Feedback Linearization for Input-saturation Nonlinear System Based on T-S Fuzzy Model

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Abstract

Considering input saturation problem of nonlinear system, a linearized model of multi-inputs nonlinear system is proposed in this paper. The final linear model has prescribed poles and has the same convergence nearby the designed equilibrium points. After this, the linear control theorem can be applied. During the calculation of linearization, T-S (Takagi Sugeno) fuzzy model and pole placement method were utilized. Pole placement just was applied only once for the final model comparing the traditional case where it was designed for every fuzzy rule. This means fewer LMIs (linear matrix inequality) will be needed and its solution will be guaranteed as much as possible. In this paper, nonlinear system will be transferred to T-S fuzzy model first. Note that the T-S fuzzy model is still nonlinear. Then, by employing a series of transfer matrix, nonlinear T-S fuzzy model will be transferred into a nearly linear form accompanied with only one nonlinear part. Finally, by designing a proper controller, linear pole placement method is used and the designed linearization controller gains can be calculated out with LMIs.

Keywords: T-S Fuzzy Control, Linearization, LMIs, Pole Placement, Saturation Nonlinear System

1. Introduction

Takagi-Sugeno (T-S) fuzzy theorem is proposed by Takagi and Sugeno in 1985¹. In the equilibrium points, measurable variables compose fuzzy rules and the corresponding membership functions. The function of system states and inputs is applied as consequent condition of if-then fuzzy rules which usully is linear models. T-S fuzzy local model is formed by the product of consequent model and its corresponding membership function. The overall T-S model is the summation of its local models. By the application of PDC (parallel distributed compensation), controllers of every local model can be combined with its corresponding membership function as the total nonlinear system controller. By this way, T-S fuzzy control is an effictive way for nonlinear system controller design. Meanwhile, many nonlinear dynamic systems can be represented by Takagi-Sugeno fuzzy models. In fact, it is proved that Takagi-Sugeno fuzzy models are universal approximators. This T-S fuzzy model is not only used

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for fuzzy controller but also used for modeling dynamic model of control object.

PDC is one of the useful control methods for T-S fuzzy model^{2,3}, especially, after LMI (Linear Matrix Inequality) can be calculated in computer. The history of the so-called parallel distributed compensation began with a model-based design procedure proposed by Kang and Sugeno e.g.⁴. The PDC offers a procedure to design a fuzzy controller from a given T-S fuzzy model⁵. The controller parameters can be directly calculated by LMI.

In this work, linear pole placement method has been utilized for the nonlinear sytem with input saturation. In the control of nonlinear systems using linear control theories, the feedback linearization technique is popular. The feedback linearization usually involves a state coordinate change and feedback. Feedback linearization via static feedback has been thoroughly studied ^{6.7}.

Feedback linearization of discrete-time nonlinear system is more difficult since the method is originally based on the partial differential operators like Lie brackets and Lie derivatives. More recently, the use of dynamic feedback has been investigated, in the hope of augmenting the class of linearizable systems^{8,9}. Since these studies are based on the profound mathematical background, they are not intuitive at all and difficult to know the exact boundary between linearizable and nonlinearizable systems. So that, finding verifiable necessary and sufficient conditions to characterize the class of such linearizable systems is still an open problem. In hardware system, there are many constraints because hardware limitations. Some systems have to change the control mode from one to another and so on. For these reasons, the input of control object and the output of controller may unequal. This can result in the response of closed-loop system worse, and it is called windup. If ignored this kind of nonlinearity problem during system design, the system overshoot cannot be suppressed well and the system may be unstable. There are many studies in this area¹⁰. As windup usually caused by integral, Buckley raised an anti-reset windup method. The error of controller output and object output is used as feedback to complement the system¹¹. Condition technique is shown in the work of Hanus and his partners first. This method re-computes reference inputs which keep controller output out of saturation area meanwhile tracking the new reference input. So the control object input will be same with controller output, and windup is eliminated¹². Still, there are anti-windup controller based on observer¹³,

internal model control¹⁴, saturation feedback control¹⁵, dynamic complement¹⁶ and so on. Variable structure control is proposed to prevent integrator windup by preset parameter¹⁷. Variable structure anti-windup controller has a good perfermance, however the preset paremeter is based on experimance. Stability and robust are difficult to discuss.

In this paper, based on T-S fuzzy theorem, inputsaturation nonlinear system is transformed to a linear system which can assign the poles in a desired disk. This linear system will keep the same convergence with the nonlinear system nearby the equillibrium points. After this procedure, linear control theorem can be applied for controller design. Proposed method has meanings in stability and robust analysis.

