Indian Journal of Science and Technology, Vol 9(42), DOI: 10.17485/ijst/2016/v9i42/104215, November 2016 ISSN (Print) : 0974-6846
ISSN (Online) : 0974-5645

Analysis of Options of Cooperative Processing of Measurements in Long-Range Multistatic Radar System

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Abstract

Background/Objectives: The study considers the options of cooperative processing of long-range measurements in multistatic radio-technical systems. The information on the sum of distances improves the precision of the estimated range. **Method:** The investigations employ simulation and statistical models describing the techniques of cooperative processing of the radar information for the case of three-station radio-technical system. **Findings:** The calculated range values include all measurements obtained by the multistatic system with their relevant weights. The investigations show the effects produced by the selected number of stations and by the precision of the original measurements on the formation of the resulting estimations under the conditions of different initial data that help achieve the preset precision within shorter intervals without the limitations of the object movement hypothesis. The new results of this study are represented by the equations that describe the slant distances in the course of cooperative processing of measurements with different types of the correlation matrix of the errors and for different options of data processing. It has been proven that the suggested option of measurement processing possesses high informational stability against any anomalous measurements. **Applications/Improvements:** The results of this study will be helpful for upgrading the existing radio navigation systems and for developing justified requirements to any prospective research studies in this area of investigation.

Keywords: Cumulative Long-Range, Cooperative Processing, Long-Range, Least Square Method, Multistatic, Root-Mean-Square Error

1. Introduction

Improving the precision of the measurements of the airborne object coordinates is one of the important problems that are solved by radio-technical systems of different types and designs. The ability to locate the object precisely is an important characteristic of radar systems that directly affects the quality of the expected functions of the system[1,2] Applying the trajectory parameter filtration algorithms makes it possible to achieve the required precision of coordinate localization. However, the overwhelming majority of such algorithms are limited in their choice of the object movement hypotheses; they take some certain time to achieve the preset precision value, which in some cases cannot be accepted. One of the studies[3] considers the possibility to create a network of radar systems for locating the targets, other works[4-6] analyze the multi-static radar system with several receivers and one transmitter and the effects produced by the locations of the stations on the precision of the object range measurements. Several studies[7-10] describe the options of determining the coordinates of the targets by the multistatic radar systems together with the equations for identifying the errors of the coordinates. One more investigation[11] highlights that multistatic radar system has several advantages compared with other present-day systems in use. Exemplified by the radar system that consists of one active and two passive stations, the opportunities

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for enhancing the amount of the signal information and for improving the characteristics of object detection have been illustrated. Besides, a part of the receiving stations in the system can be passive which makes their implementation much simpler, and also makes this system masked in a number of practical applications. One of the studies[12] describes the option of processing the data received from several pairs of bistatic stations in the passive radar multistatic system. The experimental results have shown higher probability of localizing the object and provided the equations for estimating the coordinates. Another work[13] has been dedicated to the analysis of the multistatic Doppler radar system that consists of one transmitter and three receivers that determine the space coordinates of the objects. One of the investigations[14] considers the multistatic system consisting of bistatic locators that measure the distance along the line that connects the transmitting station with the target and with the receiving station, and that also determine azimuth angles and the object location. The desired coordinates of the object are determined by the weighted method of least squares; TDOA measurement error estimation has been provided. It has been shown that the root-mean square error of localizing the coordinates is reduced considerably in the four-station system as compared to bistatic system. Special article[15] has been dedicated to the problem of measuring the movement trajectories of the ground targets assisted by one receiving and several transmitting stations that form the multi-element antenna array. It has been demonstrated that the suggested space-time processing makes it possible to undertake the procedures of localizing the object and also to take measurements of angular coordinates, whereas the rectangular coordinates are determined by applying the triangulation method. The results of simulation and statistical modeling performed for the purposes of estimating the precision of determining the rectangular coordinates have been provided.

Another article[16] studies the problem of calculating the Cramer-Rao lower bound for the bistatic radar system. It has been shown that the precision of the coordinate estimation depends on the system formation, on angular coordinates of the object, on the length of the baseline, on the distance between the target and the receiver. The procedure of selecting the optimal bistatic channel (or a set of channels) for localizing the coordinates of the target with best possible precision has been carried out.

One more study[17] considers the multistatic location system that consists of one transmitting and three receiving stations. Localization of the coordinates of the object is performed applying the suggested time difference method taking into account the fact that the transmitter is airborne and the receiving stations are on the ground. Another investigative article[18] undertakes the analysis of the multi-sensor radar in the context of sea surface control. The system consists of the receiver that uses the signal transmitted by the transducers which can be represented by UMTS base station and by FM radio stations.

Several works[19,20] prove that cooperative processing of location information in multistatic radio-technical systems ensures the highest possible precision of determining the distance and, consequently, the space coordinates of the targets; however, the effects produced by the correlation matrix of the errors on the precision of the resulting measurements have not been properly studied.

Applying the principles of multistatic radar localization with the spaced positioning of the radar stations provides the qualitatively new solutions to the problems of detecting and determining the coordinates of the targets. The principal idea of multistatic radar detection and ranging is that the information contained in the space characteristics of the electromagnetic field is used more efficiently (as compared to the conventional monostatic radio location tools).

The nonostatic radar recovers the information from a single narrow area of the field that corresponds to the aperture of the receiving antenna. Multistatic radio detection and ranging system obtains information from several spaced areas of the scattered field of the target (or of the signal source radiation field) that helps increase the information value, interference immunity and a number of other important characteristics.

Basic advantage of multistatic radio detection and ranging systems is represented by the possibility to create the effective area with required configuration taking into account the expected radio location environment. As compared to the monostatic radar stations the additional parameters that predetermine the active area of a multistatic radar station are represented by the geometric factor of the system (mutual location of the stations and the target) and the algorithms of cooperative data processing. This fact makes it possible to enhance the effective area to the preset directions. Multistatic radio detection and ranging systems equipped with movable elements possess the possibility of flexible and purposeful deformation of the detection area.

It is obvious that when any number of transmitting and (or) receiving stations are added to a monostatic radar the overall power of the system increases. Multistatic radar system also possesses other extra advantages in terms of energy. First of all, the existing power-related advantage is achieved due to the cooperative reception of the signals when the radiation energy of each of the transmitting stations is used by all receiving stations.

