

Uncertain Systems Order Reduction by Modal Analysis Approach

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Abstract

Objectives: Modeling of physical systems results in complex higher-order representation. It's somewhat strenuous to process out with these intricate systems, in such conditions these large scale systems are approximated by relegated order model. **Methods/Statistical Analysis:** A system with bounded parameters but uncertain and having constant coefficients is termed as an interval system. In this jotting Modal analysis approach, order reduction technique has been used to demote the higher order system to its relegated order. Numerical examples are solved to show the supremacy of this advanced technique. **Findings:** By using the proposed method, the step response of original and reduced order uncertain systems are closer when compared to other methods. The relative integral square error values are also less as compared to other techniques. **Application/Improvements:** The relegated model acquired by this approach has its behavior homogeneous to the original system. The stability is vowed if the original system is stable. In order to depiction a controller for a higher order system it is quite arduous, so by using the order demotion technique it becomes more facile.

Keywords: Kharitonov's Theorem, Modal Analysis Approach, Order Reduction and Stability Uncertain Systems.

1. Introduction

Today's engineering fields like control engineering, signal processing, image compression, fluid mechanics and power systems, the designing of the systems are tedious, complex and computationally uneconomic.

To get survived from these problems model order reduction techniques are playing a major role. By implementing these techniques to the higher-order systems they are demoted to lower order thereby the system designing and simulation become simple and computationally economic to retain original system essential features in its model even after reduction. There are two types of approaches namely Frequency domain and Time domain.

Abundance of model order reduction methods were posited by the researchers in frequency domain like pade approximation¹⁻³, which dwindles the order of the system by using time moment matching method, but this method has a major drawback that even a stable original model may lead to unstable reduced order model. Where as in

Time domain approach the continued fraction method⁴ does not preserve the stability in its reduced order model. Systems having uncertain with bounded parameters known as interval systems are useful in obtaining reliable and perfect solution.

Many methods had been proposed by the researchers on interval systems based on the study of stability and transient analysis in interval systems^{5,6}. To reduce an continuous interval system⁷, recently mixed methods has been proposed to reduce the order of the system, so that the accuracy of the system can be increased⁸⁻¹³. There are few other methods which are proposed to reduce the order of uncertain systems^{15,16}.

In this paper to reduce the order of interval systems, "Modal Analysis Approach" method has been used, where the system is represented in state space form.

The outline of this note includes four sections. Problem formulation will be discussed in Section 2. The proposed method procedure is explained in Section 3. The Error analysis of original and reduced order system is done in Section 4. In Section 5 Numerical example has been

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considered, to explain about the proposed method. A qualitative comparison between the proposed and other methods will be provided in Section 6. Finally, conclusion is stated in Section 7.

2. Problem Statement

Assume an original linear time invariant interval system in Controllable Canonical Form:

$$\dot{x}(t) = A_{n \times n} x(t) + B_{n \times m} u(t) \tag{1}$$

$$y(t) = C_{q \times n} x(t) + D_{q \times m} u(t) \tag{2}$$

Where

$$A_{n \times n} = \begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 1 \\ [y_0^-, y_0^+] & -[y_1^-, y_1^+] & \dots & -[y_n^-, y_n^+] \end{bmatrix}$$

$$B_{n \times m} = \begin{bmatrix} 0 \\ \vdots \\ \vdots \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

$$C_{q \times n} = [[w_0^-, w_0^+] \quad [w_1^-, w_1^+] \quad \dots \quad [w_{n-1}^-, w_{n-1}^+]]$$

n = no. of state variables; m=no. of inputs variables.

q = no. of outputs variables.

