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A Scheme for Analysis and Design of Analogue Circuits

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Abstract

Network theorems or circuit theorems are special aids, which can reduce the amount of effort involved in circuit analysis to a considerable degree. For linear networks, there exists interdependence between various network theorems. Thus, by careful analysis, one theorem can verify in terms of another theorem and it is advantageous for pedagogical reasons. A scheme for analogue circuit analysis is presented in this paper. Here piece-by-piece or part-by-part analyzing strategy is employed which can reduce the complexity associated with analyzing or biasing design of large analogue circuits. Fixator-norator pair plays the key role in this methodology. Hybrid equivalent circuit, an alternative for Thevenin's or Norton's equivalent model is realized with fixator-norator pair, and is seen to be more dynamic and flexible since it contains both voltage source and current source. Thevenin's theorem, compensation theorem and maximum power transfer theorem are analyzed in terms of fixator norator pairs, and the study indicates the importance and capabilities of fixator-norator pair and hence hybrid equivalent circuit in analogue circuit design and analysis.

Keywords: Compensation Theorem, Fixator-norator Pair, Hybrid Equivalent Circuit, Maximum Power Transfer Theorem, Thevenin's Theorem

1. Introduction

Analogue circuit design is an artwork where the skilled designers finds the interactions and inter relations between various design parameters. The design of analogue circuit is a two-step process. The AC performance design and the DC biasing design. In AC design, active devices are replaced with their linear equivalent models and here the circuit is treated entirely linear. The DC biasing design, where most of the difficulty in analogue design resides, is to get operating points fixed at the targeted critical values. As the circuit grows, the difficulty increases and in turn results in multiple or unstable operating points. The reason for this difficulty is that a traditional method treats the circuit as a whole with no separation between linear and nonlinear components. Moreover, DC sources are prefixed at certain locations of circuit and it results in reduced flexibility. Here we need to take a number of iterations to fix the operating points.

Network theorems or circuit theorems can simplify the effort in circuit analysis to a considerable degree. They are helpful to reduce a larger network containing a number of independent sources, linear dependent sources, linear resistors, capacitors and inductors to a smaller network and thus the steps involved in a targeted analysis gets reduced. Sometimes, verification of one theorem based on another theorem is advantageous[1,2] and it may give a shortcut to the required answer. Moreover, such verification results in better understanding about the so-called theorems and avoids wrong usage of equivalent models.

Thevenin's and Norton's equivalent models are the two popular linear circuit models that replace a linear circuit containing a number of sources and impedances into a source and single equivalent impedance. They are helpful for many circuit analysis operations like source transformation, DC analysis, frequency analysis etc. One of the less notified drawbacks of these models is that they depend only on source section and gives less care to load end[3]. This sometimes limits their applications, which may be discussed in following sections.

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Fixators and norators[4–7] are the emerging tools in circuit analysis. They are always used in pairs. Fixators are two terminal devices with both the voltage across and current through the component are fixed. For norators, voltage across and current through them can take any value. There is a third component nullator, which has the voltage across and current through them equals zero[8,9]. Nullor is the combination of norator and nullator. Ideal operational amplifiers are the examples of nullors and therefore in practical cases, operational amplifiers are used to model fixator-norator pairs[5,6]. Circuit simulators cannot directly model fixator-norator pairs, so ideal controlled sources with very high gain can be used to mimic the pair.

Section 2 introduces verification of Thevenin's theorem in terms of fixator norator pair. Sections 3 and 4 describe about verification of Compensation and Maximum power transfer theorem. Section 5 discuss about construction of equivalent models. Finally, Section 6 concludes the discussion.