When the stations are properly spaced, the fluctuations of echo-signals in different receiving stations (or echo-signals that are created as a result of radiating the target by different transmitting stations) are statically independent. Smoothing the fluctuations in the course of combining the information can be of some extra power-related benefit, especially if the targets are to be detected with high precision. This advantage can also be achieved in multistatic radar system with a standalone reception station and even in case of unifying the radar stations that operate at different frequencies.

Multistatic property in radio detection and ranging is understood differently: a) based on the total number of the stations; b) based on the number of transmitting stations; c) based on the number of receiving positions. When the bistatic angle is close to 180^{o} (the angle between the directions from the target to the transmitting and to receiving stations), the intensity of secondary radiation increases considerably even in cases with the targets that feature specifically low radar visibility. The system of bistatic radar stations as a chain can serve as some type of barrier that prevents the targets from being missed. The options of constructing multistatic radar systems can differ by several parameters: the degree of autonomy of each system node; mutual node positioning accuracy; the level of cooperation of the information obtained by these positions. Mutual location of the stations is considered unfixed if at least one of them is represented by a movable station.

Along with completely autonomous operations of the equipment at the stations partially autonomous and cooperative operations are also possible. Thus, the autonomy can cover only receiving the information on the target while switching, and mode selection of the basic part of the equipment is done at the control room that can be combined with one of the stations. The cooperative nature of signal reception implies the use of the secondary radiation of the target at different stations that is obtained by affecting the target from only one of the positions which considerably enhances the opportunities for radar surveillance. Cooperative nature of radiating the probing signals includes the practice of emitting the radiation from different stations in sequence or almost in parallel over time. The simplest cooperativeness of the reception is represented by combining the information on the detected or expected targets that comes from one of the stations.[2] By covering the gaps of the radar field this combination makes it possible, in case when the surveillance zones of some of the radar stations overlap, to improve the precision of coordinate measurements primarily due to comparing the sufficiently precise values of the delay time. It is also possible to combine more extensive and larger information on the parameters of the emitted video-frequency or high-frequency voltages of the receivers (quadrature-phase components or the amplitudes and phases of those voltages). The full speed vector can be found not only based on the results of trajectory processing, but in some cases and to the higher degree of precision, based on the Doppler effects in different receiving points. Multistatic radio detection and ranging systems enhance the opportunities for adaptation to the operational environment, as well as the opportunities for classifying the targets. Cooperative reception of the proper radiation of the targets, if it is arranged for at least two stations that are removed to some certain base, improves the localizing capabilities. Identifying the degree of similarity (correlation) of the received fluctuations and combining them with different mutual delays in such correlation-basic system (subsystem), it becomes possible to determine with an acceptable degree of precision the location lines of the targets as hyperbolas on the plane or on the surface of the station as the hyperboloids of spatial rotation that correspond to the differences between the distances. Thus, applying the simplest type of cooperative reception and combining the information at the level of the signals (especially, high-frequency signals), the capabilities of multistatic radar detection can be improved considerably.

Thus, multistatic radio detection and ranging system should secure the functions as follows:

1. Detection of airborne targets by randomly located receiving and transmitting stations with the preset parameters of quality.
2. Control over the airspace area of any required configuration with the preset overlapping coefficient.
3. Detecting and tracing the preset number of targets.

4. Measuring the maximal quantitative composition of the vector of phase coordinates of airborne targets with the preset precision and within the shortest possible time.
5. Implementing potential capabilities of the system for providing the information to the recipients taking into account the fast changing situation in the air.
6. Ensuring the functions of the system or of some of its stations when some of the measurements are lost.

By contrast to monostatic systems, multistatic radiotechnical systems can solve a wide spectrum of specific problems. Some of them will be discussed below. It is obvious that the real significance of that or another advantage depends on the application area of the multistatic radio detection and ranging system and on the requirements it is expected to meet. The degree of revealing the advantages differs for different types of multistatic radar system, as has been considered in much detail in some studies.[1,2] These advantages are as follows: increased probability of detecting the targets, possibility to create the effective area of the required configuration taking into account the expected radio location environment; energy-related advantages, especially in the process of cooperative reception of signals when the radiation energy of each transmitting station is used by all receiving stations; high-precision measurements of the spatial location of the target; possibility to alter the vector of speed and acceleration of the target applying the Doppler method; increased resolution; larger amount of "signal" information; improved protection from both active and passive interference; better durability of the elements of the system.

2. Methods

2.1 Applying the Least Square Method for Cooperative Processing of Measurements in Multistatic Radio-Technical System

Figure 1 shows the geometry of the multistatic radiotechnical system featuring cooperative processing of the long-range measurements and consisting of N reception-transmission stations each of which is capable of receiving its own signals as well as the signals radiated by other stations of the system. Generally, in this system it is possible to obtain two groups of measurements: N slant

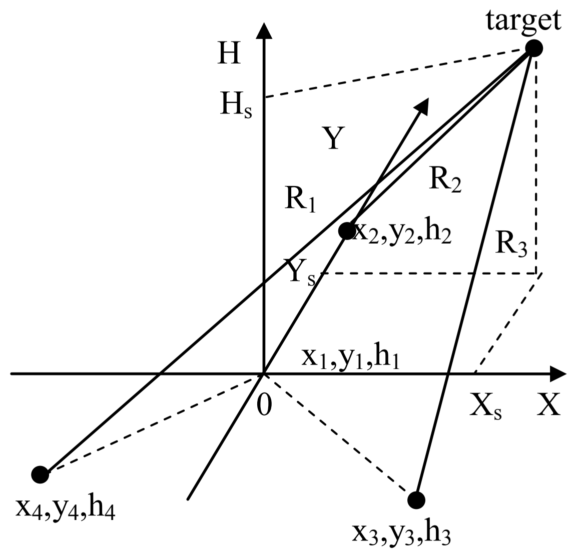

Figure 1. Geometry of multistatic radar system.

distances - $\hat{R}_1, \hat{R}_2 \ldots \hat{R}_N$ and $N(N\text{-}1)$ sums of distances - $\hat{R}_{\Sigma 12}, \hat{R}_{\Sigma 21} \ldots \hat{R}_{\Sigma N(N-1)}$.

It seems advisable that the redundant measurements obtained in this manner should be used for the purposes of improving the precision of the desired values. It is known that the measurements that are redundant relative to the number of the unknown parameters prevent any gross errors, enable obtaining more precise results and help estimate the errors of both separate measurements and final results.