Next part will introduce the handling of nonlinear system by T-S fuzzy theorem. The third part is about the solution of input-saturation problem for nonlinear system and the final results will be shown. The fourth and fifth parts are simulation example and conclusion.

2. T-S Fuzzy Transfermation For Multi-Inputs Nonlinear System

Traditional linear pole placement cannot be applied to nonlinear system directly. In this paper, T-S fuzzy acted as mediator between linear pole placement and nonlinear system. The *i-th* IF-THEN rule of the T-S fuzzy model for a nonlinear system is

Rule *i*: If $w_1(t)$ is F_{i1} and ... and $w_g(t)$ is F_{ig} , then

$$\dot{x}(t) = A_i x(t) + B_i u(t)$$
⁽¹⁾

where, i = 1, 2, ..., L, $A_i \in \mathbb{R}^{n \times n}$, $B_i \in \mathbb{R}^{n \times m}$, *L* is the number of rules, and $w_1(t), w_2(t), ..., w_g(t)$ are premise variables.

The overall fuzzy system is

$$\dot{x}(t) = \frac{\sum_{i=1}^{L} \mu_i(w(t)) \{A_i x(t) + B_i u(t)\}}{\sum_{i=1}^{L} \mu_i(w(t))}$$
where, $w(t) = [w_1(t), w_2(t), ..., w_g(t)]$. Let
(2)

$$h_i(w(t)) = \frac{\mu_i(w(t))}{\sum_{i=1}^{L} \mu_i(w(t))}$$
, nonlinear system can be described

as

$$\dot{x}(t) = \sum_{i=1}^{L} h_i(w(t)) \{A_i x(t) + B_i u(t)\}$$
(3)

Membership function $h_i(w(t))$ should satisfy:

$$\sum_{i=1}^{L} h_i(w(t)) = 1.$$
(4)

The following assumption is required for the final model calculation in this paper:

2.1 Assumption 1

If all T-S fuzzy n-dimension linear models are controllable, the following condition should be satisfied:

$$rank \begin{bmatrix} B_i & A_i B_i & \dots & A_i^{n-1} B_i \end{bmatrix} = n$$
(5)

To deal with the coordinate change, the *i*-th rule of the fuzzy model for the nonlinear system is

Rule i: If $w_1(t)$ is F_{i1} and... and $w_g(t)$ is F_{ig} , then $\dot{x}(t) = A_i x(t) + B_i u(t)$, for i = 1, 2, ..., L and

$$z(t) = T_i x(t) \tag{6}$$

for i = 1, 2, ..., L

By substitute state x(t) of (6) into (1), the linear model of *i-th* rule can be

$$Z(t) = A_{ci}Z(t) + B_{ci}u(t)$$
⁽⁷⁾

where, $A_{ci} = T_i A_i T_i^{-1}$, $B_{ci} = T_i B_i$. The overall fuzzy coordinate changed state is

$$Z(t) = \sum_{i=1}^{L} h_i(w(t))T_i x(t)$$
(8)

which is considered as the summation of the states x(t) through transformation T_i with the proportion of membership h_i . The overall fuzzy system is

$$\dot{Z}(t) = \sum_{i=1}^{L} h_i(w(t)) \{ A_{ci} Z(t) + B_{ci} u(t) \}$$
(9)

h

Now problem is to derive T_i and u(t) which make the T-S fuzzy model (9) linear. Under the assumption 1, T_i is obtained by following steps: Structure a matrix M as

 $M - \Gamma h$ 16 $\Lambda^2 h$ 1^{µ1}h

$$M_{i} = \begin{bmatrix} b_{i1} & A_{i}b_{i1} & A_{i}^{2}b_{i1} & \cdots & A_{i}^{\mu 1}b_{i1} & b_{i2} & A_{i}b_{i2} \\ A_{i}^{2}b_{i1} & \cdots & A_{i}^{\mu 2}b_{i2} & \cdots \end{bmatrix}$$
(10)

where, b_{ij} is the elements of $B_i = [b_{i1} \quad b_{i2} \quad \dots \quad b_{im}]$ in (1). For searching M_i , start from b_{i1} and end at rank(M)=n. If $A^{\mu i+1}b_{ik}$ makes vectors before this vector and this vector relative, then delete this vector and start with $b_{i(k+1)}$. The inverse of M is

$$M_{i}^{-1} = \begin{bmatrix} e_{i11}^{T} & e_{i12}^{T} & \cdots & e_{i1\mu 1}^{T} \\ e_{i21}^{T} & e_{i22}^{T} & \cdots & e_{i2\mu 2}^{T} & \cdots \end{bmatrix}^{T}.$$
 (11)

