Indian Journal of Science and Technology, *Vol 8(16), DOI: 10.17485/ijst/2015/v8i16/55529, July 2015*
ISSN (Print) : 0974-6846
ISSN (Online) : 0974-5645

Two Stage Flow Shop Scheduling under Fuzzy Environment

Kewal Krishan Nailwal[1*], Deepak Gupta[2] and Sameer Sharma[3]

[1]A.P.J. College of Fine Arts, Jalandhar – 144001, Punjab, India; kk_nailwal@yahoo.co.in
[2]M.M. University Mullana, Ambala - 133207, Haryana, India; guptadeepak2003@yahoo.co.in
[3]D.A.V. College, Jalandhar City - 144008, Punjab, India; samsharma31@yahoo.com

Abstract

Background/Objectives: The real life applications are complex in nature and demand the uncertainty as key characteristic in the problem. An algorithm incorporating the uncertain processing time and setup time of *n*-jobs on two machines to minimize rental cost of machines is described. The jobs to be processed as block and the transportation time between different units of production under particular rental situation is an addition to the existing problem. **Methods/Statistical Analysis:** The main characteristic as uncertainty in processing time and setup time is depicted with fuzzy triangular membership function. The basic set operations on fuzzy set theory are applied to obtain the mathematical solutions. A comprehensive defuzzification technique known as Yager' formula is used in the proposed algorithm. The algorithm uses a heuristic approach to obtain the desired objective with constraints of job-block, transportation time under definite rental situation. **Results/Findings:** The sequence obtained using the proposed heuristic in the sample numerical problem minimizes the total rental cost of machines by minimizing the operation time of machines. The Johnson technique used in the algorithm minimizes total elapsed time and hence ensures the total minimum operational cost of machines. **Conclusion/Findings:** The algorithm put forward minimizes the operation time and therefore the rental cost of machines using the concept of processing of jobs as block called equivalent job-block along with the time taken in transporting a job from one machine to another in fuzzy environment. This algorithm allows the dynamical nature of the processing times using fuzzy triangular membership function which is easy to understand and straight in its approach.

Keywords: Average High Ranking, Fuzzy Processing Time, Fuzzy Setup Time, Operation Time, Rental Cost

1. Introduction

Scheduling is an enduring process for assigning of resources over time to perform a set of tasks[1]. The minimization or maximization of one or several criteria's is the objective corresponding to the scheduling problem. In a general flow shop scheduling problem, set of tasks named as *n*-jobs are to be scheduled on *m*-machines in order to optimize some criteria of performance. The processing of all *n*-jobs on *m*-machines is done in some pre-defined manner since the processing requirements for all of these are same. Johnson[2] whose work is one of the earliest developed an algorithm to minimize the make span in two stage flow shop scheduling problem. The setup time of various jobs on machines are considered to be negligible and therefore could be included in processing time of jobs. However, in some applications, setup has major impact on the performance measure considered for scheduling problem so they need to be considered separately. Scheduling problems involving setup times can be divided into two classes: the first class is sequence-independent and second is sequence-dependent setup times. In this paper, we address first class i.e. the setup time does not depends on the sequence in which the jobs are processed on the machines. These scheduling problems mainly focused on deterministic type of job processing. Various theories have been put to deal with uncertain job processing times. Zadeh[3] was first to introduce fuzzy sets as a mathematical way of representing impreciseness or vagueness in everyday life induced the fuzzy set theory

**Author for correspondence*

as the most frequently used theory in intelligent control. Fuzzy set theory because of its simplicity and similarity to human reasoning has numerous applications in various fields such as engineering, medicine, manufacturing and others. Fuzzy set theory can be used to handle uncertainty inherent in actual scheduling problems. The imprecise processing time model given by Sanuja and Xueyan[4] used α-cut procedure to optimize makespan in a two-machine flow shop problem and got better results than the algorithm considered by McCahon and Lee[5] using mean values of fuzzy sets. Ishibuchi, Murata and Lee[6] described the flow shop scheduling using fuzzy numbers and gave two heuristics for small and large size problems. Besides fuzzy theory, Singh T.P. and Gupta Deepak[7] gave probabilistic model of flow shop scheduling. Further, the flow shop scheduling problem was discussed with various parameters in the studies done by Gupta and Sharma[8] and Gupta et al.[9–11]. Some of the noteworthy approaches are due to Gupta J.N.D.[12], Marin and Roberto[13].

