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Analysis of Entropy Generation Minimization in Radial Heat Sink

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Abstract

Background/Objective: In this paper, an entropy generation rateof a radial flow type heat sink model has been investigated and its minimization is the primary aim of this study to exhibit the overall performance of the model. **Methods/Statistical Analysis:** The entropy generation method (EGM), which is one of the best modeling and optimization approach, has been used to optimize the parametric variable of the heat sink. In this study, a numerical iterative technique of Newton-Raphson method has been employed to solve the multiple nonlinear equation system which occurred in the analysis. **Findings:**The optimization was conducted to study the effects of the number of fins, thermal resistance of the heat sink and the approach velocity on the rate of entropy generation for the heat sink model. From the optimized results of the model, the number of fins, $N \approx 55$ was estimated for the minimization of entropy generation rate, and its effect on the thermal resistance of the heat sink was also observed. The variation of the entropy generation rate and the number of fins at a different approach velocity were reported in this analysis, and also,three-dimensional surface plotis created by the surf command in MATLAB software. **Application/Improvements:** The entropy generation minimization method was employed to exhibit the overall performance of the optimized heat sink model and its application is more advantageous in the illumination industry.

Keywords: Analytical Model, Entropy Generation Minimization,Forced Convection, Heat Transfer, Radial Heat Sinks

1. Introduction

In the electronic cooling system and the illumination industries, researchers are facing a big challenge to develop an efficient optimization technique in which more heat dissipation and compact size is required. To achieve a goal, this methodology is more important because of the experimental setups with repeated trials is more expensive and the time consuming. The method and modeling of entropy generation minimization is better explained to discuss about a parametric study of related system, although they exhibit an impact about the thermal performance and the viscous effects on system parameters[1–3]. In the context of convective optimizations of models, the variation of heat transfer coefficient is well reported, and dependent parameters are the spacing of fins and its length in the flow direction[4].

Numerous researchers have investigated a various theoretical model, and optimization of the heat sink model has been done to exhibit the overall performance of the model. The performance of the micro channel heat sink was estimated through a dimensionless pressure drop and thermal resistance parameters using a numerical iterative method in the optimization scheme[5]. Also, in the similar way of optimization procedure, the total pressure drop and thermal resistance were reported to exhibit the performance of heat sink[6,7]. In this analysis, a numerical iterative scheme was employed. The laminar and turbulent flow conditions of considered system in which the estimation of the pressure drop and thermal resistance was investigated to optimize the channel and wall width of the system[8–10].The analytical expression of a fluid-saturated porous medium of the heat sink model was reported to estimate the friction factor and Nusselt number for the assumed model[11,12]. The temperature and velocity profile was obtained using a numerical simulation of rectangular type microchannel heat sink, and as a result, equations of the pressure drop and thermal resistance were estimated

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to exhibit the performance of the model[13–15].It is obvious from the above research work that nonehas beenconcernedwiththe optimization of the pressure drop and thermal resistance simultaneously to estimate the overall performance of the system. Then, anEntropy Generation Minimization (EGM) for a laminar flow in an air-cooled rectangular heat sink was reported very well to optimize the thermal resistance and the pressure drop simultaneously[16]. In this study, an entropy generation rate was minimized by Lagrangian polynomial method, and multi-variable equations were solved by Newton–Raphson method. Also, the optimization of the previous performance parameters of the system was reported by Genetic Algorithm (GA) due to the complexity and time consuming problem[17]. The GA and the Particle Swarm Optimization (PSO) approach were employed to optimize the design parameters of pin fin heat sink[18]. The observation of the entropy generation in fully developed laminar flow was reported to analyze the effects of the heat flux, Reynolds number and geometric parameters of a confocal elliptical duct[19]. The optimized model of multi-stream plate heat exchanger was found by using an entropy generation minimization approach[20]. In the vertical semi-infinite porous plate, the effects of Reynolds number, Prandtl number, viscous heat dissipation and suction parameter on periodic free stream and suction velocity was observed, and unsteady three-dimensional flow was considered[21]. The effects of viscous energy dissipation, heat generation and radiation on non-linear MHD boundary layer 2-D flow of grayfluid past a linear stretching porous sheet with prescribed heat flux is reported in this paper[22].