2. Analysis of Thevenin's Theorem

Figure 1(a) is a network consisting of 'n' number of independent voltage sources and 'm' number of independent current sources and Figure 1 (b) is its Thevenin's equivalent circuit. The voltage across load is marked as V_{load} and load current is I_{load}. At the next step, we remove the load R_L and instead place fixator-norator pair. As discussed in[4–6] replacement of R_L with proper fixator-norator pair makes no change to the working conditions of network to the left of R_L. Figure 2 shows the circuit arrangement. The current through voltage source V_{load} controls the norator. In this arrangement, we actually draw current I_{load} from the source network, so port L_1 is at a potential V_{load} as indicated in Figure 1. If we connect ports L_1 and L_2, it will be the equivalent circuit for network in Figure 1 in terms of fixator-norator pair. A voltage V_{load} is dropped across the norator and the norator current is I_{load}. Therefore, in this case norator is the equivalent for R_L. Addition of fixator-norator pair makes no change to the actual circuit. The added fixator does not consume or provide any power. Next, we remove all the sources by replacing them with their internal resistance, i.e., voltage sources are replaced with short circuit and current source are replaced with open circuit. The modified circuit is shown in Figure 3. Since we replaced the sources with their internal resistances, at this point when we look into the network from its output port, there we see Thevenin's equivalent resistance, R_{Th}.

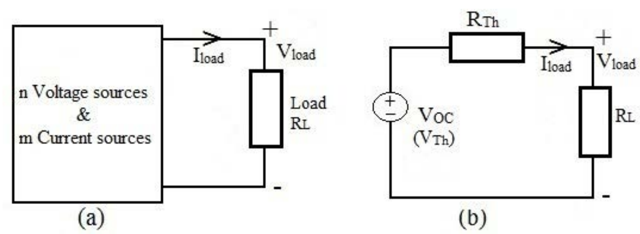

Figure 1. (a) network with 'n' number of voltage sources and 'm' number of current sources. (b) its equivalent Thevenin's model.

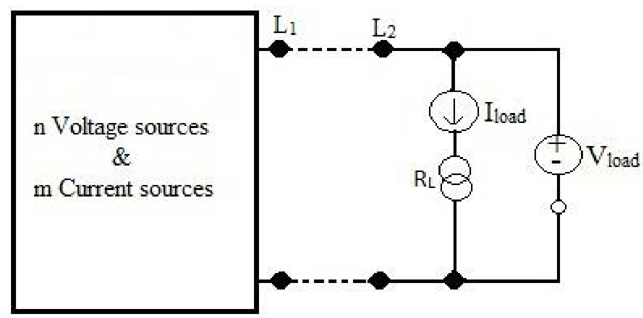

Figure 2. Applying fixator norator pair to output port.

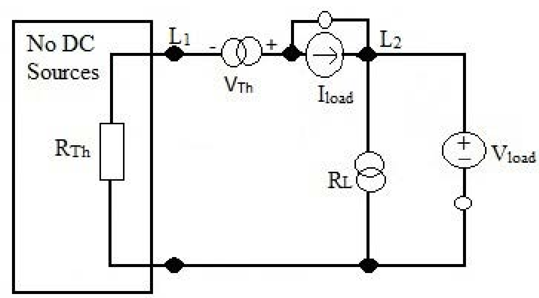

Figure 3. All the sources of circuit in Figure 1(a)are replaced with their internal resistances.

The voltage fixator Fx(V_{load},0) fixes node L_2 at a voltage V_{load}, at the same time, current fixator Fx(0,I_{load}) keeps the current through R_L fixed at I_{load}. Moreover, the norator associated with the current fixator gives the value ofV_{Th}. This is what the Thevenin's theorem states. Here we have a single equivalent source V_{Th}, equivalent resistance

R_{Th}, and load. Keep in mind that neither V_{Th} nor R_{Th} are controlled by load. Load can affect only output port values that are V_{load} and I_{load}. There are situations where we want to interchange some of the components from both sides of the ports. In such cases to meet the requirements of networks like one we discuss, we need a dynamic model like H-model. Let us see what change is made by an H-model when it is used in place of Thevenin's equivalent model. It is shown in Figure 4.