Bagga[14] considered the sequencing of jobs with specific rental situation in flow shop scheduling. The situation under which one has to acquire machines to complete one's job, the interest for him is to reduce the cost by avoiding huge investment for the purchase of such machines. For example, rental of medical equipment gives affordable, quick solution, saves working capital and allow up gradation to new technology. Also, medical practitioner suggests the patient for physiotherapy to recover faster after the surgery of fractured leg. The physiotherapist recommends hi-tech equipments and techniques for proper recovery which are generally expensive for a period of week or months. Thus, taking those things on rent make sense instead of buying them. An important and useful concept given by Maggu and Das[15] for providing priority services to the customer based on extra cost to be paid by them is known as equivalent job-block in scheduling theory. By this, the decision makers separate the priority customers from non-priority customers and therefore include priority charge from them. Also, the transportation of unfurnished goods from one place to another is of utmost importance as it adds extra cost to production.

This paper considers two-machine, *n*-job flow shop scheduling problem in which uncertain processing and setup times are represented with the help of triangular fuzzy numbers involving the restriction of transportation time and job-block. The corresponding objective is to find a job sequence which minimizes the rental cost of machines in fuzzy environment. We considered a widely used defuzzification technique known as Yager's first index[16]. The rest of paper is organized as follows: Section 2 describes the basics of fuzzy set theory. Section 3 gives the notations to be used throughout the paper. In section 4, problem is formulated. Section 5 deals with theorem for optimizing the problem under job-block. Section 6 describes the algorithm proposed to find the optimal sequence for minimizing the rental cost. In section 7, numerical illustration is given to support the proposed algorithm. The paper is concluded in section 8 followed by the references in section 9.

2. Basic Fuzzy Set Theory

A fuzzy set $\tilde{A} = x_1, x_2, x_3$ defined on the universal set of real numbers R, is said to be a fuzzy number if its characteristic (or membership) function $\mu_{\tilde{A}}$ has the following characteristics:

(i) $\mu_{\tilde{A}} : R \rightarrow [0,1]$ is continuous.

(ii) $\mu_{\tilde{A}} = 0$ for all $x \in (-\infty, x_1) \cup (x_3, \infty)$

(iii) $\mu_{\tilde{A}}$ is strictly increasing on $[x_1, x_2]$ and strictly decreasing on $[x_2, x_3]$.

(iv) $\mu_{\tilde{A}} = 1$ for $x = x_2$.

2.1 Triangular Fuzzy Number

Triangular Fuzzy Number (TFN) is a fuzzy number (Figure 1.) represented with three points as $\tilde{A} = (x_1, x_2, x_3)$, where x_1 and x_3 denote the lower and upper limits of support of a fuzzy set Ã. The membership value of the x denoted by $\mu_{\tilde{A}}(x), x \in R^+$, can be calculated according to the following formula.

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & ; x \le x_1 \\ \dfrac{x - x_1}{x_2 - x_1} & ; x_1 < x < x_2 \\ \dfrac{x_3 - x}{x_3 - x_2} & ; x_2 < x < x_3 \\ 0 & ; x \ge x_3 \end{cases}$$

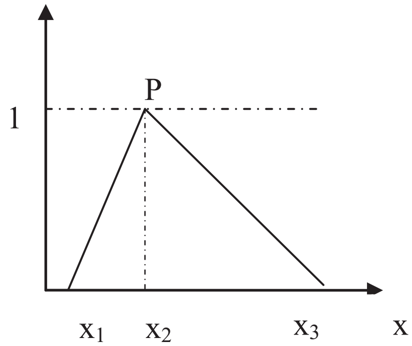

Figure 1. Triangular fuzzy number.

2.2 Average High Ranking (AHR)

The characteristic function or membership function preserves the fuzziness in the system key characteristics. But, one would prefer one crisp value for one of the system key characteristics rather than fuzzy set. In order to overcome this difficulty we defuzzify the fuzzy values of system characteristic by using the Yager's[16] formula

$$\text{Crisp}\left(\tilde{A}\right) = h(\tilde{A}) = \frac{3x_2 + x_3 - x_1}{3} \quad (1)$$

2.3 Fuzzy Arithmetic Operations

If $\tilde{A}_1 = (m_{\tilde{A}_1}, \alpha_{\tilde{A}_1}, \beta_{\tilde{A}_1})$ and $\tilde{A}_2 = (m_{\tilde{A}_2}, \alpha_{\tilde{A}_2}, \beta_{\tilde{A}_2})$ be the two triangular fuzzy numbers, then

(i) $$\tilde{A}_1 + \tilde{A}_2 = (m_{\tilde{A}_1}, \alpha_{\tilde{A}_1}, \beta_{\tilde{A}_1}) + (m_{\tilde{A}_2}, \alpha_{\tilde{A}_2}, \beta_{\tilde{A}_2}) = (m_{\tilde{A}_1} + m_{\tilde{A}_2}, \alpha_{\tilde{A}_1} + \boldsymbol{\alpha}_{\tilde{A}_2}, \beta_{\tilde{A}_1} + \beta_{\tilde{A}_1})$$

