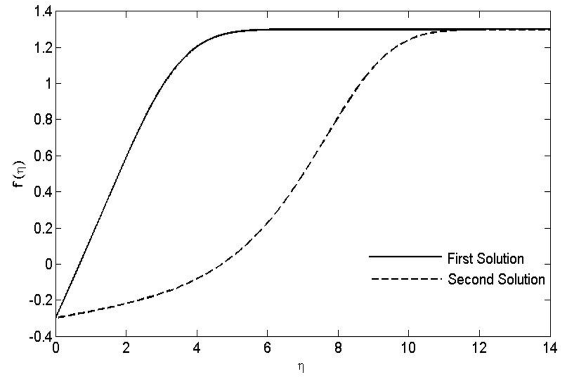

Figure 1. Velocity profile for λ = – 0.3 with Pr = 1.

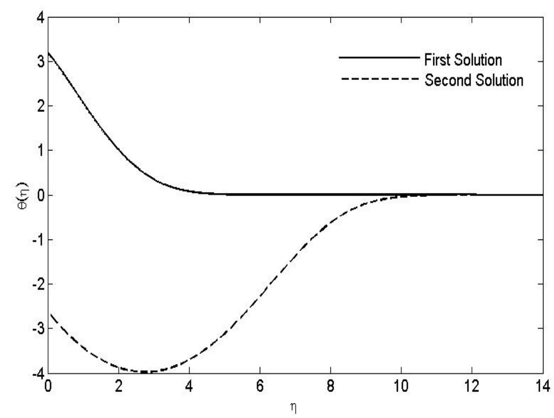

Figure 2. Temperature profile for λ = – 0.3 with Pr = 1.

5. Conclusion

In solving the positive *x*-direction of the free stream, the range of λ< 0 admits the non-unique solutions as known as upper branch and lower branch with λ_c = -0.5482. Upper branch have shown linearly stable whereas the lower branch shown linearly unstable by performing the stability analysis. This statement is supported due to the fact that the eigenvalue γ is positive on behalf of upper branch while lower branch is shown negative.

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