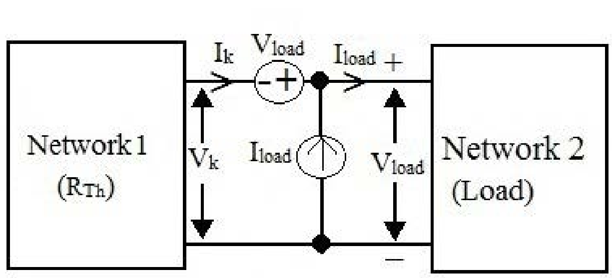

Figure 4. H-model representation of Figure 1(a).

By analyzing Figure 1 and Figure 4, we get,

$$V_{load} = V_H = V_{Th} - I_{load} R_{Th}$$

And

$$I_{load} = I_H = I_{sc} - \frac{V_{load}}{R_{Th}}$$

The null port K indicates that N_1 consume no power, hence total power consumption is reduced.

3. Analysis of Compensation Theorem

Compensation theorem can take two forms, the first and simplest form states, in a linear network, any element can be replaced by an independent voltage source, whose value is equal to voltage drop across the element. In the second form, as the resistance R of a branch carrying a current I is incremented to R+ΔR, then the compensation voltage to minimize the change V_C is given by IΔR. Consider a simple circuit as shown in Figure 5. Load voltage is found to be 2.874 V and load current is 2.874mA. Let the value of load resistor changes from 1KΩ to 1.1KΩ. If we want to kept voltage across load unchanged (2.874V in this case), we have to redesign value of the source or resistors. Now we are going to redesign value of R_2 using fixator Fx(2.874V,0). Circuit arrangement is shown in Figure 6. The pairing norator shows a drop of 3.135V and a current of 6.711mA flows through it. Thus, replacement of R_2 by a voltage source of 3.135V can solve the issue. Updating the value of R_2 as 467.1Ω or replacing R_2 by a current source of 6.711mA are other possible substitutions.

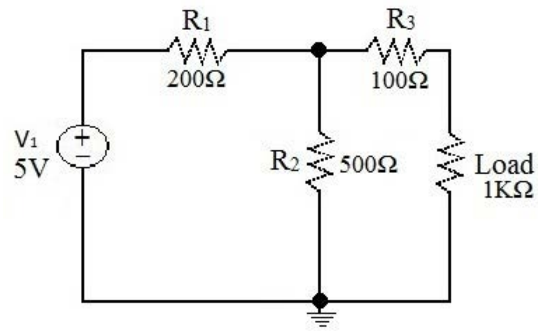

Figure 5. A simple resistor network.

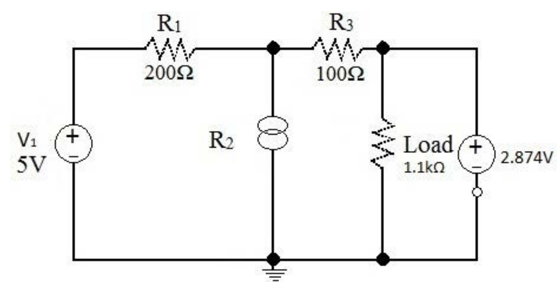

Figure 6. Redesign of resistor R_2.

Now consider the second form of compensation theorem. Here we have to find the compensation voltage V_C, which compensate the change in load. Fixator can easily find the value of V_C in a similar way as done in Figure 6. But how can we do with less power consumption? H-modeling can be done with the help of fixators. The circuit arrangement is shown in Figure 7. Simulation indicates that value of V_C = 287.4mV.

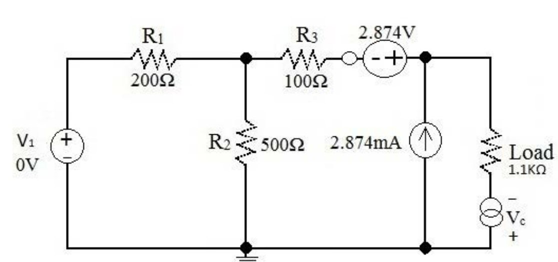

Figure 7. Finding compensation voltage V_C.