For the multiple-input case, T_i is obtained as follows:

$$T_{i} = [(e_{i1\mu1})^{T} \quad (e_{i1\mu1}A_{i})^{T} \quad \cdots \quad (e_{i1\mu1}A_{i}^{\mu1-1})^{T} (e_{i2\mu2})^{T} \quad (e_{i2\mu2}A_{i})^{T} \quad \cdots \quad (e_{i2\mu2}A_{i}^{\mu2-1})^{T} \quad \cdots]$$
(12)

Then the above state transformation T_i changed the *i-th* linear system of fuzzy system into the controllable canonical form

$$Z(t) = A_{ci}Z(t) + B_{ci}u(t)$$
⁽¹³⁾

where,

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$$B_{ci} = \begin{bmatrix} 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \\ b_{cin_1} & b_{cin_2} & \dots & b_{cin_n} \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \\ b_{cin1} & b_{cin2} & \dots & b_{cinm} \end{bmatrix}, a_{in_ln_q} \text{ presents the}$$

 n_q -th element of the n_l -th row in the *i*-th rule and \times presents the other values. This mark is same to the elements of B_{ci} .

The overall fuzzy system (9) can be reform as

$$\begin{split} \dot{Z}(t) &= \sum_{i=1}^{L} h_i(w(t)) \Big\{ A_{ci} Z(t) + B_{ci} u(t) \Big\} \\ &= \begin{bmatrix} z_2(t) \\ z_3(t) \\ \vdots \\ z_{(n_i-1)}(t) \\ \sum_{i=1}^{L} \sum_{j=1}^{n} h_i a_{in_j} z_j(t) \\ \vdots \\ z_{(n_k+2)}(t) \\ \vdots \\ z_{n}(t) \\ \vdots \\ z_n(t) \\ \sum_{i=1}^{L} \sum_{j=1}^{n} h_i a_{inj} z_j(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \sum_{i=1}^{L} \sum_{k=1}^{m} h_i b_{cin_k} u_k(t) \\ \vdots \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \\ \sum_{i=1}^{L} \sum_{k=1}^{m} h_i b_{cin_k} u_k(t) \end{bmatrix}$$
(14)

where, $u_k(t)$ is the *k*-th input and remind that $\sum_{i=1}^{L} h_i(w(t)) = 1$ from (4). The result can be represented in the following theorem:

2.1.1 Theorem 1

Under the assumption 1, if

$$\sum_{i=1}^{L} \sum_{k=1}^{m} h_{i} b_{cin_{i}k} u_{k}(t) = -\sum_{i=1}^{L} \sum_{j=1}^{n} h_{i} a_{in_{i}j} z_{j}(t)$$
(15)

for each n_l has the solution for $u_k(t)$, then (9) can be the form of

$$\dot{Z}(t) = A_{cn}Z(t) + B_{cn}v(t)$$
⁽¹⁶⁾

where,

$$A_{cn} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$
$$B_{cn} = \begin{bmatrix} 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \\ \times & \times & \dots & \times \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \\ \times & \times & \dots & \times \end{bmatrix}, Z(t) \text{ is an } n\text{-dimension vector,}$$

replace 1 or some values and anti-windup controller is included in v(t) which is a *m*-dimension vector.

×

3. The Linear Model of Input-Saturation Nonlinear System

In many equipments, there is inputs saturation problem. Theorem 1 can't work well under input saturation case because it is a feedback controller design method. Considering of this, an anti-windup controller is needed. Consider the saturation input

$$v(t) = sat(u(t)) = \begin{cases} u_{\rm lim}, & u(t) \ge u_{\rm lim} \\ u(t) & -u_{\rm lim} < u(t) < u_{\rm lim} \\ -u_{\rm lim}, & u(t) \le -u_{\rm lim} \end{cases}$$
(17)

u(t) is input before saturation segment and $u_{\text{lim}} > 0$ is input limitation. Degree of feedback saturation is defined as:

$$S(t) = \begin{cases} \frac{u_{\rm lim}}{KZ(t)}, & KZ(t) > u_{\rm lim} \\ 1, & -u_{\rm lim} < KZ(t) < u_{\rm lim} \\ -\frac{u_{\rm lim}}{KZ(t)}, & KZ(t) < -u_{\rm lim} \end{cases}$$
(18)

Define

$$d_0 = \min\{S(t) \ \forall Z \in D\}$$
⁽¹⁹⁾

where, *D* is a compact space and let a matrix S_0 is consisted by element d_0 or 1. Set

$$v(t) = KZ(t), \tag{20}$$

3.1 Corollary 1

System (16) is *D*-stable if and only if there exists a symmetric matrix *X*>0 such that

$$\begin{bmatrix} -iX & \not X + (A_a + B_a S_0 K)X \\ \not X + X(A_a + B_a S_0 K)^T & -K \end{bmatrix} < 0.$$
(21)

where, q is the pole placement disk center and r is radius.