The Corresponding relegated order model of an interval system is represented in Controllable Canonical Form (CCF) as follows:

$$\dot{x}_r(t) = A_r x_r(t) + B_r u(t) \tag{3}$$

$$y_r(t) = C_r x_r(t) + D_r u(t) \tag{4}$$

Where,

$$A_r = \begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 1 \\ -[z_0^-, z_0^+] & -[z_1^-, z_1^+] & \dots & -[z_r^-, z_r^+] \end{bmatrix}$$

$$B_r = \begin{bmatrix} 0 \\ \vdots \\ \vdots \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

$$C_r = [[x_0^-, x_0^+] \quad [x_1^-, x_1^+] \quad \dots \quad [x_{r-1}^-, x_{r-1}^+]]$$

r = reduced order

Arithmetic operations for an uncertain system are explained by¹⁴ is as follows:

Addition:

$$[c, d] + [j, k] = [c+j, d+k] \tag{5}$$

Subtraction:

$$[c, d] - [j, k] = [c-k, d-j] \tag{6}$$

Multiplication:

$$[c, d] \times [j, k] = [\min(cj, ck, dj, dk), \max(cj, ck, dj, dk)] \tag{7}$$

Division:

$$\frac{[c, d]}{[j, k]} = [c, d] \times \left[\frac{1}{j}, \frac{1}{k} \right] \tag{8}$$

3. Procedure

Step 1: The equivalent transfer function for the original uncertain plant which is expressed in Equation (1), Equation (2) is:

$$Z(s) = \frac{N(s)}{D(s)} \tag{9}$$

Where,

$$D(s) = s^n + \sum_{l=1}^{n-1} y_l * s^l, \quad y_l = [y_l^-, y_l^+]$$

$$N(s) = \sum_{l=0}^{n-1} w_l * s^l, \quad w_l = [w_l^-, w_l^+]$$

Step 2: Using Kharitonov's theorem⁵ the above interval system is transformed to four fixed transfer functions which carries the coefficients of Equation (9) this can be represented in its general form as:

$$T_p(s) = \frac{\sum_{i=0}^{n-1} N_{p_i} s^i}{\sum_{j=0}^n D_{p_j} s^j} \tag{10}$$

Where,

$$i \leq n-1; j \leq n; p=1, 2, 3, 4$$

Step 3: The above four transfer functions are converted into four fixed state models:

$$\dot{x}_{p/n}(t) = A_{p/n \times n} x(t) + B_{p/n \times m} u(t) \quad (11)$$

$$y_{p/n}(t) = C_{p/q \times n} x(t) + D_{p/q \times m} u(t) \quad (12)$$

Where,

$$p = 1, 2, 3, 4$$

Step 4: Calculate the Eigen values for the obtained four fixed state models represented in Equation (11), Equation (12) individually

Step 5: The modal matrix (V_p) is calculated for each individual state model:

$$V_p = \begin{bmatrix} V_{p1} & V_{p2} \\ V_{p3} & V_{p4} \end{bmatrix} \quad (13)$$

Where,

$$p = 1, 2, 3, 4; V_{p1} = (r \times r) \text{ and } V_{p4} = (n - r \times n - r)$$

Step 6: Then the inverse of modal matrix (\bar{V}_p) is to be reckoned from V_p as:

$$\bar{V}_p = \begin{bmatrix} \bar{V}_{p1} & \bar{V}_{p2} \\ \bar{V}_{p3} & \bar{V}_{p4} \end{bmatrix} \quad (14)$$

Where,

$$p = 1, 2, 3, 4; \bar{V}_{p1} = (r \times r) \text{ and } \bar{V}_{p4} = (n - r \times n - r)$$

Step 7: The matrix $A_{p/n \times n}$ is divided into sub matrices as:

$$A_{p/n \times n} = \begin{bmatrix} A_{p1} & \vdots & A_{p2} \\ \dots & \vdots & \dots \\ A_{p3} & \vdots & A_{p4} \end{bmatrix} \quad (15)$$

Where,

$$p = 1, 2, 3, 4; A_{p1} = (r \times r) \text{ and } A_{p4} = (n - r \times n - r)$$