Within the framework of the theory of redundant measurements the conditions of the measuring system at discrete moments in time are described by the system of linear and non-linear equations that interconnect the input and output values as well as the parameters of the known type of function of the measuring channel transformation. Thereat, not only the errors of the measuring results with generally non-linear and unstable function of transformation (sensor, measuring transformer or measuring channel in general) decrease automatically, but also so do the errors of determining the current values of the transformation function parameters.

Investigating the ways of improving the accuracy of estimating the unknown parameters is of special interest in case when they are measured simultaneously in the system of the spaced stations of the multistatic detection and ranging station.

The physical sense of the specified measurements that are obtained independently makes it possible to formulate a system of N^2 non-liner algebraic equations for N unknown distance estimations which at $N \geq 3$ represents a considerably redundant system of equations[19-22]:

$$\begin{cases} \begin{cases} \hat{R}_1 = 1 \cdot R_1 + 0 \cdot R_2 + 0 \cdot R_3 + \ldots + 0 \cdot R_N \\ \hat{R}_2 = 0 \cdot \widetilde{R}_1 + 1 \cdot \widetilde{R}_2 + 0 \cdot \widetilde{R}_3 + \ldots + 0 \cdot \widetilde{R}_N \\ \vdots \\ \hat{R}_N = 0 \cdot R_1 + 0 \cdot R_2 + 0 \cdot R_3 + \ldots + 1 \cdot R_N \end{cases} \\ \begin{cases} \hat{R}_{\Sigma 12} = 1 \cdot R_1 + 1 \cdot R_2 + 0 \cdot R_3 + \ldots + 0 \cdot R_N \\ \hat{R}_{\Sigma 21} = 1 \cdot R_1 + 1 \cdot R_2 + 0 \cdot R_3 + \ldots + 0 \cdot R_N \\ \hat{R}_{\Sigma 13} = 1 \cdot R_1 + 0 \cdot R_2 + 1 \cdot R_3 + \ldots + 0 \cdot R_N \\ \hat{R}_{\Sigma 31} = 1 \cdot R_1 + 0 \cdot R_2 + 1 \cdot R_3 + \ldots + 0 \cdot R_N \\ \vdots \\ \hat{R}_{\Sigma N} = 0 \cdot R_1 + 0 \cdot R_2 + 0 \cdot R_3 + \ldots + 1 \cdot R_N \end{cases} \end{cases} \quad (1)$$

In the matrix notation the system of equations (1) will read as follows:

$$\hat{H} = A\widetilde{S}, \quad (2)$$

where: $\hat{H}^T = \left\| \hat{R}_1 \hat{R}_2 \ldots \hat{R}_N \hat{R}_{\Sigma 12} \hat{R}_{\Sigma 21} \hat{R}_{\Sigma 13} \ldots \hat{R}_{\Sigma N} \right\|$ – matrix (vector-line) of the original measurements with dimension of $1 \times N^2$;

A – matrix of the quotients with the unknown values with dimension of $N^2 \times N$, which values are equal to one if such unknown values are present in this equation, and are equal to zero in the opposite case;

$\widetilde{S}^T = \left\| \widetilde{R}_1 \widetilde{R}_2 \ldots \widetilde{R}_N \right\|$ – matrix (vector-line) of the unknown estimations of distances with dimension of $1 \times N$.

Solving the vector-matrix equation (2) and applying the known least square method,[19,20] obtain the following:

$$\widetilde{S} = \left[\left(A^T \Lambda W^{-1} A \right)^{-1} A^T \Lambda W^{-1} \right] \hat{H}, \quad (3)$$

where

$$W = \begin{Vmatrix} \sigma_{R1}^2 & \cdots & \cdots & \cdots & \cdots & \cdots \\ \vdots & \sigma_{R2}^2 & & & & \vdots \\ \vdots & & \ddots & & & \vdots \\ \vdots & & & \sigma_{R\Sigma 12}^2 & & \vdots \\ \vdots & & & & \ddots & \vdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \sigma_{R\Sigma_{N^2}}^2 \end{Vmatrix}, \quad (4)$$

Matrix of the precision of the original measurements with dimension of $N^2 \times N^2$, whose principal diagonal contains the dispersions (expected mean squares) of the errors of distance measurements and sums of distances, and the connotations reflect the possible correlations between them.

$$\Lambda = \begin{Vmatrix} j & 0 & 0 & 0 \\ 0 & j & 0 & 0 \\ 0 & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & j \end{Vmatrix}, \quad (5)$$

Variation diagonal matrix of the quotients with dimension of $N^2 \times N^2$, whose diagonal elements are equal to one if the corresponding measurements, are involved in the cooperative processing, and are equal to zero in the opposite case.

Assuming that the dispersions of the errors of the original measurements of distances and sums of distances are equal, i.e. $\sigma_{R1}^2 = \sigma_{R2}^2 = \ldots = \sigma_{RN}^2 = \sigma_{R\Sigma 12}^2 = \sigma_{R\Sigma 21}^2 = \ldots = \sigma_{R\Sigma_{N^2}}^2 = \sigma_{R0}^2$ (the sign ^ above the original measurements at the relevant dispersions has been omitted to simplify the notation), the covariant matrix of the errors of determining the unknown estimations of the distances can be presented as follows:

$$K_{\widetilde{S}} = \left(A^T A \right)^{-1} \sigma_{R0}^2. \quad (6)$$

For two- and three-station systems the matrices A, vectors $\hat{H}^{\grave{O}}$, $\widetilde{S}^{\grave{O}}$ at $N=2$ and $N=3$ are respectively equal to the following:

$$A^T = \begin{Vmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{Vmatrix},$$

$$A^T = \begin{Vmatrix} 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 \end{Vmatrix}, \quad (7)$$

$$H^T = \left\| \hat{R}_1 \hat{R}_2 \hat{R}_{\Sigma 12} \hat{R}_{\Sigma 21} \right\|, \; H^T = \left\| \hat{R}_1 \hat{R}_2 \hat{R}_3 \hat{R}_{\Sigma 12} \hat{R}_{\Sigma 21} \hat{R}_{\Sigma 13} \hat{R}_{\Sigma 31} \hat{R}_{\Sigma 23} \hat{R}_{\Sigma 32} \right\|, \quad (8)$$

$$\widetilde{S}^T = \left\| \widetilde{R}_1 \widetilde{R}_2 \right\|, \; \widetilde{S}^T = \left\| \widetilde{R}_1 \widetilde{R}_2 \widetilde{R}_3 \right\|. \quad (9)$$