(ii) $$\tilde{A}_1 - \tilde{A}_2 = (m_{\tilde{A}_1}, \alpha_{\tilde{A}_1}, \beta_{\tilde{A}_1}) - (m_{\tilde{A}_2}, \alpha_{\tilde{A}_2}, \beta_{\tilde{A}_2}) = (m_{\tilde{A}_1} - m_{\tilde{A}_2}, \alpha_{\tilde{A}_1} - \boldsymbol{\alpha}_{\tilde{A}_2}, \beta_{\tilde{A}_1} - \beta_{\tilde{A}_2})$$

if the following condition is satisfied $DP\left(\tilde{A}_1\right) \geq DP\left(\tilde{A}_2\right)$, where $DP(\tilde{A}_1) = \frac{\beta_{\tilde{A}_1} - m_{\tilde{A}_1}}{2}$ and $DP(\tilde{A}_2) = \frac{\beta_{\tilde{A}_2} - m_{\tilde{A}_2}}{2}$. Here, *DP* denotes difference point of a Triangular Fuzzy Number (TFN).

Otherwise;

$$\tilde{A}_1 - \tilde{A}_2 = (m_{\tilde{A}_1}, \alpha_{\tilde{A}_1}, \beta_{\tilde{A}_1}) - (m_{\tilde{A}_2}, \alpha_{\tilde{A}_2}, \beta_{\tilde{A}_2}) = (m_{\tilde{A}_1} - \beta_{\tilde{A}_2}, \alpha_{\tilde{A}_1} - \boldsymbol{\alpha}_{\tilde{A}_2}, \beta_{\tilde{A}_1} - m_{\tilde{A}_2})$$

(iii) $k\tilde{A}_1 = k(m_{\tilde{A}_1}, \alpha_{\tilde{A}_1}, \beta_{\tilde{A}_1}) = (km_{\tilde{A}_1}, k\alpha_{\tilde{A}_1}, k\beta_{\tilde{A}_1})$; if $k > 0$.

(iv) $k\tilde{A}_1 = k(m_{\tilde{A}_1}, \alpha_{\tilde{A}_1}, \beta_{\tilde{A}_1}) = (k\beta_{\tilde{A}_1}, k\alpha_{\tilde{A}_1}, km_{\tilde{A}_1})$; if $k < 0$.

3. Notations

i : Job index, $i = 1,2,3\ldots\ldots n$

j : Machine index , $j = 1,2$

σ : Sequence of jobs

σ_k : Sequence by implementing Johnson's technique, $k = 1, 2, 3,$ -------

M : Minimum elapsed time

p_{ij} : Processing time in fuzzy of the job i on machine j

q_{ij} : Set up time in fuzzy of the job i on machine j

p'_{ij} : Average high ranking of processing time of job i on machine j

q'_{ij} : Average high ranking of setup time of job i on machine j

p''_{ij} : Average high ranking of the fuzzy flow time of job i on machine j

$T_{i1\to2}$: Time in fuzzy for transporting job i from machine 1 to machine 2

$T'_{i1\to2}$: Average high ranking of time in transporting job i from machine 1 to machine 2.

β : Single equivalent job as block.

r_j : Rental cost per unit time of machine j.

$t_{ij}(\sigma_k)$: Time in fuzzy to complete job i on machine j for sequence σ_k

$R_c(\sigma_k)$: Total rental cost in fuzzy of all machines for the sequence σ_k

O_k : Operation time in fuzzy of machine 2 for sequence σ_k

3.1 Assumptions

- The jobs to be processed are independent of each other.
- Pre-emption of job is not allowed, i.e. a job once started on a machine, cannot be stopped in between unless the job is completed.
- All the jobs and machines are available at the beginning of the processing.
- Each job is processed through each of the machine once and only once. A job is not available to the next

machine until and unless processing on the current machine is completed.

- Machines never breakdown and are available throughout the scheduling process.

3.2 Rental Situation (S)

Since as per our assumption the machines are available at the beginning of the processing so we under our rental situation take the first machine on rent at the start of production and second machine when first job is in ready mode for processing on it after it got processed on machine 1. The machines are returned back as soon as the requirement of job processing on them is not required.

4. Problem Formulation

A two stage fuzzy flow shop problem with jobs (i = 1,2,……..,n) having fuzzy processing time p_{ij} and setup q_{ij} on machines (j=1,2) under particular rental situation S can mathematically be formulated as,

Minimize $O_k(\sigma_k)$ or Minimize $R_c(\sigma_k)=t_{n1}\times C_1+O_k(\sigma_k)\times C_2$.