Step 8: The matrix $B_{p/n \times m}$ is partitioned into the sub matrices as:

$$B_{p/n \times m} = \begin{bmatrix} B_{p1} \\ B_{p2} \end{bmatrix} \quad (16)$$

Where,

$$p = 1, 2, 3, 4; B_{p1} = (r \times r) \text{ and } B_{p2} = (n - r) \times 1$$

Step 9: Then the matrix $C_{p/q \times n}$ is divided into the sub matrices as expressed below:

$$C_{p/q \times n} = [C_{p1} C_{p2}] \quad (17)$$

Where,

$$p = 1, 2, 3, 4; C_{p1} = (1 \times r) \text{ and } C_{p2} = 1 \times (n - r)$$

Step 10: Now the four demoted order state models are generalized as:

$$\dot{x}_{p/r}(t) = A_{p/r \times r} x_r(t) + B_{p/r \times m} u(t) \quad (18)$$

$$y_{p/r}(t) = C_{p/q \times r} x_r(t) + D_{p/q \times m} u(t) \quad (19)$$

Where

$$A_{p/r \times r} = A_{p1} + A_{p2} V_{p3} \bar{V}_{p1}$$

$$B_{p/r \times m} = B_{p1}$$

$$C_{p/q \times r} = C_{p1}$$

$$p = 1, 2, 3, 4; r = \text{order of reduced system}$$

Step 11: From the above obtained four reduced r^{th} order state models which is expressed in Equation (18), Equation (19) and its corresponding four reduced r^{th} order transfer functions in its general form is expressed as:

$$R_p(s) = \frac{\sum_{a=0}^{r-1} x_{0p_a} s^a}{\sum_{b=0}^r z_{0p_b} s^b} \quad (20)$$

Where,

$$a \leq r-1; b \leq r; p=1, 2, 3, 4;$$

$$r = \text{order of reduced system.}$$

Step 12: Now the equivalent transfer function for relegated uncertain system is equated below:

$$R(s) = \frac{[x_0^-, x_0^+] + [x_1^-, x_1^+]s + \dots + [x_{r-1}^-, x_{r-1}^+]s^{r-1}}{[z_0^-, z_0^+] + [z_1^-, z_1^+]s + \dots + [z_r^-, z_r^+]s^r} \quad (21)$$

Step 13: Ultimately the reduced model of the uncertain system is expressed in Controllable Canonical Form (CCF)

$$\dot{X}_r(t) = \begin{bmatrix} 0 & 1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 1 \\ -[z_0^-, z_0^+] & -[z_1^-, z_1^+] & \dots & -[z_r^-, z_r^+] \end{bmatrix} x_r(t) + \begin{bmatrix} 0 \\ \vdots \\ \vdots \\ 0 \\ 1 \end{bmatrix} u(t) \quad (22)$$

$$y_r(t) = [x_0^-, x_0^+] [x_1^-, x_1^+] \dots [x_{r-1}^-, x_{r-1}^+] x_r(t) \quad (23)$$

4. Relative and Integral Square Error

The Relative Integral Square Error between transient responses of original and reduced systems is also determined as formulated below:

$$\text{Relative ISE} = \int_0^\infty [z(t) - r(t)]^2 dt / \int_0^\infty [z(t) - r(\infty)]^2 dt$$

$$\text{ISE} = \int_0^\infty [z(t) - r(t)]^2 dt$$

Where,

$\mathbf{z}(t)$ and $\mathbf{r}(t)$ are the unit step responses of original Z(s) and reduced R(s) systems, $\mathbf{r}(\infty)$ final value of original system.