2.2 Analysis of the Effects Produced by the Precision of the Original Long-Range Measurements on the Formation of the Resulting Distance Measurements

If all measurements participate in the cooperative processing then directly based on (3) and taking into account (5), (7) and (8) it is possible to obtain the following developed formulae for the estimations of the distances relative to each of the stations for the case when N=2 (two-station system), assuming that in (6) the dispersions of coordinate measurements are ones:

$$\tilde{R}_1 = \frac{1}{5}\left(3\hat{R}_1 - 2\hat{R}_2 + \hat{R}_{\Sigma 12} + \hat{R}_{\Sigma 21}\right),$$
$$\tilde{R}_2 = \frac{1}{5}\left(3\hat{R}_2 - 2\hat{R}_1 + \hat{R}_{\Sigma 12} + \hat{R}_{\Sigma 21}\right), \quad (10)$$

And also assuming that the values of summary measurement dispersions are different from ones and are equal to, for instance, 4 and 8 respectively:

$$\tilde{R}_1 = \frac{1}{8}\left(6\hat{R}_1 - 2\hat{R}_2 + \hat{R}_{\Sigma 12} + \hat{R}_{\Sigma 21}\right),$$
$$\tilde{R}_2 = \frac{1}{8}\left(6\hat{R}_2 - 2\hat{R}_1 + \hat{R}_{\Sigma 12} + \hat{R}_{\Sigma 21}\right), \quad (11)$$

$$\tilde{R}_1 = \frac{1}{12}\left(10\hat{R}_1 - 2\hat{R}_2 + \hat{R}_{\Sigma 12} + \hat{R}_{\Sigma 21}\right),$$
$$\tilde{R}_2 = \frac{1}{12}\left(10\hat{R}_2 - 2\hat{R}_1 + \hat{R}_{\Sigma 12} + \hat{R}_{\Sigma 21}\right), \quad (12)$$

And also taking into account definite values of $\sigma_{R1}^2 = \sigma_{R2}^2 = \sigma_R^2$ и $\sigma_{R\Sigma 12}^2 = \sigma_{R21}^2 = \sigma_{R\Sigma}^2$

$$\tilde{R}_1 = \hat{R}_1 + \frac{\sigma_R^2}{4\sigma_R^2 + \sigma_{R\Sigma}^2}\left(\hat{R}_{\Sigma 12} + \hat{R}_{\Sigma 21} - 2\left(\hat{R}_1 + \hat{R}_2\right)\right),$$
$$\tilde{R}_2 = \hat{R}_2 + \frac{\sigma_R^2}{4\sigma_R^2 + \sigma_{R\Sigma}^2}\left(\hat{R}_{\Sigma 12} + \hat{R}_{\Sigma 21} - 2\left(\hat{R}_1 + \hat{R}_2\right)\right). \quad (13)$$

The dispersions of distance measurements at the cooperative processing for (9) given (6) at different dispersions of the measurements of the total range will be as follows:

$$\sigma_{RK}^2 = diag\left(A^T W^{-1} A\right)^{-1} = \frac{1}{5}\begin{Vmatrix} 3 & -2 \\ -2 & 3 \end{Vmatrix}\sigma_R^2, \quad (14)$$

$$\sigma_{RK}^2 = diag\left(A^T W^{-1} A\right)^{-1} = \frac{1}{4}\begin{Vmatrix} 3 & -1 \\ -1 & 3 \end{Vmatrix}\sigma_R^2, \quad (15)$$

$$\sigma_{RK}^2 = diag\left(A^T W^{-1} A\right)^{-1} = \frac{1}{6}\begin{Vmatrix} 5 & -1 \\ -1 & 5 \end{Vmatrix}\sigma_R^2, \quad (16)$$

$$\sigma_{RK}^2 = diag\left(A^T W^{-1} A\right)^{-1} = \frac{1}{4\sigma_R^2 + \sigma_{R\Sigma}^2} \times$$
$$\times \begin{Vmatrix} 2\sigma_R^4 + \sigma_R^2\sigma_{R\Sigma}^2 & -2\sigma_R^4 \\ -2\sigma_R^4 & 2\sigma_R^4 + \sigma_R^2\sigma_{R\Sigma}^2 \end{Vmatrix}\sigma_R^2. \quad (17)$$

In the same manner for the case when N=3 (three-station system) obtain the following:

$$\tilde{R}_1 = \frac{1}{27}\left(7\hat{R}_1 - 2\hat{R}_2 - 2\hat{R}_3 + 5\hat{R}_{\Sigma 12} + 5\hat{R}_{\Sigma 21} + \right.$$
$$\left. + 5\hat{R}_{\Sigma 13} + 5\hat{R}_{\Sigma 31} - 4\hat{R}_{\Sigma 23} - 4\hat{R}_{\Sigma 32}\right),$$
$$\tilde{R}_2 = \frac{1}{27}\left(-2\hat{R}_1 + 7\hat{R}_2 - 2\hat{R}_3 + 5\hat{R}_{\Sigma 12} + \right.$$
$$\left. + 5\hat{R}_{\Sigma 21} - 4\hat{R}_{\Sigma 13} - 4\hat{R}_{\Sigma 31} + 5\hat{R}_{\Sigma 23} + 5\hat{R}_{\Sigma 32}\right),$$
$$\tilde{R}_3 = \frac{1}{27}\left(-2\hat{R}_1 - 2\hat{R}_2 + 7\hat{R}_3 - 4\hat{R}_{\Sigma 12} - \right.$$
$$\left. - 4\hat{R}_{\Sigma 21} + 5\hat{R}_{\Sigma 13} + 5\hat{R}_{\Sigma 31} + 5\hat{R}_{\Sigma 23} + 5\hat{R}_{\Sigma 32}\right)$$
$$\tilde{R}_1 = \frac{1}{18}\left(10\hat{R}_1 - 2\hat{R}_2 - 2\hat{R}_3 + 2\hat{R}_{\Sigma 12} + 2\hat{R}_{\Sigma 21} + \right.$$
$$\left. + 2\hat{R}_{\Sigma 13} + 2\hat{R}_{\Sigma 31} - \hat{R}_{\Sigma 23} - \hat{R}_{\Sigma 32}\right), \quad (18)$$