Subject to the constraint: Rental Situation *(S)*.

Let $T_{i1\to 2}$ be the fuzzy time taken in transporting job i (Table 1). Our objective is to find a sequence $\{\sigma_k\}$ of jobs which minimizes the rental cost of the machines (j = 1,2).

5. Theorem

The following theorem supports the finding of optimal sequence with job-block of jobs processing.

If passing of jobs is not allowed on machine j (j = 1,2) for a schedule. σ = {1,2,3,…,k,k+1,k+2,…n} of n jobs processed strictly in the order from machine 1 to machine 2 only and the job block *(k,k+1)* having processing times $\{p_{k,1}, p_{k,2}, p_{(k+1),1}, p_{(k+1),2}\}$ is equal to the single job β then the processing time of job block β on machine 1 and machine 2 denoted by $p_{\beta,1}$ and $p_{\beta,2}$ respectively are given by

$$p_{\beta,1}=p_{k,1}+p_{k+1,1}-\min\{p_{k,2},p_{k+1,1}\}$$

$$p_{\beta,2}=p_{k,2}+p_{k+1,2}-\min\{p_{k,2},p_{k+1,1}\}$$

Table 1. The matrix form of the problem

	Machine 1			Machine 2	
i	p_{i1}	q_{i1}	$T_{i1\to 2}$	p_{i2}	q_{i2}
1	p_{11}	q_{11}	$T_{11\to 2}$	p_{12}	q_{12}
2	p_{21}	q_{21}	$T_{21\to 2}$	p_{22}	q_{22}
3	p_{31}	q_{31}	$T_{31\to 2}$	p_{32}	q_{32}
-	-	-	-	-	-
m	p_{m1}	q_{m1}	$T_{m1\to 2}$	p_{m2}	q_{m2}
-	-	-	-	-	-
n	p_{n1}	q_{n1}	$T_{n1\to 2}$	p_{n2}	q_{n2}

Proof: For the processing of n jobs in a given sequence σ, let $t_{k,l}$ be the completion time of job k (k =1, 2, 3,---, n) on machine l (l = 1, 2).

By definition, we have

$$t_{k,2}=\max\left(t_{k,1},t_{k-1,2}\right)+p_{k,2}=\max\left(t_{k,1}+p_{k,2},t_{k-1,2}+p_{k,2}\right)$$

$$\text{Now, } t_{k+1,2}=\max\{t_{k+1,1},t_{k,2}\}+p_{k+1,2}$$
$$=\max\{t_{k+1,1},t_{k,1}+p_{k,2},t_{k-1,2}+p_{k,2}\}+p_{k+1,2}$$
$$=\max\{t_{k+1,1}+p_{k+1,2},t_{k,1}+p_{k,2}+p_{k+1,2},t_{k-1,2}+p_{k,2}+p_{k+1,2}\}$$

$$\text{As, } t_{k+1,1}=t_{k,1}+p_{k+1,1}$$

$$\Rightarrow t_{k+1,2}=\max\{t_{k,1}+p_{k+1,1}+p_{k+1,2},t_{k,1}+p_{k,2}+p_{k+1,2},$$
$$t_{k-1,2}+p_{k,2}+p_{k+1,2}\}$$

$$\text{Also, } t_{k+2,2}=\max\{t_{k+2,1},t_{k+1,2}\}+p_{k+2,2}$$

$$=\max\{t_{k+2,1},t_{k,1}+p_{k+1,1}+p_{k+1,2},t_{k,1}+p_{k,2}+p_{k+1,2},$$
$$t_{k-1,2}+p_{k,2}+p_{k+1,2}\}+p_{k+2,2}$$

$$\text{As, } t_{k+2,1}=t_{k,1}+p_{k+1,1}+p_{k+2,1}$$

Therefore, we have

$$t_{k+2,2}=\max\{t_{k,1}+p_{k+1,1}+p_{k+2,1},t_{k,1}+p_{k+1,1}+p_{k+1,2},$$
$$t_{k,1}+p_{k,2}+p_{k+1,2},t_{k-1,2}+p_{k,2}+p_{k+1,2}\}+p_{k+2,2}$$

Since,

$$\max\{t_{k,1}+p_{k+1,1}+p_{k+1,2},t_{k,1}+p_{k,2}+p_{k+1,2}\}=$$
$$t_{k,1}+\max\{p_{k+1,1},p_{k,2}\}+p_{k+1,2}$$