5. Numerical Evaluation

5.1 Example 1

Let us consider an interval system having state model as followed below:

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -[6.8333, 10.75] \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -[11.667, 18] & -[5.667, 9] \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t) \quad (24)$$

$$\mathbf{y}(t) = [-[5, 8] \quad -[5.8333, 9.25] \quad -[0.667, 1.5]] \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \quad (25)$$

1. The equivalent transfer function of an uncertain is as follows:

$$Z(s) = \frac{[0.6667, 1.5]s^2 + [5.8333, 9.25]s + [5, 8]}{[1, 1]s^3 + [5.667, 9]s^2 + [11.667, 18]s + [6.8333, 10.75]} \quad (26)$$

2. Evaluate the four 3rd order transfer functions by using Kharitonov's theorem as expressed in Equation (10).

3. The above four transfer functions are converted into four state models by using Equation (11), Equation (12) are:

$$\dot{\mathbf{x}}_1(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} -9 & -11.667 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t) \quad (27)$$

$$\mathbf{y}_1(t) = [1.5 \quad 5.8333 \quad 5] \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \quad (28)$$

$$\dot{\mathbf{x}}_2(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} -9 & -18 & -6.8333 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t) \quad (29)$$

$$\mathbf{y}_2(t) = [1.5 \quad 9.25 \quad 5] \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \quad (30)$$

$$\dot{\mathbf{x}}_3(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} -5.667 & -11.667 & -10.75 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t) \quad (31)$$

$$\mathbf{y}_3(t) = [0.667 \quad 5.8333 \quad 8] \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \quad (32)$$

$$\dot{\mathbf{x}}_4(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} -5.667 & -18 & -10.75 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u(t) \quad (33)$$

$$y_4 = \frac{1}{3} [0.667 \quad 9.25 \quad 8] \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} \quad (34)$$

4. The corresponding Eigen values are calculated individually for the above four state models.
5. Then modal matrix, inverse of modal matrix and $A_{p/n \times n}, B_{p/n \times m}, C_{p/q \times n}$ matrices are obtained for the four state models individually by using Equation (13) to Equation (17).
6. Next, four reduced order state models are obtained from Equation (18), Equation (19) as given below:

$$\dot{x}_{1/2}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -9.106 & -13.46 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) \quad (35)$$

$$y_{1/2}(t) = \frac{1}{2} [1.5 \quad 5.833] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad (36)$$

$$\dot{x}_{2/2}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -8.304 & -12.08 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) \quad (37)$$

$$y_{2/2}(t) = \frac{1}{2} [1.5 \quad 9.25] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad (38)$$

$$\dot{x}_{3/2}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -7.072 & -17.07 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) \quad (39)$$

$$y_{3/2}(t) = \frac{1}{2} [0.667 \quad 5.833] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad (40)$$

$$\dot{x}_{4/2}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -4.705 & -13.27 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) \quad (41)$$

$$y_{4/2}(t) = \frac{1}{2} [0.667 \quad 9.25] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad (42)$$

5. Using four reduced state models from Equation (35) to Equation (42) the corresponding reduced order transfer functions are acquired as expressed in Equation (20).

6. Now the equivalent reduced order transfer function for uncertain system will be as expressed in Equation (21):

$$R(s) = \frac{[0.667, 1.5]s + [5.833, 9.25]}{[1, 1]s^2 + [4.705, 9.106]s + [12.08, 17.07]} \quad (43)$$

Under steady state condition $s \rightarrow 0$ then

$$R(s) = \frac{[1.0105, 2.0595]s + [8.837, 12.70025]}{[1, 1]s^2 + [4.705, 9.106]s + [12.08, 17.07]} \quad (44)$$

7. The CCF of the reduced order interval system

$$\dot{x}_r(t) = \begin{bmatrix} 0 & 1 \\ -[12.08, 17.07] & -[4.705, 9.106] \end{bmatrix} x_r(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \quad (45)$$

$$y_r(t) = [8.837, 12.70025] [1.0105, 2.0595] x_r(t) \quad (46)$$

The step responses of both original and reduced 3rd order systems are shown in Figure 1.