4. Analysis of Maximum Power Transfer Theorem

In many instance we need to adjust load impedance for maximum power transfer from source to load. Maximum power transfer theorem defines the relationship between load impedance and internal impedance of source for maximum power transfer. We can verify the Maximum power transfer theorem by taking a sample network and analysing it. It is important to keep in mind that this theorem is defined for networks with sources having fixed source impedance. Figure 8(a) shows a sample network and Figure 8(b) is its Thevenin's equivalent circuit.

According to Maximum power transfer theorem, maximum power transfer or 50% efficiency results if the internal resistance of source (or R_{Th} in this case) is equal to the load resistance. We try to prove that if efficiency is 50%, then load resistance equals to R_{Th}.

From Figure 8(a), we have V_{OC} at load end = 10.91V, which is the V_{Th}. For 50% efficiency,

$$V_L = \frac{V_{OC}}{2}$$

Or,

$$I_L = \frac{I_{SC}}{2}$$

Take the first condition and proceed, remove load from the circuit as shown in Figure 8(a) and instead place a voltage fixator to fix load voltage at $V_{OC}/2$. It is shown in Figure 9. Here current through the norator is controlled by the current through fixator.

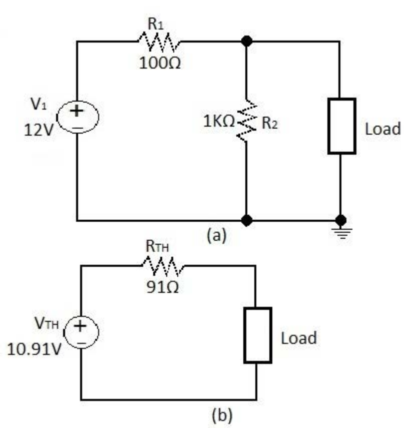

Figure 8. (a) A sample network. (b) its Thevenin equivalent.

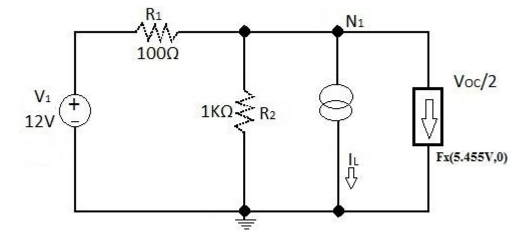

Figure 9. Load is replaced by fixator-norator pair at 50% efficiency condition.

After simulation, the norator current I_{load} is observed as 60mA, which results in a load resistance of 91Ω. From the analysis of compensation theorem, we have,

$$R_{Th} = \frac{V_{load}}{I_{SC} - I_{load}}$$

Here, V_{load} =5.455V, I_{SC}-I_{load}= 60mA. Therefore, R_{Th} must be 91 Ω. These results indicate that, for maximum power transfer, load in Figure 8(a) must be 91Ω. It is now easy to calculate V_{Th} as 10.91V. This verifies maximum power transfer theorem in terms of fixator norator pair. Again, H-modeling may also be employed to find new value of load for maximum power transfer, with $V_H = V_{OC}/2$ and $I_H = I_{SC}/2$. In addition, it is now easy to find value of load for a given efficiency provided fixators of required values.

5. Construction of Equivalent Models

Equivalent models are necessary for the analysis of analogue circuits. Consider a common emitter (C.E) amplifier with voltage divider biasing as shown in Figure 10. To construct its Thevenin's equivalent model, at first, we have to make its linear model. Figure 11 shows the simplified small signal equivalent circuit with virtual biasing sources.

For the base side, i.e., input port of amplifier, the biasing can be provided with Thevenin's equivalent of voltage divider network and the network reduces to a 6V DC source and a series 10KΩ resistor. Now, using fixator-norator pair, we can set various amplifier parameters like V_C, V_E etc. by re-designing the value of V_{Th}, R_{Th}, DC sources or other resistors.