Thus, all the above studies were related to the micro channel heat sinks, none has been related to radial flow type heat sink which is composed of a circular base and arrays of the rectangular fins at regular intervals in a radial manner. In our work, the study of the effects of the number of fins, thermal resistance of the heat sink and the approach velocity on the entropy generation rate for the considered model has been reported to exhibit the overall performance of the optimized model.

2. Assumptions

The following assumptions made in this analysis are given.

(1) The steady and laminar flow is considered.
(2) he heat flux on the bottom surface of the circular base plate is considered as uniform.
(3) The incompressible fluid with constant properties is assumed.
(4) The approach velocity is considered as uniform.
(5) The material of the fin is homogeneous and isotropic nature.
(6) The adiabatic fin tips are assumed.
(7) The contact resistance at the fin to base plate is neglected.
(8) The heat and fluid flow is considered as fully developed.
(9) Radiation heat transfer is neglected.
(10) The changes in kinetic and potential energies are neglected.

3. Mathematical Modeling

Figure 1 shows the model of a radial heat sink in which the rectangular fins have been mounted on a circular base at regular intervals. The inner and outer diameter of the heat sink is D_i and D_o respectively. Also, the thickness of circular base and the fin are $t_b$and t, respectively. In addition, the number of fins in the heat sink model is N , and each fin has a length l and height H. The heat dissipation from the system model occurs, when the air is passing through a number of fins.

A model has been developed which relates between the heat sink design parameters and the rate of entropy generation, provided that the best possible optimized heat sink design is produced. For the expression of the entropy generation rate, this expression had been well reported by using the mass and energy rate balance with entropy rate balance for a fluid flowing through a model[3], and is given by

$$S_{gen} = \left(\frac{Q^2 R_{th}}{T_a T_b}\right) + \left(\frac{\dot{m}}{\rho T_a}\Delta P\right) \tag{1}$$

This expression shows that the dependent parameters of the rate of entropy generation are the total thermal resistance R_{th} of the heat sink and the pressure drop,

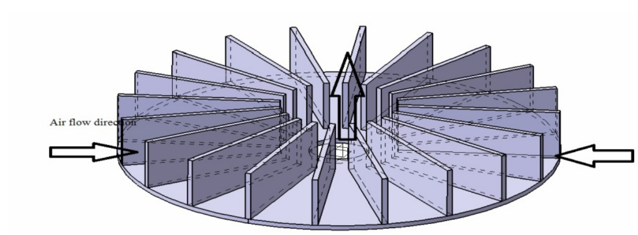

Figure 1. Radial heat sink geometry.

although the heat load and the ambient condition are given.

From balancing of force, the total drag force is expressed as

$$F_D = -(P_{out} - P_{in})A \tag{2}$$

where, A is the cross-sectional area of free stream. Also, the mass flow rate is written as

$$\dot{m} = \rho \, A \, U_{app} \tag{3}$$

where, ρ ρ is the fluid density at ambient temperature. Using Gibb`s equation in terms of entropy and pressure difference which is written as

$$h_{out} - h_{in} = T_a \, (s_{out} - s_{in}) + \frac{1}{\rho} \, (P_{out} - P_{in}) \tag{4}$$

The combining equation (2)-(4), the rate of entropy generation is expressed as

$$S_{gen} = \left(\frac{Q^2 \, R_{th}}{T_a T_b}\right) + \left(\frac{F_D U_{app}}{T_a}\right) \tag{5}$$

The complete expression of the rate of entropy generation has been mentioned in the equation (5) which demonstrates that the dependent parameters of the rate of entropy generation are the total thermal resistance R_{th} of the heat sink and the drag force, although the heat load and the ambient condition are given.