$$\tilde{R}_2 = \frac{1}{18}\left(-2\hat{R}_1 + 10\hat{R}_2 - 2\hat{R}_3 + 2\hat{R}_{\Sigma 12} + 2\hat{R}_{\Sigma 21} - \right.$$
$$\left. - \hat{R}_{\Sigma 13} - \hat{R}_{\Sigma 31} + 2\hat{R}_{\Sigma 23} + 2\hat{R}_{\Sigma 32}\right),$$
$$\tilde{R}_3 = \frac{1}{18}\left(-2\hat{R}_1 - 2\hat{R}_2 + 10\hat{R}_3 - \hat{R}_{\Sigma 12} - \hat{R}_{\Sigma 21} + \right.$$
$$\left. + 2\hat{R}_{\Sigma 13} + 2\hat{R}_{\Sigma 31} + 2\hat{R}_{\Sigma 23} + 2\hat{R}_{\Sigma 32}\right),$$
$$\tilde{R}_1 = \frac{1}{40}\left(28\hat{R}_1 - 4\hat{R}_2 - 4\hat{R}_3 + 3\hat{R}_{\Sigma 12} + 3\hat{R}_{\Sigma 21} + \right.$$
$$\left. + 3\hat{R}_{\Sigma 13} + 3\hat{R}_{\Sigma 31} - \hat{R}_{\Sigma 23} - \hat{R}_{\Sigma 32}\right), \quad (19)$$

$$\tilde{R}_2 = \frac{1}{40}\left(-4\hat{R}_1 + 28\hat{R}_2 - 4\hat{R}_3 + 3\hat{R}_{\Sigma 12} + 3\hat{R}_{\Sigma 21} - \right.$$
$$\left. - \hat{R}_{\Sigma 13} - \hat{R}_{\Sigma 31} + 3\hat{R}_{\Sigma 23} + 3\hat{R}_{\Sigma 32}\right)$$
$$\tilde{R}_3 = \frac{1}{40}\left(-4\hat{R}_1 - 4\hat{R}_2 + 28\hat{R}_3 - \hat{R}_{\Sigma 12} - \hat{R}_{\Sigma 21} + \right.$$
$$\left. + 3\hat{R}_{\Sigma 13} + 3\hat{R}_{\Sigma 31} + 3\hat{R}_{\Sigma 23} + 3\hat{R}_{\Sigma 32}\right). \quad (20)$$

Taking into account the real values of the dispersions of the measured coordinates, obtain the equations for the distances as follows:

$$\begin{aligned}\tilde{R}_1 =& \frac{\hat{R}_1\left(\sigma_{R\Sigma}^4+6\sigma_R^2\sigma_{R\Sigma}^2\right)}{\left(2\sigma_R^2+\sigma_{R\Sigma}^2\right)\left(8\sigma_R^2+\sigma_{R\Sigma}^2\right)}-\frac{2\hat{R}_2\sigma_R^2\sigma_{R\Sigma}^2}{\left(2\sigma_R^2+\sigma_{R\Sigma}^2\right)\left(8\sigma_R^2+\sigma_{R\Sigma}^2\right)}\\ &-\frac{2\hat{R}_3\sigma_R^2\sigma_{R\Sigma}^2}{\left(2\sigma_R^2+\sigma_{R\Sigma}^2\right)\left(8\sigma_R^2+\sigma_{R\Sigma}^2\right)}+\frac{\left(4\sigma_R^4+\sigma_R^2\sigma_{R\Sigma}^2\right)\hat{R}_{\Sigma12}}{\left(2\sigma_R^2+\sigma_{R\Sigma}^2\right)\left(8\sigma_R^2+\sigma_{R\Sigma}^2\right)}+\\ &+\frac{\left(4\sigma_R^4+\sigma_R^2\sigma_{R\Sigma}^2\right)\hat{R}_{\Sigma21}}{\left(2\sigma_R^2+\sigma_{R\Sigma}^2\right)\left(8\sigma_R^2+\sigma_{R\Sigma}^2\right)}+\frac{\left(4\sigma_R^4+\sigma_R^2\sigma_{R\Sigma}^2\right)\hat{R}_{\Sigma13}}{\left(2\sigma_R^2+\sigma_{R\Sigma}^2\right)\left(8\sigma_R^2+\sigma_{R\Sigma}^2\right)}+\\ &+\frac{\left(4\sigma_R^4+\sigma_R^2\sigma_{R\Sigma}^2\right)\hat{R}_{\Sigma31}}{\left(2\sigma_R^2+\sigma_{R\Sigma}^2\right)\left(8\sigma_R^2+\sigma_{R\Sigma}^2\right)}-\frac{\left(4\sigma_R^4+\sigma_R^2\sigma_{R\Sigma}^2\right)\hat{R}_{\Sigma23}}{\left(2\sigma_R^2+\sigma_{R\Sigma}^2\right)\left(8\sigma_R^2+\sigma_{R\Sigma}^2\right)}\\ &-\frac{\left(4\sigma_R^4+\sigma_R^2\sigma_{R\Sigma}^2\right)\hat{R}_{\Sigma32}}{\left(2\sigma_R^2+\sigma_{R\Sigma}^2\right)\left(8\sigma_R^2+\sigma_{R\Sigma}^2\right)},\end{aligned} \quad (21)$$