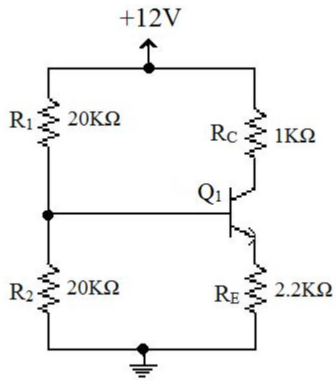

Figure 10. A Common emitter amplifier with voltage divider biasing.

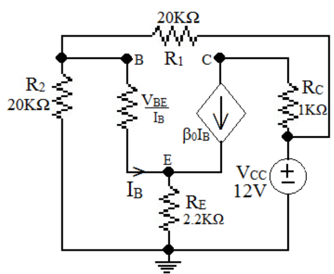

Figure 11. Small signal equivalent model of the C.E amplifier.

We can construct Thevenin's equivalent model of the above C.E amplifier looking from the load end by using fixator-norator pair. The open circuited (no load) voltage at the collector node of Figure 11 gives the value of V_{Th}. By applying maximum power transfer theorem, a fixator Fx(V_{Th}/2, 0) connected to collector node along with its pairing norator provides the value of R_{Th}. For a selected transistor, equivalent circuit and port characteristic curve are shown in Figure 12 (a) and Figure 12 (b).

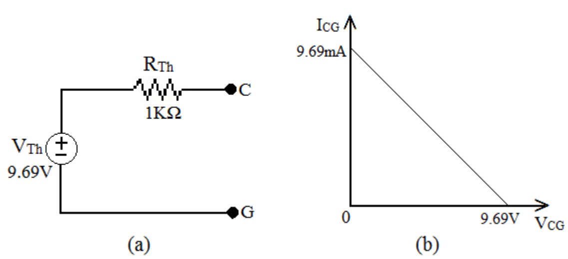

Figure 12. (a) Thevenin's equivalent circuit. (b) Port characteristic curve.

Maximum voltage drop occurs across the load when $R_L = R_{Th}$. I.e., maximum efficiency is 50%. But for a linear network with fixed R_{Th}, how can we make the effect of zero R_{Th} for the load without removing/altering any component in source network? If it is so happened, we get $V_{load} = V_{Th}$ and $I_{load} = I_{SC}$ (where $I_{SC} = V_{Th} / R_{Th}$). A method similar to H-modeling can be used for this purpose with $V_H = V_{Th}/2$ and $I_H = V_H/R_{Th}$. It is shown in Figure 13.

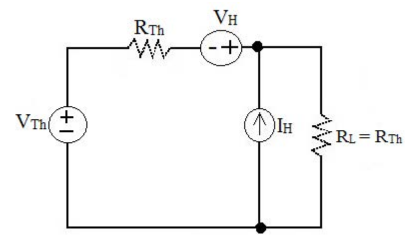

Figure 13. Making R_{Th} = 0 for the load.

Similiarly, for a linear network, we can find the value of R_{Th} by replacing R_L in Figure 13 by fixator Fx(V_{Th},0) and I_H by its pairing norator. R_{Th} is V_H/I_H or $V_{Th}/(2I_H)$. The circuit arrangement is shown in Figure 14.

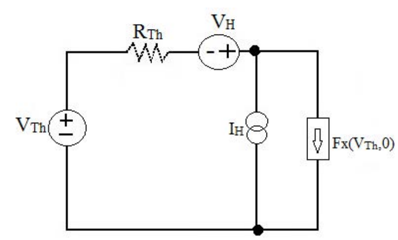

Figure 14. Finding the value of R_{Th}.

6. Conclusion

A scheme for analysis and biasing design of analogue circuit is presented. Thevenin's theorem has been verified using fixator-norator pair. In addition, Compensation theorem and Maximum power transfer theorem have also been analysed. Here we can see the interdependence between Thevenin's theorem and other network theorems. Hybrid equivalent circuit is presented and it is seen to be a good alternative for Thevenin's and Norton's equivalent circuits. Two forms of compensation theorem have been discussed and verified in this article. The method adopted to verify Maximum power transfer theorem is useful to design the value of R_L as per the required efficiency. At the end, construction of equivalent models using fixators have been worked out.

7. References

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