Since, the contact resistances and thermal spreading are negligible. Therefore, the total thermal resistance R_{th} is given as

$$R_{th} = \frac{1}{\dfrac{N}{R_{fin}} + \dfrac{1}{R_b}} \tag{6}$$

where

$$R_{fin} = \frac{1}{\sqrt{h_{fin} P k A_c} \tanh(mH)} \tag{7}$$

$$R_b = \frac{1}{h_b\left[\pi \left(R_o^2 - R_i^2\right) + 2\pi t_b (R_o + R_i) - Ntl\right]} \tag{8}$$

with

$$m = \sqrt{\frac{h_{fin} P}{k A_c}} \tag{9}$$

Assuming $h_{fin} = h_b = h_{avg}$, so

$$h_{avg} = \frac{Nu \, k_f}{l} \tag{10}$$

For finding heat transfer coefficient, a correlation used for rectangular heat sink has been employed in the analysis in terms of the modified Rayleigh number and average spacing between fins[23,24].

$$Nu = 0.195 \, (Ra^*)^{0.263} \left(\frac{N \, b_{avg}}{H}\right)^{1.35} \left(\frac{R_o}{l}\right)^{0.444} \left(\frac{R_o}{b_{avg}}\right)^{-0.142} \left(\frac{R_o}{H}\right)^{-1.4} \tag{11}$$

where,

$$Ra^* = \frac{\rho^2 g \beta c_p \pi \left(R_o^2 - R_i^2\right) \dot{q} \, l^3}{\mu l k^2} \tag{12}$$

$$b_{\downarrow}avg = \{((2\pi R_{\downarrow} o) \,/\, N - t) \dashv + ((2\pi \square (R\square_{\downarrow} o - 1)) \,/\, N - t)\}/2 \tag{13}$$

The drag force can written as

$$F_D = C_D \left(\frac{1}{2} \, \rho \, U_{app}^2\right) A_p \tag{14}$$

where, C_D is the drag coefficient, which is expressed as[25]

$$C_D = \frac{C_1}{Re_L} + C_2 + \frac{C_3}{Re_L} \tag{15}$$

where, C_1, C_2 and C_3 are the constants which depends on the geometry. Also, Re_L is the Reynolds number which is based on the characteristic length of the fin and is given by

$$Re_L = \frac{U_{app} L}{\upsilon} \tag{16}$$

Also, the final form of total thermal resistance is written as

$$R_{th} = \frac{1}{1.242 \, N \sqrt{C_4 C_5} \tanh\left(8.551\sqrt{C_4 C_5}\right) + \dfrac{}{637.261 C_4 C_5 \, (0.0184 - 0.00011N)}} \tag{17}$$

where, the terms C_4 and C_5 are given as

$$C_4 = (0.235 - 0.002N)^{1.35} \tag{18}$$

$$C_5 = \left[\frac{N}{0.235 - 0.002N}\right]^{-0.142} \tag{19}$$

Therefore, the simplified expression of equation (5) for the entropy generation rate can be found and numerical iterative method is applied for parametric study of radial heat sink design.

4. Optimization Procedure

The simplified expression of equation (5) for the entropy generation rate has been minimized to optimize the performance parameter of the considered model. For optimization of parametric variable,a numerical iterative technique of Newton-Raphson method has been employed to solve the multiple nonlinear equation system which occurred in the analysis. The approach used in this method has been discussed to solve the system of nonlinear equations[26] and is given as

$$S_{gen} = f(x_1, x_2, \dots\dots\dots\dots x_n) \tag{20}$$

and the partial derivative of all the functions as

$$\frac{\partial S_{gen}}{\partial x_i} = y_i = 0, for\ i = 1,2, \dots\dots\dots\dots n \tag{21}$$

where, x_i is the unconstrained type variables.