The proof is clearly based on the work of²⁰. Equation (21) can be solved by LMI, then v(t) in (20) can be designed with anti-windup ability. The final result can be gived as:

3.1.1 *Theorem 2*

Under the assumption 1, nonlinear system can be linearized by the controller of (15). If there is input saturation of (17), (21) can be used to design the v(t) of (16). The final controller is

$$u(t) = \sum_{k=1}^{m} u_k(t) + v(t)$$
(22)

which can be applied to nonlinear system to guarantee it stable.

4. Numerical Example

Based on the mathematic model of an IPMSM in the d-q synchronously rotating reference frame mentioned in²¹. Parameters are shown in Table 1.

| Table 1. IPM | SM parameters |
|--------------|---------------|
|--------------|---------------|

| Pole pair number P | 2 |
|------------------------------|---------------------|
| d-axis inductance L_d | 42.44 mH |
| q-axis inductance L_q | 79.57 mH |
| Stator resistance R | 1.93 Ω |
| Motor inertia J_m | $0.003 kgm^2$ |
| Friction coefficient B_m | 0.001 Nm/rad/sec |
| Magnetic flux constant $ø_f$ | 0.311 volts/rad/sec |

Set $x_1(t) = i_q(t)$, $x_2(t) = i_d(t)$, $x_3(t) = w(t)$, $u_1(t) = u_q(t)$, $u_2(t) = u_d(t)$, $e_1(t) = x_1(t) - x_{1r}(t)$, $e_2(t) = x_2(t) - x_{2r}(t)$, $e_3(t) = x_3(t) - x_{3r}(t)$, $u_{1e}(t) = u_1(t) - u_{10}(t)$, and $u_{2e}(t) = u_2(t) - u_{20}(t)$. Here reference values are $x_{1r} = 0.0209$, $x_{2r} = 1.5817$ and $x_{3r} = 15.8144$. The error dynamic system is derived out as

$$\begin{split} \dot{e}_{1}(t) &= -\frac{R}{L_{q}}e_{1}(t) - \frac{PL_{d}}{L_{q}}x_{3r}e_{2}(t) - \frac{P\psi_{f} + PL_{d}x_{2}(t)}{L_{q}}e_{3}(t) \\ &+ \frac{1}{L_{q}}u_{10}(t) \\ \dot{e}_{2}(t) &= \frac{PL_{q}}{L_{d}}x_{3r}e_{1}(t) - \frac{R}{L_{d}}e_{2}(t) + \frac{PL_{q}x_{1}(t)}{L_{d}}e_{1}(t) \\ &+ \frac{1}{L_{d}}u_{20}(t) \\ \dot{e}_{3}(t) &= \frac{3P(\psi_{f} + (L_{d} - L_{q})x_{2r}(t))}{2J_{m}}e_{1}(t) \\ &+ \frac{3P(L_{d} - L_{q})x_{1}(t)}{2J_{m}}e_{2}(t) - \frac{B_{m}}{J_{m}}e_{3}(t) \end{split}$$

where, $x_1(t)$ and $x_2(t)$ are selected as premise variables.

The input saturation degree is 0.8. There are 4 fuzzy rules for this system. The coordinate change matrixes T_i can be get from (12), and linearization controller can be calculated from (15). Set the poles in a disk with origin (-2000,0) and radius r=1000. The anti-windup controller gains of (20) is K=[-362.1858,-40.4408,0;0,0,-22.0059]. Nominal system eigenvalues are -13.3883, -27.0525 and -22.0059. A set of random change values from -0.5 to 0.5 is used as disturbance which is added in state $x_1(t)$ before the integral. Two saturation inputs are both of form (17) where $u_{\text{lim}} = 15$ for the error system. The final controller form is (22). The simulation results of $e_1(t)$ compared with traditional Lyapunov PDC T-S fuzzy control result

is in Figure 1. The other satae-results are absent here for economizing space. System output errors are almost zero. For the existance of input saturation limitation, settling time is about 0.2s.



Figure 1. The performance of $e_1(t)$.

5. Conclusions

Based on T-S fuzzy and pole placement, an anti-windup controller is proposed for saturation nonlinear multiinputs system in this paper and the controller parameters can be calculated by LMIs. The proposed controller was applied for a PMSM model as a test. The comparing results showed that the proposed controller of this paper gave a good control performance.

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