$$\begin{aligned}\tilde{R}_2 =& -\frac{2\hat{R}_1\sigma_R^2\sigma_{R\Sigma}^2}{\left(2\sigma_R^2+\sigma_{R\Sigma}^2\right)\left(8\sigma_R^2+\sigma_{R\Sigma}^2\right)}+\frac{\hat{R}_2\left(\sigma_{R\Sigma}^4+6\sigma_R^2\sigma_{R\Sigma}^2\right)}{\left(2\sigma_R^2+\sigma_{R\Sigma}^2\right)\left(8\sigma_R^2+\sigma_{R\Sigma}^2\right)}-\\ &-\frac{2\hat{R}_3\sigma_R^2\sigma_{R\Sigma}^2}{\left(2\sigma_R^2+\sigma_{R\Sigma}^2\right)\left(8\sigma_R^2+\sigma_{R\Sigma}^2\right)}+\frac{\left(4\sigma_R^4+\sigma_R^2\sigma_{R\Sigma}^2\right)\hat{R}_{\Sigma12}}{\left(2\sigma_R^2+\sigma_{R\Sigma}^2\right)\left(8\sigma_R^2+\sigma_{R\Sigma}^2\right)}+\\ &+\frac{\left(4\sigma_R^4+\sigma_R^2\sigma_{R\Sigma}^2\right)\hat{R}_{\Sigma21}}{\left(2\sigma_R^2+\sigma_{R\Sigma}^2\right)\left(8\sigma_R^2+\sigma_{R\Sigma}^2\right)}-\frac{4\sigma_R^4\hat{R}_{\Sigma13}}{\left(2\sigma_R^2+\sigma_{R\Sigma}^2\right)\left(8\sigma_R^2+\sigma_{R\Sigma}^2\right)}-\\ &-\frac{4\sigma_R^4\hat{R}_{\Sigma31}}{\left(2\sigma_R^2+\sigma_{R\Sigma}^2\right)\left(8\sigma_R^2+\sigma_{R\Sigma}^2\right)}+\frac{\left(4\sigma_R^4+\sigma_R^2\sigma_{R\Sigma}^2\right)\hat{R}_{\Sigma23}}{\left(2\sigma_R^2+\sigma_{R\Sigma}^2\right)\left(8\sigma_R^2+\sigma_{R\Sigma}^2\right)}+\\ &+\frac{\left(4\sigma_R^4+\sigma_R^2\sigma_{R\Sigma}^2\right)\hat{R}_{\Sigma32}}{\left(2\sigma_R^2+\sigma_{R\Sigma}^2\right)\left(8\sigma_R^2+\sigma_{R\Sigma}^2\right)},\end{aligned}$$

$$\begin{aligned}\tilde{R}_3 =& -\frac{2\hat{R}_1\sigma_R^2\sigma_{R\Sigma}^2}{\left(2\sigma_R^2+\sigma_{R\Sigma}^2\right)\left(8\sigma_R^2+\sigma_{R\Sigma}^2\right)}-\frac{2\hat{R}_2\sigma_R^2\sigma_{R\Sigma}^2}{\left(2\sigma_R^2+\sigma_{R\Sigma}^2\right)\left(8\sigma_R^2+\sigma_{R\Sigma}^2\right)}\\ &+\frac{\hat{R}_3\left(\sigma_{R\Sigma}^4+6\sigma_R^2\sigma_{R\Sigma}^2\right)}{\left(2\sigma_R^2+\sigma_{R\Sigma}^2\right)\left(8\sigma_R^2+\sigma_{R\Sigma}^2\right)}-\frac{4\sigma_R^4\hat{R}_{\Sigma12}}{\left(2\sigma_R^2+\sigma_{R\Sigma}^2\right)\left(8\sigma_R^2+\sigma_{R\Sigma}^2\right)}-\\ &-\frac{4\sigma_R^4\hat{R}_{\Sigma21}}{\left(2\sigma_R^2+\sigma_{R\Sigma}^2\right)\left(8\sigma_R^2+\sigma_{R\Sigma}^2\right)}+\frac{\hat{R}_{\Sigma13}\left(4\sigma_R^4+\sigma_R^2\sigma_{R\Sigma}^2\right)}{\left(2\sigma_R^2+\sigma_{R\Sigma}^2\right)\left(8\sigma_R^2+\sigma_{R\Sigma}^2\right)}-\\ &+\frac{\hat{R}_{\Sigma31}\left(4\sigma_R^4+\sigma_R^2\sigma_{R\Sigma}^2\right)}{\left(2\sigma_R^2+\sigma_{R\Sigma}^2\right)\left(8\sigma_R^2+\sigma_{R\Sigma}^2\right)}+\frac{\left(4\sigma_R^4+\sigma_R^2\sigma_{R\Sigma}^2\right)\hat{R}_{\Sigma23}}{\left(2\sigma_R^2+\sigma_{R\Sigma}^2\right)\left(8\sigma_R^2+\sigma_{R\Sigma}^2\right)}+\\ &+\frac{\left(4\sigma_R^4+\sigma_R^2\sigma_{R\Sigma}^2\right)\hat{R}_{\Sigma32}}{\left(2\sigma_R^2+\sigma_{R\Sigma}^2\right)\left(8\sigma_R^2+\sigma_{R\Sigma}^2\right)}.\end{aligned}$$

2.3 Precision of Range Measurements in Case of Cooperative Processing

For three-station system the dispersions of distances at cooperative processing will be as follows: $\sigma_{RK}^2 = diag\left(A^T W^{-1} A\right)^{-1}$, then:

$$\sigma_{RK}^2 = \frac{1}{27}\begin{Vmatrix} 7 & -2 & -2 \\ -2 & 7 & -2 \\ -2 & -2 & 7 \end{Vmatrix}\sigma_R^2, \quad (22)$$

$$\sigma_{RK}^2 = \frac{1}{9}\begin{Vmatrix} 5 & -1 & -1 \\ -1 & 5 & -1 \\ -1 & -1 & 5 \end{Vmatrix}\sigma_R^2, \quad (23)$$

$$\sigma_{RK}^2 = \frac{1}{10}\begin{Vmatrix} 7 & -1 & -1 \\ -1 & 7 & -1 \\ -1 & -1 & 7 \end{Vmatrix}\sigma_R^2, \quad (24)$$

$$\begin{aligned}\sigma_{RK}^2 =& \frac{1}{16\sigma_R^4+10\sigma_R^2\sigma_{R\Sigma}^2+\sigma_{R\Sigma}^4}\times\\ &\times\begin{Vmatrix} 6\sigma_R^4\sigma_{R\Sigma}^2+\sigma_R^2\sigma_{R\Sigma}^4 & -2\sigma_R^4\sigma_{R\Sigma}^2 & -2\sigma_R^4\sigma_{R\Sigma}^2 \\ -2\sigma_R^4\sigma_{R\Sigma}^2 & 6\sigma_R^4\sigma_{R\Sigma}^2+\sigma_R^2\sigma_{R\Sigma}^4 & -2\sigma_R^4\sigma_{R\Sigma}^2 \\ -2\sigma_R^4\sigma_{R\Sigma}^2 & -2\sigma_R^4\sigma_{R\Sigma}^2 & 6\sigma_R^4\sigma_{R\Sigma}^2+\sigma_R^2\sigma_{R\Sigma}^4 \end{Vmatrix}\sigma_R^2\end{aligned} \quad (25)$$