The set of nonlinear multiple equations can be written as

$$\begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \cdots & \frac{\partial y_1}{\partial x_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial y_n}{\partial x_1} & \cdots & \frac{\partial y_n}{\partial x_n} \end{bmatrix} \begin{bmatrix} \delta x_1 \\ \vdots \\ \delta x_n \end{bmatrix} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \tag{22}$$

For solving this equation, an appropriate initial value of unconstrained variables in the equation has been considered that leads to improve the estimation. Since, δx_i is the successive value difference of the unconstrained type variables, and it becomes zero in such a manner that

$$y_{i\,(guess)} \approx y_i(actual) + y'_i(guess).\delta x_i \tag{23}$$

The solution of this method has been converged satisfactorily after a few iterations which leads an adequate initial guess. This is the simplest method for solving nonlinear equations, and it is widely used for research work due to the easily assessable to the desired results by using a mathematical software. This is shown in Figure 2 in flowchart form.

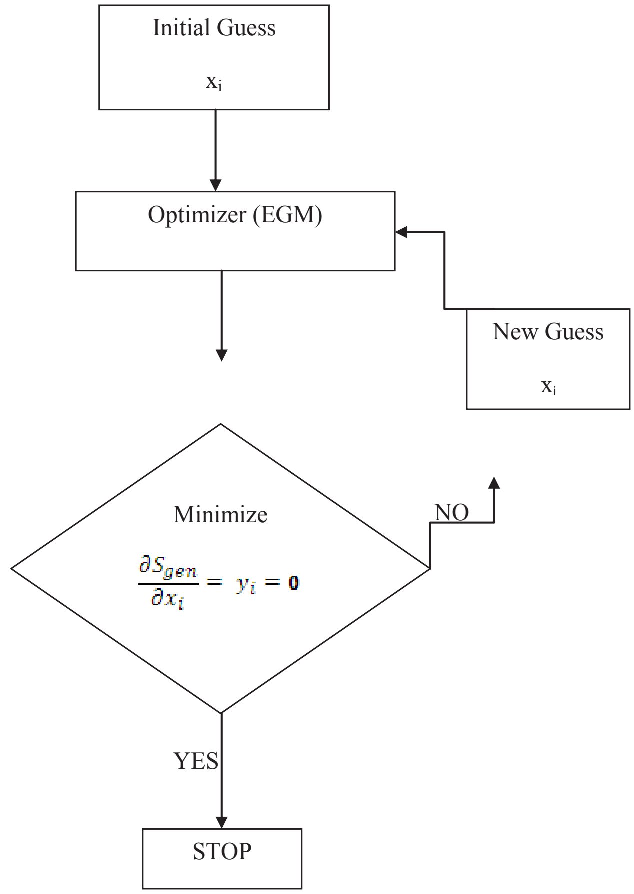

Figure 2. Flowchart for optimized parametric design variables.

5. Results and Discussion

For the optimization approach, the main objective has been selected the best heat sink to optimize the overall performance of the considered model. The assumption made in this analysis is that the uniformly distributed heat dissipation load of 11.351 W is used on the entire circular base plate, and its thickness is taken as 2mm. Also, the temperature of the surrounding medium is constant i.e. 30^{0}C. The other quantities are given in Table 1. The required parameters are taken in optimum manner for this heat sink[27]. The variation of design parameters in this analysis has been included the total number of fins, N , heat sink resistance, R_{th} , and approach velocity, U_{app} corresponding to the entropy generation rate. With reference to this, the analysis has been done that leads to optimize the overall performance of the model where the heat transfer as well as the viscous effects is assumed in the analysis.

The effect of the number of fins on the rate of entropy generation is shown in Figure 3. It has been shown that decreasing the entropy generation rate as increases the number of fins, since the gain in reduced temperature

Table 1. Dimensions used to find performance of heat sinks

Quantity	Dimension
Circular base plate thickness, t_b (mm)	2
Outer diameter of circular base, D_o (mm)	150
Inner diameter of circular base, D_i (mm)	20
Length of rectangular fin, l (mm)	50
Height of rectangular fin, H (mm)	21.3
Thickness of rectangular fin, t (mm)	2
Approach velocity, U_{app} (m/s)	3
Thermal conductivity of solid, k (W/m-k)	193
Thermal conductivity of air, k_f (W/m-k)	0.0278
Density of air, ρ (kg/m^3)	1.116
Specific heat of air, C_p (J/kg-k)	1005
Kinematic viscosity, υ (m^2/s)	1.731 x 10^{-5}
Dynamic viscosity, μ (N/m^{2}s)	1.929 x 10^{-5}
Prandtl number(air), Pr	0.71
Heat load, Q (W)	11.351
Ambient temperature, T_a (K)	303
Base plate temperature, T_b (K)	330
Coefficient of thermal expansion, β (K^{-1})	0.00316