Therefore, we get

$$t_{k+2,2} = \max\left\{t_{k,1} + p_{k+1,1} + p_{k+2,1}, t_{k,1} + \max\left\{p_{k+1,1}, p_{k,2}\right\} + p_{k+1,2}, t_{k-1,2} + p_{k,2} + p_{k+1,2}\right\} + p_{k+2,2} \quad (2)$$

Also, $t_{k+2,1} = t_{k-1,1} + p_{k,1} + p_{k+1,1} + p_{k+2,1} = t_{k,1} + p_{k+1,1} + p_{k+2,1}$ (3)

Now, let the new sequence σ' of jobs be defined as

$$\sigma' = \{1, 2, 3, \text{---}, k-1, \beta, k+2, \text{---}, n\}$$

Where, $p_{\beta,1} = p_{k,1} + p_{k+1,1} - c$ (4)

$p_{\beta,2} = p_{k,2} + p_{k+1,2} - c$; c is a constant. (5)

Let $t'_{k,l}$ denote the completion time of job k (k =1, 2, 3,---, n) on machine l (l = 1, 2) for the sequence σ' of jobs.
By definition we have,

$$t'_{\beta,2} = \max\left(t'_{\beta,1}, t'_{\beta-1,2}\right) + p_{\beta,2} = \max\left(t'_{\beta,1} + p_{\beta,2}, t'_{\beta-1,2} + p_{\beta,2}\right)$$

$$t'_{k+1,2} = \max\left\{t'_{k+1,1}, t'_{\beta,2}\right\} + p_{k+2,2}$$

$$= \max\left\{t'_{k+2,1}, t'_{\beta,1} + p_{\beta,2}, t'_{\beta-1,2} + p_{\beta,2}\right\} + p_{k+2,2} \quad (6)$$

As $t'_{k+2,1} = t'_{k-1,1} + p_{\beta,1} + p_{k+2,1}$

$$= t_{k-1,1} + (p_{k,1} + p_{k+1,1} - c) + p_{k+2,1}$$

$$= t_{k,1} + p_{k+1,1} - c + p_{k+2,1} \quad \left[\because t_{k,1} = t_{k-1,1} + p_{k,1}\right] \quad (7)$$

Also, $t'_{\beta,1} = t'_{k-1,1} + p_{\beta,1} = t_{k-1,1} + p_{k,1} + p_{k+1,1} - c$

$$= t_{k,1} + p_{k+1,1} - C \quad (8)$$

On combining the results (4), (5), (6), (7) and (8), we have

$$t'_{k+2,2} = \max\left\{t_{k,1} + p_{k+1,1} - c + p_{k+2,1}, t_{k,1} + p_{k+1,1} - c + p_{k,2} + p_{k+1,2} - c, t_{k,1} + p_{k+1,1} - c + t_{k,2} + t_{k+1,2} - c\right\} + p_{k+2,2} \quad (9)$$

Let $c = \min\left\{p_{k+1,1}, p_{k,2}\right\}$, then (10)

$$p_{k+1,1} - c + p_{k,2} = p_{k+1,1} - \min\left\{p_{k+1}, p_{k,2}\right\} + p_{k,2} = \max\left\{p_{k+1,1}, p_{k,2}\right\} \quad (11)$$

Also, $t'_{k-1,2} = t_{k-1,2}$ (12)

On combining results (9), (10), (11) and (12), we have

$$t'_{k+2,2} = \max\left\{t_{k,1} + p_{k+1,1} + p_{k+2,1} - c, t_{k,1} + \max\left\{p_{k+1,1}, p_{k,2}\right\} + p_{k+1,2} - c, t_{k-2,2} + p_{k,2} + p_{k+1,2} - c\right\} + p_{k+2,2}$$

$$= \max\left\{t_{k,1} + p_{k+1,1} + p_{k+2,1}, t_{k,1} + \max\left\{p_{k+1}, p_{k,2}\right\} + p_{k+1,2}, t_{k-1,2} + p_{k,2} + p_{k+1,2}\right\} + p_{k+2,2} - c \quad (13)$$

From (2) and (13), we have

$t'_{k+2,2} = t_{k+2,2} - c$ (14)

From (3) and (7), we conclude that

$t'_{k+2,1} = t_{k+2,1} - c$ (15)

From results (14) and (15), we observe that when jobs ($k,k+1$) as block are replaced by single job β in a sequence σ, diminishes the completion time of the later job $k+2$ on both the machines by the same constant in job sequence σ' in comparison to the job $k+2$ in σ i.e. it means that $t=t'-c$, where t and t' be the completion times of the job sequence σ and σ', then we have i.e. the completion times of the jobs on both the machines is independent of σ. This means that the relative importance of distinct job sequences is not altered with the inclusion of job-block. Hence, for the job ($k,k+1$) as job-block there corresponds β as an equivalent job.