Replacing the values of the dispersions of distance measurements and of sums of distances in (13) and (21) with their estimations, it becomes possible to obtain the optimal algorithm for estimating distances with the unknown dispersions of errors. At different correlations between the dispersions of distance measurements and of sums of distances it is possible to demonstrate that:

$$\lim_{\sigma_R\to\infty}\left(A^T W^{-1} A\right)^{-1} = \frac{1}{8}\begin{Vmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{Vmatrix}\sigma_{R\Sigma}^2,$$

$$\lim_{\sigma_{R\Sigma}\to\infty}\left(A^T W^{-1} A\right)^{-1} = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{Vmatrix}\sigma_R^2, \quad (26)$$

$$\lim_{\sigma_R\to 0}\left(A^T W^{-1} A\right)^{-1} = 0, \lim_{\sigma_{R\Sigma}\to 0}\left(A^T W^{-1} A\right)^{-1} = 0, \quad (27)$$

$$\lim_{\sigma_R\to\sigma_{R\Sigma}}\left(A^T W^{-1} A\right)^{-1} = \frac{1}{27}\begin{Vmatrix} 7 & -2 & -2 \\ -2 & 7 & -2 \\ -2 & -2 & 7 \end{Vmatrix}\sigma_{R\Sigma}^2,$$

$$\lim_{\sigma_{R\Sigma}\to\sigma_R}\left(A^T W^{-1} A\right)^{-1} = \frac{1}{27}\begin{Vmatrix} 7 & -2 & -2 \\ -2 & 7 & -2 \\ -2 & -2 & 7 \end{Vmatrix}\sigma_R^2. \quad (28)$$

Equations (26) – (28) reflect the specific features of the cooperative processing that are represented by the fact that the increased numbers of the dispersions of the distance measurement errors and of the sums of distances up to the limit values will not lead to any disturbances in the functionality of the options of the cooperative processing. Figure 2 presents the values of the root-mean-square errors of determining the coordinates when in matrix (4) there are correlated changes with the correlation factor of ρ=±1 (line 1 is related to N=2; line 2 is related to N=3).

3. Results of Modeling

The modeling was performed applying visual programming language VisSim designed for simulating dynamic systems. The geometry of the multistatic radio-technical system was the same as the one presented in Figure 1. The distance between the reception and transmission stations was assumed to be equal to 50 km. The distance to the object from the origin of coordinates was equal to 200 km, the period of information updates amounted to 1 sec. The errors of measurements of the long-range parameters were assumed to be redistributed according to the normal law with zero mathematical expectation.

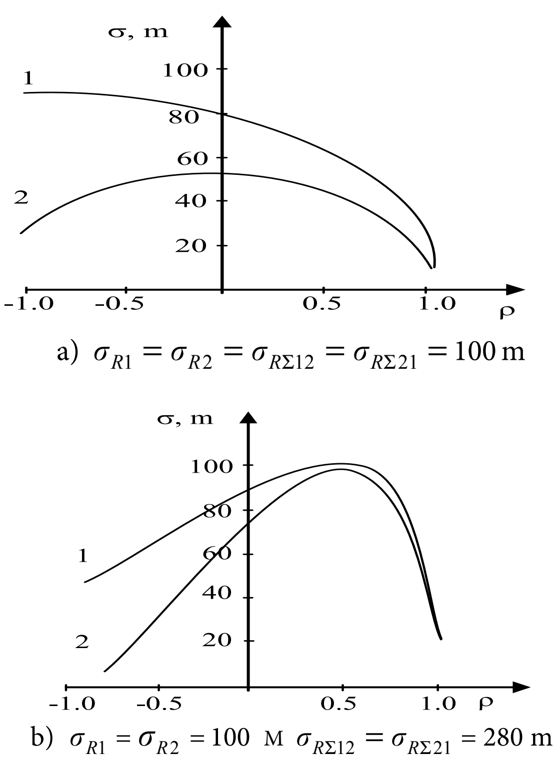

Figure 2. Mean-root-square error of range estimation.

Figures 3 and 4 show RMS errors of determining the coordinates calculated based on equations (21).

It is possible to improve the precision of determining the coordinates considerably by using M additional channels of reception or by performing S sequential measurements of the relevant parameters. In this case equations (4), (7), (8) upon accumulating S measurements in each channel for three-station system will be as follows:

$$W = \begin{Vmatrix} \sum_{i=1}^{S}\sigma_{R1}^2 & \cdots & \cdots & \cdots & \cdots & \cdots \\ \vdots & \sum_{i=1}^{S}\sigma_{RN}^2 & & & & \vdots \\ \vdots & & \ddots & & & \vdots \\ \vdots & & & \sum_{i=1}^{S}\sigma_{R\Sigma12}^2 & & \vdots \\ \vdots & & & & \ddots & \vdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \sum_{i=1}^{S}\sigma_{R\Sigma_{N^2}}^2 \end{Vmatrix}, \qquad (29)$$

$$A^T = \begin{Vmatrix} \sum_{i=1}^{S}1 & 0 & 0 & \sum_{i=1}^{S}1 & \sum_{i=1}^{S}1 & \sum_{i=1}^{S}1 & \sum_{i=1}^{S}1 & 0 & 0 \\ 0 & \sum_{i=1}^{S}1 & 0 & \sum_{i=1}^{S}1 & \sum_{i=1}^{S}1 & 0 & 0 & \sum_{i=1}^{S}1 & \sum_{i=1}^{S}1 \\ 0 & 0 & \sum_{i=1}^{S}1 & 0 & 0 & \sum_{i=1}^{S}1 & \sum_{i=1}^{S}1 & \sum_{i=1}^{S}1 & \sum_{i=1}^{S}1 \end{Vmatrix}, \qquad (30)$$

$$H^T = \left\Vert \sum_{i=1}^{S}\hat{R}_1 \sum_{i=1}^{S}\hat{R}_2 \sum_{i=1}^{S}\hat{R}_3 \sum_{i=1}^{S}\hat{R}_{\Sigma12} \sum_{i=1}^{S}\hat{R}_{\Sigma21} \sum_{i=1}^{S}\hat{R}_{\Sigma13} \sum_{i=1}^{S}\hat{R}_{\Sigma31} \sum_{i=1}^{S}\hat{R}_{\Sigma23} \sum_{i=1}^{S}\hat{R}_{\Sigma32} \right\Vert. \qquad (31)$$