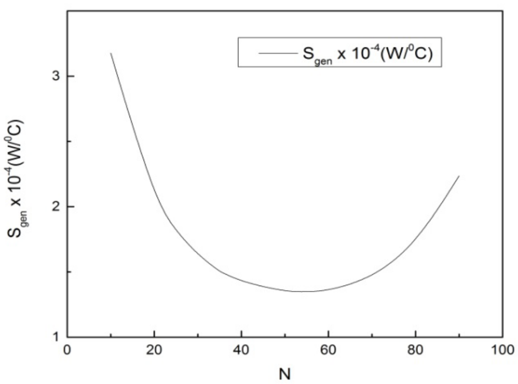

Figure 3. Variation of the entropy generation rate versus number of fins.

excess of heat sinks that leads to a decrease in the rate of entropy generation of the heat sink model. However, when the number of fins is increasing beyond the optimized limit, the heat sink temperature excess and the thermal resistance have been decreased, and as a result the entropy generation rate increases. Thus, the optimized results of the number of fins, N ≈ 55 have been estimated for the minimum entropy generation rate, and the variation of this result is given in Table 2.

Figure 4 shows the effect of the number of fins on the thermal resistance of the considered model. As the number of fins increased, the thermal resistance decreased because of the increased surface area of heat transfer. Also, it has been shown that decreasing the number of fins as increases the thermal resistance of the model, since the gain in heat sink temperature excess, and as a result the rate of entropy generation increases.

Figure 5 indicates the variation of the rate of entropy generation versus approach velocity, when the height of the fin, H=21.3mm, the number of fins, N=35, and the thermal conductivity of aluminum alloy material, k=193W/mk have been considered. Under the same condition, it can be seen that the rate of entropy generation first decreases significant manner and then increases with the increase in approach velocity, and as a result the optimized approach velocity has been found in these conditions.

Table 2. Performance parameter results

N	Performance Parameters	
	R_{th} (K/W)	S_{gen} x 10^{-4} (W/^{0}C)
10	0.245	3.178
20	0.158	2.057
30	0.125	1.632
40	0.109	1.426
50	0.103	1.349
60	0.1035	1.355
70	0.111	1.452
80	0.131	1.709
90	0.172	2.237

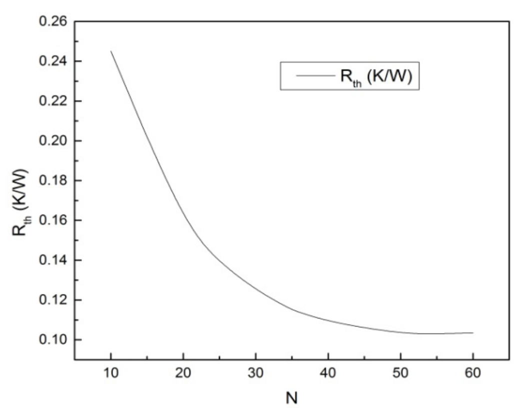

Figure 4. Variation of the thermal resistance versus number of fins.

The effect of the number of fins on the rate of entropy generation for three different velocities is shown in Figure 6. It is observed that an optimized number of fins have been found at each approach velocity, which results that the lower entropy generation rate occurs at higher approach velocities and lowerfin densityis better for this condition.

The three-dimensional surface plot is shown in Figure 7 which exhibits a surface defined by the entropy generation rate equation and this is created by the surf command in MATLAB.