6. Algorithm

The following algorithm is proposed for two flow shop scheduling to minimize the rental cost of machines when fuzzy processing time, setup time and transportation of jobs are considered involving equivalent job block.

Step 1: Find Average High Ranking (AHR) using equation (1) for the fuzzy processing time, setup time of various jobs on different machines and the fuzzy transportation time from machine 1 to machine 2.

Step 2: Using the formulae $p''_{i1} = p'_{i1} - q'_{i2}$ and $p''_{i2} = p'_{i2} - q'_{i1}\ \forall i$, find the average high ranking of fuzzy flow time for machines.

Step 3: Deploy two machines G and H which are fictitious with respective processing times as G_{i1} and H_{i2} for job i, as:

$$G_{i1} = p''_{i1} + T'_{i1\to2}, H_{i2} = T'_{i1\to2} + p''_{i2}$$

Step 4: The processing times defined as $G_{\beta 1}$ and $H_{\beta 2}$ for equivalent job *β(k,m)* are calculated processing using the criteria of Maggu and Das[15] as:

$$G_{\beta 1} = G_{k2} + G_{m2} - \min(G_{m1}, H_{k2}), H_{\beta 2} = H_{k2} + H_{m2} - \min(G_{m1}, H_{k2})$$

Step 5: Define a new reduced problem having two fictitious machines *G* and *H* with the processing times G_{i1} and H_{i2} as mentioned in step 3 and jobs (*k, m*) as job-block replaced by single equivalent job *β* with processing time $G_{\beta 1}$ and $H_{\beta 2}$ as mentioned in step 4.

Step 6: Using Johnson's technique[2] find all the sequences σ_k with minimum elapsed time *M*. Let these sequences be $\sigma_1, \sigma_2, \ldots$

Step 7: Let α be the processing time of first job of the sequence σ_1 on machine 1.

Step 8: Observe all the jobs on machine 1 having processing time greater than α defined in the step 7. Put these jobs one by one in the first position of the sequence σ_1 in the same order. Let these sequences be $\sigma_2, \sigma_3, \ldots \sigma_r$.

Step 9: Prepare flow table only for those sequences σ_p (p = 1,2,…r) which have job block *β(k,m)* and find total elapsed time of last job on machines for each sequence i.e. $t_{n1}(\sigma_p)$ and $t_{n2}(\sigma_p)$.

Step 10: Evaluate completion time of first job on machine 1 i.e. $t_{11}(\sigma_p)$ for each of above obtained sequence σ_p in step 8. **Step 11:** Calculate operation time O_p of machine 2 for

$$O_p(\sigma_p) = t_{n2}(\sigma_p) - t_{11}(\sigma_p) \quad \text{for} \quad p = 1, 2, 3, \ldots r.$$

each of the sequence σ_p obtained as in step 8.

Step 12: Find $\min\{O_p(\sigma_p)\}$, p=1, 2 …r. let it corresponds to $p = q$, then σ_q is the optimal sequence for minimum rental cost.

Minimum rental cost $R_c(\sigma_q) = t_{n1}(\sigma_q) \times r_1 + O_p(\sigma_p) \times r_2$

7. Numerical Illustration

Consider 5 jobs, 2 machine flow shop scheduling problem with processing time and setup time of jobs represented by triangular fuzzy numbers (Table 2). The jobs (3, 4) are to be processed in priority as equivalent group job. Let us take the rental costs per unit time for machine 1 and machine 2 as 10 and 8 units respectively. The optimization criterion is to minimize the total rental cost of machines for the sequence of jobs thus becoming the optimal sequence.

Solution: Calculate the AHR of fuzzy processing time, setup time of jobs on machine *j* (*j* = 1,2) and transportation time. The various values as per step 1 of algorithm are shown (Table 3).

Table 2. Data set for numerical illustration

	Machine 1			Machine 2	
i	p_{i1}	q_{i1}	$T_{i1\to 2}$	p_{i2}	q_{i2}
1	(9,10,11)	(1,2,3)	(3,4,5)	(9,10,11)	(1,2,3)
2	(12,13,14)	(4,5,6)	(4,5,6)	(11,12,13)	(3,4,5)
3	(13,14,15)	(3,4,5)	(2,3,4)	(10,11,12)	(4,5,6)
4	(15,16,17)	(5,6,7)	(3,4,5)	(14,15,16)	(2,3,4)
5	(17,18,19)	(3,4,5)	(5,6,7)	(13,14,15)	(5,6,7)

Table 3. Average high ranking for the data set

	Machine 1			Machine 2	
i	p'_{i1}	q'_{i1}	$T'_{i1\to 2}$	p'_{i2}	q'_{i2}
1	32/3	8/3	14/3	32/3	8/3
2	41/3	17/3	17/3	38/3	14/3
3	44/3	14/3	11/3	35/3	17/3
4	50/3	20/3	14/3	47/3	11/3
5	56/3	14/3	20/3	44/3	20/3

Find the AHR of fuzzy flow time as per step 2 of algorithm (Table 4).