5.2 Example 2

Let the system be:

$$\dot{x}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \\ \dot{x}_5(t) \\ \dot{x}_6(t) \\ \dot{x}_7(t) \\ \dot{x}_8(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -4800,9601 & -14440,28881 & -18746,37493 & -13735,27471 & -5935,11871 & -1508,5,3018 & -218,5,438 & -16,5,34 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \\ x_6(t) \\ x_7(t) \\ x_8(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u(t) \quad (47)$$

$$y(t) = [[97240, 194481] \quad [241482, 482965] \quad [255906, 511813] \quad [139188, 278377] \quad [41201, 82403] \quad [6642.5, 13286]] \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \\ x_6(t) \\ x_7(t) \\ x_8(t) \end{bmatrix} \quad (48)$$

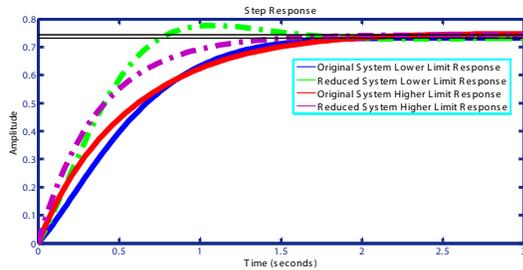


Figure 1. Step response of original 3rd order and reduced 2nd order system using Proposed Method.

1. The equivalent transfer function of an uncertain plant is as follows:

$$Z(s) = \frac{[17.5,36]s^7 + [543,1087]s^6 + [6642.5,13286]s^5 + [41201,82403]s^4 + [139188,278377]s^3 + [255906,511813]s^2 + [241482,482965]s + [97240,194481]}{[1,1]s^8 + [16.5,34]s^7 + [218.5,438]s^6 + [1508.5,3018]s^5 + [5935,11871]s^4 + [1373527471]s^3 + [18746,37493]s^2 + [14440,28881]s + [4800,6901]} \quad (49)$$

2. Evaluate four fixed 8th order transfer functions by using Kharitonov’s theorem as in Equation (10).

3. The above four transfer functions are converted into four state models by using Equation (11), Equation (12) are:

$$\dot{x}_{1/8}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \\ \dot{x}_5(t) \\ \dot{x}_6(t) \\ \dot{x}_7(t) \\ \dot{x}_8(t) \end{bmatrix} = \begin{bmatrix} -34 & -438 & -1509 & -5935 & -27471 & -37493 & -14440 & -4800 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \\ x_6(t) \\ x_7(t) \\ x_8(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u(t) \quad (50)$$

$$y_{\frac{1}{8}}(t) = [40 \quad 1090 \quad 6640 \quad 41200 \quad \dots]$$

$$[278380 \quad 511810 \quad 241480 \quad 97240] \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \\ x_6(t) \\ x_7(t) \\ x_8(t) \end{bmatrix} \quad (51)$$

$$\dot{x}_{2/8}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \\ \dot{x}_5(t) \\ \dot{x}_6(t) \\ \dot{x}_7(t) \\ \dot{x}_8(t) \end{bmatrix} = \begin{bmatrix} -17 & -438 & -3018 & -5935 & -13735 & -37493 & -28881 & -4800 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \\ x_6(t) \\ x_7(t) \\ x_8(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u(t) \quad (52)$$

$$y_{\frac{2}{8}}(t) = [20 \quad 1090 \quad 13290 \quad 41200 \quad \dots] \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \\ x_6(t) \\ x_7(t) \\ x_8(t) \end{bmatrix} \quad (53)$$

$$\dot{x}_{3/8}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \\ \dot{x}_5(t) \\ \dot{x}_6(t) \\ \dot{x}_7(t) \\ \dot{x}_8(t) \end{bmatrix} = \begin{bmatrix} -34 & -219 & -1509 & -11871 & -27471 & -18746 & -14440 & -9600 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \\ x_6(t) \\ x_7(t) \\ x_8(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u(t) \tag{54}$$