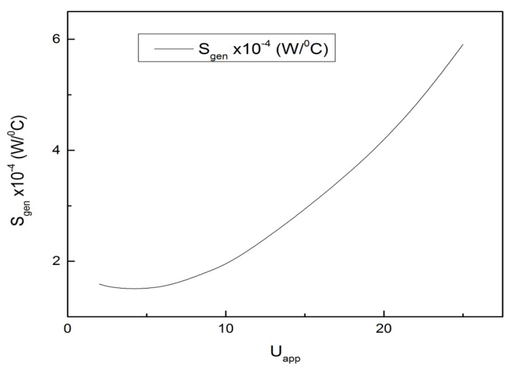

Figure 5. Variation of the entropy generation rate versus approach velocity.

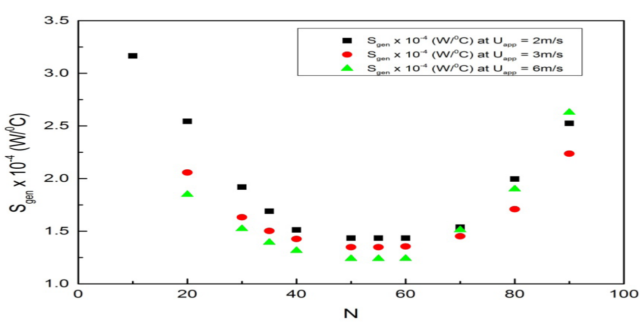

Figure 6. Variation of the entropy generation rate versus number of fins at different approach velocities.

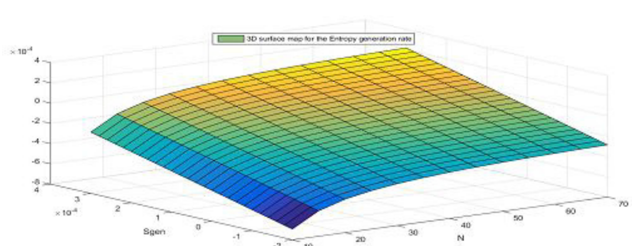

Figure 7. Three-dimensional surface plot for the Entropy generation rate.

6. Conclusion

The analysis of the results has been presented to optimize the performance parameters in a radial heat sink. This has been analyzed by the entropy generation minimization approach which includes both effects such as the heat transfer as well as the viscous dissipation effects. The effects of the number of fins, thermal resistance and approach velocity on the rate of entropy generation are reported to optimize the design parameters and the overall performance of the model. As the number of fins increases, the thermal resistance decreases and also entropy generation rate is found which leads to an optimum condition for that parameter. In the parameter optimization, the effect of the approach velocity on the rate of entropy generation is also examined.

7. Nomenclature

A_c	Cross sectional area of the fin, (m^2)
A_p	Plan form area for drag force, (m^2)
b	Fin spacing, (mm)
C_D	Total drag coefficient
D	Circular base diameter, (mm)
F_D	Drag force, (N)
H	Height of rectangular fin, (mm)
h	Heat transfer coefficient, (W/m^{2}k)
k	Thermal conductivity, (W/mk)
l	Fin length, (mm)
m	Fin performance parameter, (m^{-1})
$\dot{m}$	Mass flow rate, (kg/s)
N	Number of fins
ΔP	Pressure drop, (N/m^2)
Q	Heat load, (W)
$\dot{q}$	Heat flux, (W/m^2)
R_{th}	Thermal resistance, (K/W)
S_{gen}	Total entropy generation rate, (W/^{0}C)
T	Temperature, (K)
t	Fin thickness, (mm)
U_{app}	Approach velocity, (m/s)

Dimensionless parameters

Nu	Nusselt number
Re	Reynolds number
Ra^*	Modified Rayleigh number

Greek symbols

ρ	Air density, (kg/m^3)
υ	Kinematic viscosity, (m^2/s)

μ Dynamic viscosity, (N/m^2s)
β Thermal expansion coefficient, (K^{-1})

Subscripts

avg Average
f Fluid
i Inner
o Outer
a Ambient
b Base plate

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