Table 4. Average high ranking of fuzzy flow time for data set

	Machine 1		Machine 2
i	p''_{i1}	$T'_{i1\to 2}$	p''_{i2}
1	24/3	14/3	24/3
2	27/3	17/3	21/3
3	27/3	11/3	21/3
4	39/3	14/3	27/3
5	36/3	20/3	30/3

Find the processing time of jobs on fictitious machines *G* and *H* as per step 3 of algorithm (Table 5).

Table 5. Processing time of fictitious machines *G* and *H*

i	G_{i1}	H_{i2}
1	38/3	38/3
2	44/3	38/3
3	38/3	32/3
4	53/3	41/3
5	56/3	50/3

As per step 4 of the algorithm, the processing times of equivalent job block β = (3,4) using Maggu and Dass[15] criteria is given by,

$G_{\beta 1} = 38/3 + 53/3 - 32/3 = 59/3$

and $H_{\beta 2} = 32/3 + 41/3 - 32/3 = 41/3$

The new reduced problem as per step 5 of algorithm is shown (Table 6).

Table 6. Reduced problem of data set

i	G_{i1}	H_{i2}
1	38/3	38/3
2	44/3	38/3
β	59/3	41/3
5	56/3	50/3

Using Johnson's two machine algorithm[2] for minimum elapsed time using step 6, the optimal sequence is

$\sigma_1 = 1 - 5 - \beta - 2 = 1 - 5 - 3 - 4 - 2$

As per step 7, the processing time of first job on machine 1 for sequence σ_1 is = 38/3, .i.e.

$\alpha = 38/3$.

The rest optimal sequences for minimizing rental cost as per step 8 are

$\sigma_2 = 5 - 1 - 3 - 4 - 2$, $\sigma_3 = 3 - 1 - 5 - 4 - 2$, $\sigma_4 = 4 - 1 - 5 - 3 - 2$, $\sigma_5 = 2 - 1 - 5 - 3 - 4$

The flow tables for sequences so obtained above i.e. σ_1, σ_2 and σ_5 having job block (3, 4) are as shown (Table 7, Table 8, Table 9).

For $\sigma_1 = 1 - 5 - 3 - 4 - 2$

Table 7. Flow table for sequence σ_1

	Machine 1	Machine 2
i	FLOW IN – FLOW OUT	FLOW IN – FLOW OUT
1	(0,0,0) – (9,10,11)	(12,14,16) –(21,24,27)
5	(10,12,14)– (27,30,33)	(32,36,40) –(45,50,55)
3	(30,34,38) –(43,48,53)	(50,56,62) –(60,67,74)
4	(46,52,58) –(61,68,75)	(64,72,80) –(78,87,96)
2	(66,74,82) –(78,87,96)	(82,92,102) –(93,104,115)

Total elapsed time for sequence σ_1 on machine 1 = $t_{n1}(\sigma_1)$ = (78,87,96).

Total elapsed time for sequence σ_1 on machine 2 = $t_{n2}(\sigma_1)$ = (93,104,115).

Operation time of machine 2 for sequence $\sigma_1 = O_1(\sigma_1)$= (93,104,115) – (12,14,16) = (81,90,99).

with defuzzified value as 96 units.

For $\sigma_2 = 5 - 1 - 3 - 4 - 2$

Table 8. Flow table for sequence σ_2

	Machine 1	Machine 2
i	FLOW IN – FLOW OUT	FLOW IN – FLOW OUT
5	(0,0,0) – (17,18,19)	(22,24,26) – (35,38,41)
1	(20,22,24) – (29,32,35)	(40,44,48) – (49,54,59)
3	(30,34,38) – (43,48,53)	(50,56,62) – (60,67,74)
4	(46,52,58) – (61,68,75)	(64,72,80) – (78,87,96)
2	(66,74,82) – (78,87,96)	(82,92,102) – (93,104,115)

Total elapsed time for sequence $\sigma_2$on machine 1 = $t_{n1}(\sigma_2)$ = (78,87,96).

Total elapsed time for sequence $\sigma_2$on machine 2 = $t_{n2}(\sigma_2)$ = (93,104,115).

Operation time of machine 2 for sequence $\sigma_2 = O_2(\sigma_2)$= (93,104,115)– (22,24,26) = (71,80,89).

with defuzzified value as 86 units.