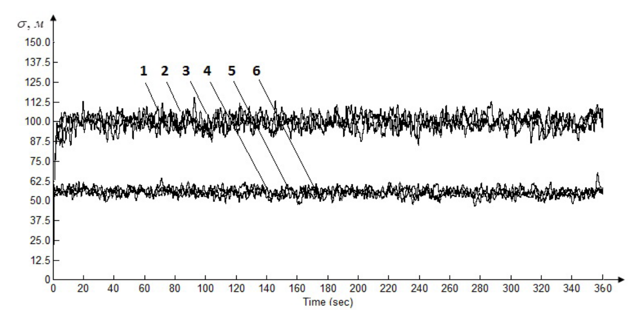

Figure 3. RMS error of estimating ranges with cooperative processing for the cases of equal observations with RMS errors of $\sigma_{R1} = \sigma_{R2} = \ldots \sigma_{R\Sigma12} = \sigma_{R\Sigma21} = \ldots \sigma_{R\Sigma23} = \sigma_R$ =100 m., (1-3 RMS errors of range measurements, 5-6 range measurements applying cooperative processing).

Figures (5) and (6) show RMS errors of determining distances, taking into account (29)-(31)

One of the studies[23] describes the equation for RMS error of determining the range in case when the measuring system was equipped with N sensors and with M independent channels of processing upon accumulating S measurements $\sigma_{RK} = \sigma_R \sqrt{\dfrac{4N-5}{(2N-3)(4N-3)SM}}$.

Thus, at M=1, S=1 and N=3 the processing can be performed with 9 measurements which will result in $\sigma_{RK} = 0.509\,\sigma_R$, i.e. almost two times better estimation precision. Analyzing the dependency $\sigma_{RK} = \sigma_R \sqrt{\dfrac{2(2S-1)}{S(S+1)}}$,[24] it is possible to draw the conclusion that to achieve the same precision in monostatic system 14 measurements would be required; thereat, the time balance of three-station system would be 14 times lower as compared to the monostatic one.

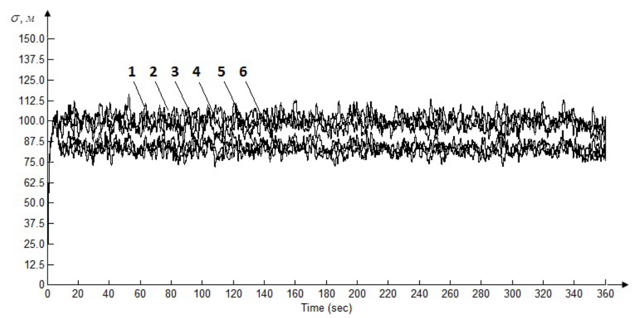

Figure 4. RMS error of estimating ranges with cooperative processing for the cases of equal observations with RMS errors of $\sigma_{R\Sigma 12} = \sigma_{R\Sigma 21} = \ldots \sigma_{R\Sigma 23}$=280 m (1-3 RMS errors of range measurements, 5-6 range measurements applying cooperative processing).

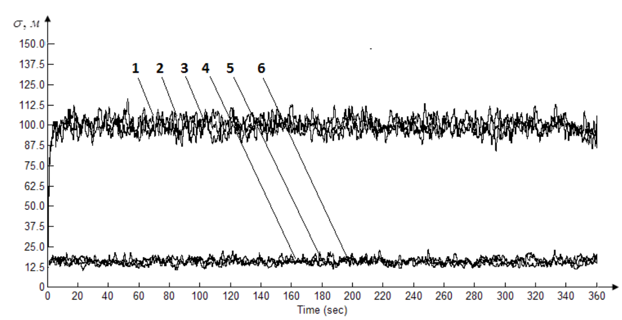

Figure 5. RMS error of estimating ranges with cooperative processing for the cases of equal observations with RMS errors of $\sigma_{R1} = \sigma_{R2} = \sigma_{R3} = \sigma_{R\Sigma 12} = \sigma_{R\Sigma 21} = \ldots \sigma_{R\Sigma 23}$ =100 m, (1-3 RMS errors of range measurements, 5-6 range measurements applying cooperative processing) upon accumulating 10 measurements.

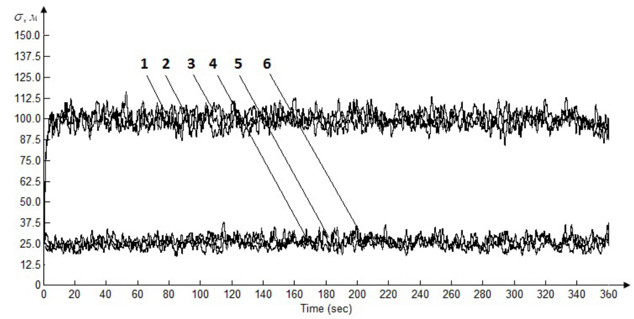

Figure 6. RMS error of estimating ranges with cooperative processing for the cases of equal observations with RMS errors of $\sigma_{R1} = \sigma_{R2} = \sigma_{R3} = \sigma_{R\Sigma 12} = \sigma_{R\Sigma 21} = \ldots \sigma_{R\Sigma 23}$ =280 m, (1-3 RMS errors of range measurements, 5-6 range measurements applying cooperative processing) upon accumulating 10 measurements.

4. Conclusion

1. The long-range radar system under consideration applies all measurements of distances and sums of distances to formulate the resulting estimations.
2. Increased dispersion of the measurements of the sums of distances decreases the precision of the resulting range estimations; thereat, the measurements with greater errors are assigned lower weights.
3. For optimal estimation of the dispersion in the covariant matrix of the errors, its real value should be aimed at, which implies the availability of the algorithm for formulating the estimations of the dispersions.
4. Data accumulation results in considerable improvement of the estimation precision on the one hand; however, on the other hand, it leads to the traditional trade-off between dynamic and fluctuation errors.
5. In the process of estimating the dispersions there is some "delay" in formulating the resulting measurement which is caused by the time spent on the dispersion estimation;
6. Increased number of the errors that occur in the process of measuring distance or the sums of distances, when they occur separately, does not interfere with the relevant options (algorithms) of processing.

5. Acknowledgements

The study has been developed with financial support from the Ministry of Education and Science of the Russian Federation in the framework of developing the applied scientific investigations under lot number 2014-14-579-0112: "Developing experimental prototype of multistatic standalone radio-technical fast deployed system of the aircraft ground infrastructure for rough field landing" (application code: 2014-14-579-0112-030) RFMEFI60714X0057.

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