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A Simple Proof on Coloring of Dominated Special Graphs

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Abstract
In this simple survery paper we prove a simple result for a coloring on dominated Special graphs.

1. Introduction

Definition 1

A proper coloring of a graph G = (V (G),E(G)) is an assignment of colors to the vertices of the graph, such that any two adjacent vertices have different colors.

Definition 2

A dominator coloring of a graph G is a proper coloring in which each vertex of the graph dominates every vertex of some color class.

Definition 3

A k-coloring of G is a coloring that uses at most k colors. The chromatic number of G is β(G) = min{k / G has a proper k-coloring}. A coloring of G can also be thought of as a partition of V (G) into color classes $V_1, V_2, ..., V_q$, and a proper coloring of G is then a coloring in which each V_i, $1 \le i \le q$ is an independent set of G, i.e., for each i, the subgraph of G induced by V_i contains no edges.

Definition 4

A dominating set S is a subset of the vertices in a graph such that every vertex in the graph either belongs to S or has a neighbor in S. The domination number is the order of a minimum dominating set.

2. Theorem

For the path P_n on n vertices, $\beta_{cpl}(P_{8j}) = 2j$, $\beta_{cpl}(P_{8j+1}) = \beta_{cpl}(P_{8j+2}) = 2j + 1$, $\beta_{cpl}(P_{8j+3}) = \beta_{cpl}(P_{8j+4}) = \beta_{cpl}(P_{8j+5}) = \beta_{cpl}(P_{8j+6}) = 2j+ 2$, and $\beta_{cpl}(P_{8j+7}) = 2j + 3$.

Proof

Let n = 8j + r where $0 \le r \le 7$. Clearly $\beta_{cpl}(P_1) = 1$, and $\beta_{cpl}(P_2) = 1$. If $3 \le r \le 6$ we can choose the two vertices in any color class of order two or any two on adjacent singleton color classes to get $\beta(P_r; S)\ \beta \ge 2$ for any coupled proper coloring, and it is easy to find a particular S for which $\beta(P_r; S) = 2$, so $\beta_{cpl}(P_r) = 2$. For P_7 the proper coloring (1,2,3,4,1,3,2) (that is, $\{\{v_1, v_5\}, \{v_2, v_7\}, \{v_3, v_6\}, \{v_4\}\}$) shows that $\beta_{cpl}(P_7)\ \beta \ge 3$. Let S be any coupled proper coloring of P_7. If S has one singleton color class and three pairs, let the singleton be v_i, then at least one of the colored pairs has neither vertex adjacent to v_i, so $\beta(P_7; S) \ge 3$. Suppose S has at least three singleton color classes $\{v_i\}$, $\{v_j\}$, and $\{v_k\}$ with i < j < k.

If $i \le j-2$ and $k \ge j + 2$, then$\{v_i, v_j, v_k\}$ shows that $\beta(P_7; S) \ge 3$. If, for example,

i = j − 1 then there are at least three vertices whose colors are not those of v_{j-1}, v_j , and v_{j+1}, so one can use a color pair or two singletons from these three vertices along with v_j to see that $\beta(P_7; S) \ge 3$.

Hence, $\beta_{cpl}(P_7) = 3$. For n = 8j + r with $j \ge 1$, 8j consecutive vertices will be colored as L :

$(1, 2, 4, 3, 2, 3, 1, 4, 5, 6, 8, 7, 6, 7, 5, 8, \ldots, 4_{i-3}, 4_{i-2}, 4_i, 4_{i-1}, 4_{i-2}, 4_{i-1}, 4_{i-3}, 4_i, \ldots, 4_{j-3}, 4_{j-2}, 4_j, 4_{j-1}, 4_{j-2}, 4_{j-1}, 4_{j-3}, 4_j)$. For each group of four colors, at most one pair can be used in any independent set. Use L for P_{8j}, $(4_{j+1}, L)$ for P_{8j+1}, $(4_{j+1}, 4_{j+2}, L)$ for P_{8j+2}, $(4_{j+1}, 4_{j+2}, L, 4_{j+1})$ for P_{8j+3}, $(4_{j+2}, 4_{j+1}, L, 4_{j+1}, 4_{j+2})$ for P_{8j+4}, $(4_{j+1}, 4_{j+2}, 4_{j+3}, 4_{j+1}, 4_{j+2}, L)$ for P_{8j+5}, $(4_{j+3}, 4_{j+2}, 4_{j+1}, 4_{j+3}, 4_{j+2}, 4_{j+1}, L)$ for P_{8j+6}, and $(4_{j+1}, 4_{j+2}, 4_{j+3},$

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$4_{j+4}, 4_{j+1}, 4_{j+3}, 4_{j+2}$,L) for P_{8j+7} to see that $\beta_{cpl}(P_{8j}) \leq 2j$, $\beta_{cpl}(P_{8j+1}) \leq 2j + 1, \ldots, \beta_{cpl}(P_{8j+7}) \leq 2j + 3$.

Let S be any proper coupled coloring of P_n with $n \geq 8$. We can find a sufficiently large S -independent set as follows. Start with S = Φ and repeat the following until fewer than eight vertices remain. Choose a vertex v of degree at most one. Put v in S and if there is another vertex v' with {v, v'} ϵ S, then also put v' in S. Delete v, v', and any vertex in N(v) ϵ N(v') or of the same color as a vertex in N(v) ϵ N(v'). If {v, v'} ϵ S, then two vertices are added to S and at most eight deleted. If {v} ϵ S, then one vertex in placed into S and at most three are deleted. At the first point where fewer than eight vertices remain, at least one-fourth of the deleted vertices are in S, that is $|S| \geq 2j$. From what remains we can add the required number of vertices to S in the same manner as we did for $P_1, P_2, \ldots, P_7$. Hence, $\beta_{cpl}(P_{8j}) \geq 2j$, $\beta_{cpl}(P_{8j+1}) \geq 2j + 1, \ldots, \beta_{cpl}(P_{8j+7}) \geq 2j + 3$, and the proof is complete.

To conclude, it is noted that along with further study of colored-domination and colored independence,

We have a colored-problem associated with essentially every graph parameter. 196 Colored Problems in Graphs

3. References

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