For $\sigma_5 = 2 - 1 - 5 - 3 - 4$

Table 9. Flow table for sequence σ_5

	Machine 1	Machine 2
i	FLOW IN – FLOW OUT	FLOW IN – FLOW OUT
2	(0,0,0) – (12,13,14)	(16,18,20) – (27,30,33)
1	(16,18,20) – (25,28,31)	(30,34,41) – (39,44,52)
5	(26,30,34) – (43,48,53)	(48,54,60) – (61,68,75)
3	(46,52,58) – (59,66,73)	(66,74,82) – (76,85,94)
4	(62,70,78) – (77,86,95)	(80,90,100) – (94,105,116)

Total elapsed time for sequence σ_5 on machine 1 = $t_{n1}(\sigma_5)$ = (77,86,95).

Total elapsed time for sequence σ_5 on machine 2 = $t_{n2}(\sigma_5)$ = (94,105,116).

Operation time of machine 2 for sequence $\sigma_5 = O_5(\sigma_5)$ = (94,105,116)– (16,18,20) = (78,87,96).

with defuzzified value as 93 units.

From the above calculations, the total minimum operation of machine 1 is (78,87,96) units and minimum operation of machine 2 is (71,80,89) units for the sequence σ_2. Therefore, the optimal sequence is $\sigma_2 = 5 - 1 - 3 - 4 - 2$ and the minimum rental cost $R_c(\sigma_2)$ = (78,87,96) × 11 +(71,80,89) × 7 = (858,957,1056) + (497,560,623) = (1355,1517,1679)

with defuzzified value as 1625 units.

8. Conclusion

In real life practical situation of production management the processing time of each job changes dynamically may due to human factor, operating error etc. which are generally assumed to be deterministic in nature. To incorporate the changing environment due to lack of complete information or uncertainty or vagueness, the concept of fuzzy processing time is introduced. This paper considers two stage fuzzy flow shop scheduling to minimize rental cost of machines under the constraints of job-block and fuzzy transportation time. The present work using triangular fuzzy number with job-block and transportation time can further be extended to trapezoidal fuzzy numbers with different parameters like breakdown.

9. References

1. Baker KR. Introduction to Sequencing and Scheduling. Wiley New York; 1974.
2. Johnson SM. Optimal two and three stage production schedule with set up times included. Naval Research Logistics Quarterly. 1954; 1(1):61–8.
3. Zadeh LA. Fuzzy sets. Information and Control. 1965; 8(3):338–53.
4. Sanuja P, Xueyan S. A new approach to two machine flow shop problem with uncertain processing time. Optimization and Engineering. 2006; 7(3):329–43.
5. McCahon CS, Lee ES. Job sequencing with fuzzy processing times. Computer and Mathematics with Applications. 1990; 19(7):31–41.
6. Ishibuchi H, Murata T, Lee KH. Formulation of fuzzy flow shop scheduling problem with fuzzy processing time. Proceedings of IEEE International Conference on Fuzzy System; 1996. p. 199–205.
7. Singh TP, Gupta D. Minimizing rental cost in two stage flow shop, the processing time associated with probabilities including job block. Reflections de ERA. 2006; 1(2):107–20.
8. Gupta D, Sharma S. Minimizing rental cost under specified rental situation in two stage flow shop, the processing time associated with probabilities including breakdown interval and job-block criteria. European Journal of Business and Management. 2011; 2(3):85–103.
9. Gupta D, Singla P, Bala S. Two stage flow shop scheduling problem including transportation time and weightage of jobs with branch and bound method. International Journal of Applied Operational Research. 2013; 3(4):1–6.
10. Gupta D, Sharma S, Aggarwal S. Flow shop scheduling on two machines with setup time and single transport facility under fuzzy environment. Opsearch. 2013; 50 (1):14–23.
11. Gupta D, Aggarwal S, Sharma S. A fuzzy logic based approach to minimize rental cost of machines for specially structured three stage flow shop scheduling. Advances in Applied Science Research. 2012; 3(2):1071–6.
12. Gupta JND. Optimal schedules for special structure flow shops. Naval Research Logistics Quarterly. 1975; 26(2):255–69.
13. Marin L, Roberto T. Fuzzy scheduling with application to real time systems. Fuzzy Sets and Systems. 2001; 121(3): 523–35.
14. Bagga PC. Sequencing in a rental situation. Journal of Canadian Operation Research Society. 1969; 7:152–3.
15. Maggu PL, Das G. Equivalent jobs for job block in job scheduling. Opsearch. 1977; 14(4):277–81.
16. Yager RR. A procedure for ordering fuzzy subsets of the unit interval. Information Sciences. 1981; 24:143–61.