$$y_3(t) = \begin{bmatrix} 36 & 543 & 13286 & 82403 \end{bmatrix}$$

$$\begin{bmatrix} 139188 & 255906 & 482965 & 194481 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \\ x_6(t) \\ x_7(t) \\ x_8(t) \end{bmatrix} \tag{55}$$

$$\dot{x}_{4/8}(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \\ \dot{x}_4(t) \\ \dot{x}_5(t) \\ \dot{x}_6(t) \\ \dot{x}_7(t) \\ \dot{x}_8(t) \end{bmatrix} = \begin{bmatrix} -17 & -219 & -3018 & -11871 & -13735 & -18746 & -28881 & -9601 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \\ x_6(t) \\ x_7(t) \\ x_8(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u(t) \tag{56}$$

$$y_4(t) = \begin{bmatrix} 20 & 540 & 13290 & 82400 \end{bmatrix}$$

$$\begin{bmatrix} 139190 & 255910 & 482970 & 194480 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \\ x_6(t) \\ x_7(t) \\ x_8(t) \end{bmatrix} \tag{57}$$

4. Follow the procedural Steps (4 to 9).
5. Next, four reduced order state models are obtained from Equation (18), Equation (19) as given below:

$$\dot{x}_1(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -30.04 & -295.6 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) \tag{58}$$

$$y_1(t) = \begin{bmatrix} 40 & 1090 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \tag{59}$$

$$\dot{x}_2(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -8.24 & -348.2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) \tag{60}$$

$$y_2(t) = \begin{bmatrix} 20 & 1090 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \tag{61}$$

$$\dot{x}_3(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -34.27 & -266.29 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) \tag{62}$$

$$y_3(t) = \begin{bmatrix} 36 & 543 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \tag{63}$$

$$\dot{x}_4(t) = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} -7.58 & -214.1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t) \tag{64}$$

$$y_4(t) = \begin{bmatrix} 20 & 540 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \tag{65}$$

6. Using four reduced state models from Equation (57) to Equation (64) the corresponding reduced

order transfer functions are obtained as expressed in Equation (20).

7. Now the equivalent reduced order transfer function for uncertain system will be as expressed in Equation (21):

$$R(s) = \frac{[20, 40]s + [540, 1090]}{[1, 1]s^2 + [7.58, 34.27]s + [214.1, 348.2]} \quad (66)$$

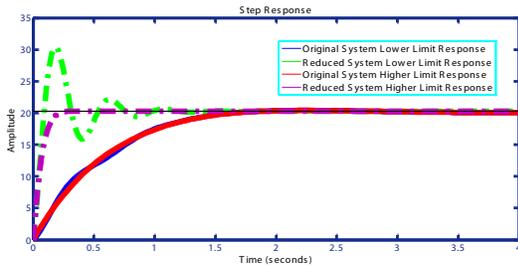


Figure 2. Step response of original 8th order and reduced 2nd order system using proposed method.

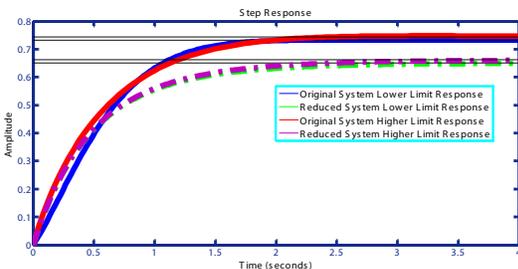


Figure 3. Step response of original 8th and reduced 2nd order system using differentiation method.

Under steady state condition $s \rightarrow 0$ then

$$R(s) = \frac{[160.641, 258.836]s + [4337.307, 7053.281]}{[1, 1]s^2 + [7.58, 34.27]s + [214.1, 348.2]} \quad (67)$$

8. The CCF of the reduced order interval system:

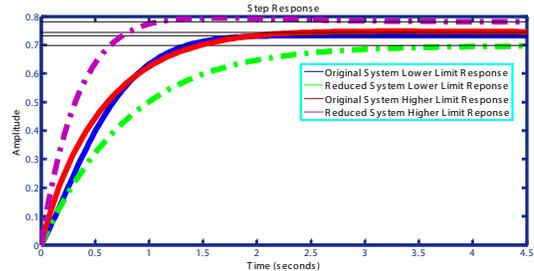


Figure 4. Step response of original 8th order and reduced 2nd order system using differentiation and factor division method.

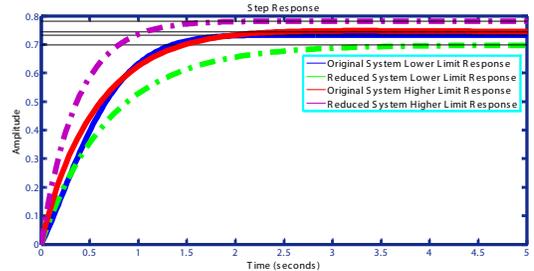


Figure 5. Step response of original 8th order and reduced 2nd order system using differentiation and cauer second form.

Table 1. Comparison of reduced order models for 3rd order system

S.no.	Methods	Reduced Order Systems	Step Response of Lower Limit		Step Response of Higher Limit	
			ISE Values	Relative ISE Values	ISE Values	Relative ISE Values
1.	Proposed Method	$R(s) = \frac{[1.0105, 2.0595]s + [8.837, 12.70025]}{[1, 1]s^2 + [4.705, 9.106]s + [12.08, 17.07]}$	0.0285	0.1649	0.0090	0.0605
2.	Differentiation Method	$R(s) = \frac{[5.8333, 6.1667]s + [10.0005, 10.6672]}{[4.25, 4.5]s^2 + [17, 18]s + [15.375, 16.125]}$	0.0531	0.0396	0.0617	0.0350
3.	Differentiation and Factor Division Method	$R(s) = \frac{[3.4809, 10.7884]s + [10.7267, 12.5854]}{[4.25, 4.5]s^2 + [17, 18]s + [15.375, 16.125]}$	0.0094	0.0511	0.0074	0.1309
4.	Differentiation and Cauer Second Form	$R(s) = \frac{[4.2604, 9.6099]s + [10.7271, 12.5856]}{[4.25, 4.5]s^2 + [17, 18]s + [15.375, 16.125]}$	0.0094	0.0241	0.0073	0.0603

$$\dot{\mathbf{x}}_r(t) = \begin{bmatrix} \mathbf{0} & \mathbf{1} \\ -[214.1, 348.2] & -[7.58, 34.27] \end{bmatrix} \mathbf{x}_r(t) + \begin{bmatrix} \mathbf{0} \\ \mathbf{1} \end{bmatrix} u(t) \quad (68)$$

$$y_r(t) = [[4337.307, 7053.281] \ [160.641, 258.836]] \mathbf{x}_r(t) \quad (69)$$

The step responses of both original and reduced 8th order systems are shown in Figure 2.

6. Comparison of Methods

The relegated order models acquired by present method are compared with other methods and their corresponding ISE and Relative ISE values of reduced models and their step response are shown below

The original and reduced model step responses obtained by proposed method which is shown in Figure 1 and Figure 2 are closer when compared to the responses of other methods which are shown in Figure 3, 4 and 5. The ISE values of proposed technique are less as compared to other methods which is shown in Table 1.

7. Conclusion

The reduction procedure is done for higher order systems so that the complexity of the system can be decreased. In this paper the order reduction for an uncertain system which is represented in state space form is numerically evaluated by using the proposed method. The reduced order models obtained by proposed method and their ISE and Relative ISE values of step response are compared. Hence the proposed method preserves stability with low ISE values compared to other existing methods.

8. References

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